$$* \; F(x) = u^n$$

$$F'(x) = n\,u^{n-1}\,du$$

For example

$u = (x+1)$

$u = (x^2+2x+1)$

مشتقة -- الدالة الكسرية

$$F'(x) = \frac{dy}{dx} = \frac{\text{مشتقة المقام} * \text{البسط} \ominus \text{مشتقة البسط} * \text{المقام}}{(\text{المقام})^2}$$

Example :

$$y = \frac{x^2}{(x+1)}$$

$$\frac{dy}{dx} = \frac{(x+1) * 2x - x^2 * 1}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 + 2x - x^2}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2 + 2x)}{(x+1)^2}$$

Ex Find $\frac{dy}{dx}$ for the following functions:

a) $y = (x^2+1)^5$ (b) $y = [(5-x)(4-2x)]^2$

(c) $y = (2x^3 - 3x^2 + 6x)^{-5}$ (d) $y = \frac{12}{x} - \frac{4}{x^3} + \frac{3}{x^4}$

(E) $y = \frac{(x^2+x)(x^2-x+1)}{x^3}$ (f) $y = \frac{x^2-1}{x^2+x-2}$

Sol:

(a) $\frac{dy}{dx} = 5(x^2+1)^{5-1} * 2x = 5(x^2+1)^4 * 2x = 10x(x^2+1$

(b) $\frac{dy}{dx} = 2[(5-x)(4-2x)] * [(5-x) * -2 + (4-2x) * -1]$

$= 2[(5-x)(4-2x)][-2(5-x) - (4-2x)]$

$= 2[20 - 10x - 4x + 2x^2][(-10 + 2x) - 4 + 2x]$

$= 2[2x^2 - 14x + 20][4x - 14]$

$= 2[8x^3 - 56x^2 + 80x - 28x^2 + 196x - 280]$

$= 2[8x^3 - 84x^2 + 276x - 280]$

(c) $$\frac{dy}{dx} = -5(2x^3-3x^2+6x)^{-6} * (6x^2-6x+6)$$

$$= -5(2x^3-3x^2+6x)^{-6} * 6(x^2-x+1)$$

$$= -30(2x^3-3x^2+6x)^{-6}(x^2-x+1)$$

(d) $$\frac{dy}{dx} = -12x^{-2} + 12x^{-4} - 12x^{-5}$$

$$\frac{dy}{dx} = \frac{-12}{x^2} + \frac{12}{x^4} - \frac{12}{x^5}$$

(E) $$y = \frac{\cancel{x}(x+1)(x^2-x+1)}{x^{\cancel{3}}} = \frac{(x+1)(x^2-x+1)}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2[(x+1)(2x-1)+(x^2-x+1)*1] - (x+1)(x^2-x+1)*2x}{x^4}$$

$$\frac{dy}{dx} = \frac{x^2[(2x^2-\cancel{x}+\cancel{2x}-\cancel{1})+x^2-\cancel{x}+\cancel{1}] - 2x(x^3-\cancel{x^2}+\cancel{x}+\cancel{x^2}-\cancel{x}+1)}{x^4}$$

$$\frac{dy}{dx} = \frac{x^2[3x^2] - 2x(x^3+1)}{x^4}$$

$$\frac{dy}{dx} = \frac{3x^4-2x^4-2x}{x^4}$$

$$\boxed{\frac{dy}{dx} = \frac{x^4-2x}{x^4}}$$

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(f) $$\frac{dy}{dx} = \frac{(x^2+x-2)*2x-(x^2-1)*(2x+1)}{(x^2+x-2)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{2x^3}+2x^2-4x-\cancel{2x^3}-x^2+2x+1}{(x^2+x-2)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2-2x+1}{(x^2+x-2)^2}}$$

The Chain Rule :

① $y=g(t)$, $x=f(t)$

$$\boxed{\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}} \quad \text{— ①}$$

② $y=g(t)$, $t=f(x)$

$$\boxed{\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx}} \quad \text{— ②}$$

EX

(a) $y=\frac{t^2}{t^2+1}$ and $t=\sqrt{2x+1}$ ②

(b) $y=\frac{1}{t^2+1}$ and $x=\sqrt{4t+1}$

(c) $y=\left(\frac{t-1}{t+1}\right)^2$ and $x=\frac{1}{t^2}-1$ at $t=2$

(d) $y = 1 - \frac{1}{t}$ and $t = \frac{1}{1-x}$ at $x = 2$

<u>Sol</u>

(a) $$\frac{dy}{dt} = \frac{(t^2+1) * 2t - t^2 * 2t}{(t^2+1)^2}$$

$$\frac{dy}{dt} = \frac{\cancel{2t^3} + 2t - \cancel{2t^3}}{(t^2+1)^2} = \frac{2t}{(t^2+1)^2}$$

$$\frac{dt}{dx} = \frac{1}{\cancel{2}}(2x+1)^{-\frac{1}{2}} * \cancel{2} = \frac{1}{\sqrt{2x+1}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx} = \frac{2t}{(t^2+1)^2} * \frac{1}{\sqrt{2x+1}}$$

$$= \frac{2\sqrt{2x+1}}{\left((\sqrt{2x+1})^2+1\right)^2} * \frac{1}{\sqrt{2x+1}} = \frac{\cancel{2\sqrt{2x+1}}}{((2x+1)+1)^2} \cdot \frac{1}{\cancel{\sqrt{2x+1}}}$$

$(2x+2)^2$

$(2(x+1))^2$

$= 4(x+1)^2$

$$\frac{dy}{dx} = \frac{\cancel{2}}{\cancel{4}(x+1)^2} = \frac{1}{2(x+1)^2}$$

(b) $y = \dfrac{1}{t^2+1}$ $\qquad x = \sqrt{4t+1}$

$$\frac{dy}{dt} = \frac{(t^2+1)*0 - 1*2t}{(t^2+1)^2} = \frac{-2t}{(t^2+1)^2}$$

$$\frac{dx}{dt} = \frac{1}{\cancel{2}}(4t+1)^{-\frac{1}{2}} * \cancel{4}^{\,2} = \frac{2}{\sqrt{4t+1}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-2t}{(t^2+1)^2} \div \frac{2}{\sqrt{4t+1}}$$

$$= \frac{-\cancel{2}t}{(t^2+1)^2} * \frac{\sqrt{4t+1}}{\cancel{2}} = \frac{-t\sqrt{4t+1}}{(t^2+1)^2}$$

where $x = \sqrt{4t+1} \Longrightarrow t = \dfrac{x^2-1}{4}$

where $y = \dfrac{1}{t^2+1} \Longrightarrow t^2+1 = \dfrac{1}{y} \Longrightarrow t^2 = \dfrac{1}{y} - 1$

$t = \sqrt{\dfrac{1}{y} - 1}$

(c) $y = \left(\dfrac{t-1}{t+1}\right)^2 \Longrightarrow \dfrac{dy}{dt} = 2\left(\dfrac{(t-1)}{(t+1)}\right) * \dfrac{(\cancel{t}+1)*1 - (\cancel{t}-1)*1}{(t+1)^2}$

$$\frac{dy}{dt} = 2\left(\frac{(t-1)}{(t+1)}\right) * \frac{2}{(t+1)^2} = \boxed{\frac{4(t-1)}{(t+1)^3}}$$

$x = \dfrac{1}{t^2} - 1 \Rightarrow \dfrac{dx}{dt} = \dfrac{t^2*0 - 2t*1}{t^4} - 0$

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$$\frac{dx}{dt} = \frac{-2\cancel{t}}{t^{\cancel{4}}} = \boxed{\frac{-2}{t^3}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{4(t-1)}{(t+1)^3} \div \frac{-2}{t^2}$$

$$= \frac{\cancel{4}^{2}(t-1)}{(t+1)^3} * \frac{-t^3}{\cancel{2}} = \frac{-2t^3(t-1)}{(t+1)^3}$$

at $t = 2$

$$\frac{dy}{dx} = \frac{-2 * 2^3 (2-1)}{(2+1)^3} = \boxed{\frac{-2 * 8}{27}} = \boxed{\frac{-16}{27}}$$

(d) $y = 1 - \frac{1}{t}$ $\quad t = \frac{1}{1-x}$ $\quad$ at $x = 2$

$t = \frac{1}{1-2} = -1$

$$\frac{dy}{dt} = 0 - \frac{t * 0 - 1 * 1}{t^2} = \frac{+1}{t^2}$$

$$\frac{dt}{dx} = \frac{(1-x) * 0 - 1 * -1}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx} = \frac{-1}{t^2} * \frac{1}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{+1}{(-1)^2} * \frac{1}{(1-x)^2} = \frac{+1}{(1-x)^2} = \frac{+1}{(1-2)^2} = \boxed{1}$$