2-14 Thermal stresses

Consider an elastic bar as shown in Fig. 2-9 under the effect of temperature, increasing or decreasing the temperature of a material produces elongation or contraction. By the principles of superposition the resulting axial strain ε is

$$\varepsilon = \varepsilon t + \varepsilon s$$

Where

$$\varepsilon_t = \alpha . t$$

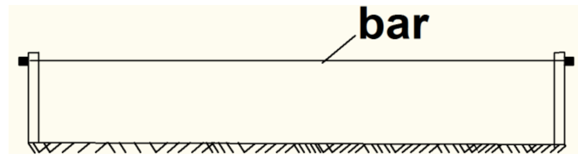

Fig. 2.9

is the thermal strain (strain due to changes of temperature) where α is the coefficient of thermal expansion (material property)

and

$$\varepsilon_s = \frac{\sigma}{E}$$

is the strain due to induced stress σ

If the rise of temperature provided the body concerned is free to expand in this case stains have no stresses associated with them. In the case where both applied stresses and temperature change , thermal strains are calculated by the following equations :

$$\varepsilon_x = \frac{1}{E}[\sigma x - \mu(\sigma y + \sigma z)] \pm \alpha \Delta t$$

$$\varepsilon_y = \frac{1}{E}[\sigma y - \mu(\sigma x + \sigma z)] \pm \alpha \Delta t$$

$$\varepsilon_z = \frac{1}{E}[\sigma z - \mu(\sigma x + \sigma y)] \pm \alpha \Delta t$$

while the normal strains in a body are affected by changes in temperature shear strains are not.

Exercises

2-6. Two parallel walls 7m apart(Fig. 2-11), are held by steel bar of 25mm diameter, the bar passes through a metal plate and nut at each end, the nuts are screwed up to the plates which the bar is at 150^{o}c. Find the pull exerted by the bar after cooling to 16$\overset{o}{c}$. If **a-** the ends do not yield . **b-** the total yield at the two ends is 6.25mm . Given Es = 220 GPa and α = 11x10^{-6}/$\overset{o}{c}$

solution

a) No yield

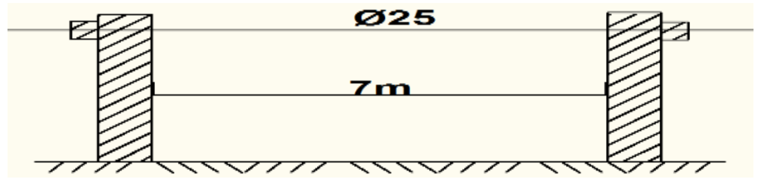

Fig.2-11

Total axial strain in:

$\varepsilon = \varepsilon_T + \varepsilon_S$

But:

$\varepsilon = 0 \quad (no\ change\ of\ length)$

$\varepsilon_T = \alpha\Delta T = 11 \times 10^{-6}(16 - 150)$

$= -1474 \times 10^{-6}\ \ (contraction\ to\ produce\ tension)$

Substituting the strain due to constraints, in

$\varepsilon_S = \varepsilon - \varepsilon_T$

$= 0 - (-1474 \times 10^{-6})$

$= 1474 \times 10^{-6}\ (tensile)$

But:

$$\varepsilon_S = \frac{\sigma}{E}$$

Then:

$\sigma = E \cdot \varepsilon_S = 220 \times 10^9 \times 1474 \times 10^{-6}$

$= 324.28\ MPa$

The pull in

$P = \sigma A$

$$= 324.28 \times \frac{\pi d^2}{4}$$

$$= 324.28 \times \frac{\pi \times (0.025)^2}{4} = 159180.57\ N = 159.2\ kN$$

a) When yielding in ΔL=6.25 mm

The total axial strain in:

$\varepsilon = \varepsilon_T + \varepsilon_S$, but:

$$\varepsilon = \frac{6.25}{7000} = -893 \times 10^{-6} \quad (contraction)$$

Thermal strain in

$$\varepsilon_T = \alpha \Delta T = 11 \times 10^{-6}(16 - 150)$$

$$= -1474 \times 10^{-6} \ (contraction)$$

Therefore the induced strain in:

$$\varepsilon_S = \varepsilon - \varepsilon_T$$

$$= -893 \times 10^{-6} - (-1474 \times 10^{-6})$$

$$= 581 \times 10^{-6} \ (tensile)$$

The axial stress in:

$$\sigma = E \cdot \varepsilon_S = 220 \times 10^9 \times 581 \times 10^{-6}$$

$$= 127.82\ MPa$$

$$P = \sigma A$$

$$= 127.82 \times \frac{\pi \times (0.025)^2}{4} = 62743.5\ N = 62.7434\ kN$$

2-7. A solid brass of length L=100mm of diameter D= 15mm is used to fix two rigid surfaces. Find the stresses induced into it if the temperature of the bar is raised by 20°c. Given α= 19x10^{-6}/°c and E=103GPa/m².

Solution:

$\sigma = -E\alpha\Delta T \ (or - E\alpha T)$

$\sigma x = -103 \times 10^{9} \times 19 \times 10^{-6} \times 20$

$= -39.1 \ MPa$

It is interesting to note that this result does not depend on either the length or diameter of the cylinder