Mathimatics 1

Lecture 2

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The Derivative

Definition: The derivative of the function $y = f(x)$ with respect to the variable x is the function y' **or** (f') whose value at x is:

$$y' = \frac{dy}{dx} = \frac{df(x)}{dx} = f'(x)$$

$$\frac{dy}{dx} = \lim_{h \to o} \frac{f(x+h) - f(x)}{h}$$

⇨ Definition ofDerivative

$h = \Delta x$

Example: Use definition to find $\frac{dy}{dx}$ if $y = f(x) = x^3$.

Solution:

$$\frac{dy}{dx} = \lim_{h \to o} \frac{f(x+h) - f(x)}{h} \quad \frac{dy}{dx} = \lim_{h \to o} \frac{f(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to o} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \quad = \lim_{h \to o} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$= \lim_{h \to o} (3x^2 + 3xh + h^2) = 3x^2$$

(Derivative Rules)

General formulas	Trigonometric functions
(1) $\frac{d}{dx}(c) = 0$ (The derivative of a constant function is zero.) (2) $\frac{d}{dx}(x) = 1$ (The derivative of the identity function is 1.) (3) $\frac{d}{dx}(cu) = c\frac{du}{dx}$ (Constant Multiple) (4) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$ (Sum Rule) (5) $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$ (Difference Rule) (6) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ (Product Rule) (7) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ provided that $v \neq 0$ (Quotient Rule) (8) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$ provided that $x \neq 0$ (9) $\frac{d}{dx}(x^m) = mx^{m-1}$ (Power Rule)	(1) $\frac{d}{dx}(\sin x) = \cos x$ (2) $\frac{d}{dx}(\cos x) = -\sin x$ (3) $\frac{d}{dx}(\tan x) = \sec^2 x$ (4) $\frac{d}{dx}(\cot x) = -\csc^2 x$ (5) $\frac{d}{dx}(\sec x) = \sec x \tan x$ 6) $\frac{d}{dx}(\csc x) = -\csc x \cot x$ **Chain rule** Let $u = g(x)$ and $y = f(u)$. Then, $y = f(u) = f(g(x))$ $y' = f'(g(x)) * g'(x)$ *or* $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dg}$ Where $\frac{dy}{dx}$ is evaluated at $u = g(x)$

<u>Example</u> : derivative the functions:

1) $y = 8 \quad \frac{dy}{dx} = \frac{d}{dx}(8) = 0$

2) $y=x^3 \quad \frac{dy}{dx} = \frac{d}{dx}(x^3) = 3x^{3-1}=3x^2$

3) $y=3x^3 \quad \frac{dy}{dx} = \frac{d}{dx}(3x^3) = 3*3x^{3-1}=9x^2$

4) $y=x^3 +\frac{4}{3} x^2 - 5x +1 \quad \frac{dy}{dx}=\frac{d}{dx}(x^3) + \frac{d}{dx}(\frac{4}{3} x^2) - \frac{d}{dx}(5x)+ \frac{d}{dx}(1)$

$=3x^2 + \frac{4}{3} * 2x – 5 + 0 = 3x^2 +\frac{8}{3} x -5$

5) $y = (x^2+1)(x^3+3)$

From product rule with $u=x^2+1$ and $v= x^3+3$, we find

$\frac{dy}{dx}=\frac{d}{dx}[(x^2+1)(x^3+3)] = (x^2+1) (3x^2) +(x^3+3)(2x)$

$=3x^4 + 3x^2 +2x^4+6x \quad = 5x^4 + 3x^2 +6x.$

6) $y= \frac{x^2-1}{x^2+1}$ We apply the quotient rule with $u=x^2 - 1$ and $v = x^2 +1$:

$$\frac{dy}{dx} = \frac{(x^2+1)*2x - (x^2-1)*2x}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+ 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

7) $y = \frac{4}{x^3} \quad \frac{dy}{dx} = \frac{d}{dx}(\frac{4}{x^3}) \quad = 4 \frac{d}{dx}(x^{-3})= 4(-3) x^{-4} =\frac{-12}{x^4}$

8) $y = (3x^2 +1)^2$(chain rule)

Let $y = f(u) = f(g(x))$ $y'= f'(g(x)) * g'(x)$

$=2(3x^2 +1)^{2-1}.(3*2 x^{2-1}+0) =2(3x^2 +1) *6x=36x^3+12x$

Chain Rule:

Example: if $y = u^3 - 1$ and $u = 2x$, find $\frac{dy}{dx}$. $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = 3u^2 * 2 = 6(2x)^2 = 24x^2$	Example : if $y = 5u^3$ and $x = u^2$, find $\frac{dy}{dx}$. (U= $\sqrt{x}$) $\frac{dy}{dx} = \frac{dy}{du} / \frac{dx}{du} = \frac{15u^2}{2u} = \frac{15u}{2} = \frac{15\sqrt{x}}{2}$

Example: derivative the functions:

1) $y = x^2 - \sin x$ $\quad y' = 2x - \cos x$
2) $y = x^2 \sin x$ $\quad y' = x^2 * \cos x + \sin x * 2x = x^2 \cos x + 2x \sin x$
3) $y = \sin 2x$ $\quad y' = \cos 2x * 2 = 2 \cos 2x$
4) $y = \sin (x^2 - x)$ $\quad y' = \cos(x^2 - x) * (2x-1) = (2x-1) \cos (x^2 - x)$
5) $y = \sin^5 3x$ $\quad y' = 5 \sin^{5-1} 3x * \cos 3x * 3 = 15 \cos 3x \sin^4 3x$
6) $y = \frac{4}{\cos x} + \frac{1}{\tan x}$ $\quad y = 4 \sec x + \cot x$ $\quad y' = 4 \sec x \tan x - \csc^2 x$
7) $y = (\sin x + \cos x) \sec x$

$y' = (\sin x + \cos x) \frac{d}{dx}(\sec x) + \sec x \frac{d}{dx}(\sin x + \cos x)$

$= (\sin x + \cos x) \sec x \tan x + \sec x (\cos x - \sin x)$

$= \frac{(\sin x + \cos x) \sin x}{\cos^2 x} + \frac{(\cos x - \sin x)}{\cos x}$

$\frac{\sin^2 x + \cos x \sin x + \cos^2 x - \cos x \sin x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

Y =(sin x +cos x) $\frac{1}{\cos x}$
Y = tan x + 1
Y' = $\sec^2$ x

Implicit differentiation:

Example :

1) $y^2 - x = 0$ $\quad y^2 = x$ $\quad$ [y'] $\quad 2y\,y' = 1$ $\quad y' = \frac{1}{2y}$ $\quad y' = \frac{1}{2\sqrt{x}}$
2) $x^2 + y^2 - 25 = 0$ $\quad$ [y'] $\quad yy' = 0$ $\quad 2y\,y' = -2x$

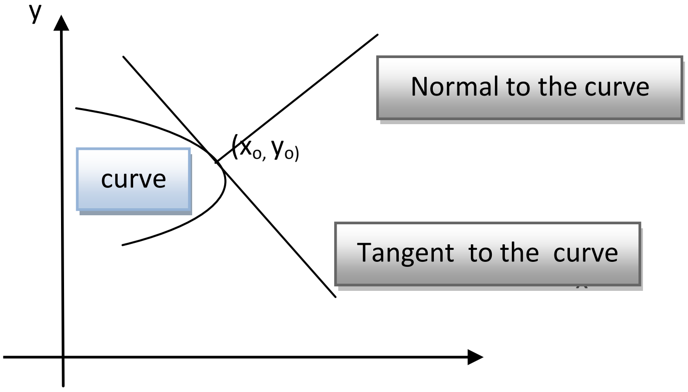
$y' = \frac{-x}{y} = \frac{-x}{\sqrt{25 - x^2}}$

3) $x^3 + y^3 - 9yx = 0$ $\quad$ [y'] $\quad 3x^2 + 3y^2 y' - (9y * 1 + x * 9y') = 0$

$3y^2 y' - 9xy' = -3x^2 + 9y$ $\Rightarrow$ $(3y^2 - 9x) y' = 9y - 3x^2$

$y' = \frac{9y - 3x^2}{3y^2 - 9x}$ $\Rightarrow$ $= \frac{3y - x^2}{y^2 - 3x}$

Tangent and Normal equation to the curve:



Tangent equation to the curve	**Normal equation to the curve**
The equation of tangent to the curve y=f(x) at the point (x_o , y_o) is : **($y-y_o$) = m ($x-x_o$)** Where m is the slope of the curve the point (x_o , y_o) $m = y'$	The equation of normal to the curve y=f(x) at the point (x_o , y_o) is : **($y-y_o$) = m_1 ($x-x_o$)** Where m_1 is the slope of the curve the point (x_o , y_o) $m_1 = \frac{-1}{m}$

Example :Find an equations for the tangent and normal to the curve

$y = x + \frac{2}{x}$ at the point (1,3) .

Solution: the slope of the curve is m = $y' = 1 - (2/x^2)$

the slope at the point (1,3) $y'(1) = 1 - (2/x^2) = -1$

-tangent equation (y-3) = (-1) (x-1) y+x-4=0	-normal equation (y-3) = (−1/−1) (x-1) y-x-2=0

Higher Derivatives:

Higher Derivatives:	**Distance , Velocity and Acceleration :**
If y =f(x) ,then	Time : -t
First derivative: y', $f'(x)$, $\frac{dy}{dx}$	Distance - s(t)
Second derivative: y'', $f''(x)$, $\frac{d^2y}{dx^2}$	Velocity - v(t) , $\frac{ds}{dt}$
Third derivative: y''', $f'''(x)$, $\frac{d^3y}{dx^3}$	Acceleration - a(t) , $\frac{dv}{dt}$ or $\frac{d^2s}{dt^2}$
*n*th derivative: y^n, $f^n(x)$, $\frac{d^ny}{dx^n}$	

Example :Finding higher derivatives. $y = x^3-3x^2 +2$

Solution:
$y' = 3x^2-6x$ ⇨ $y'' = 6x-6$

$y''' = 6$ ⇨ $y'''' = 0$

Example: A body moves along a straight line according to the law $s=\frac{1}{2}t^3-2t$. Determine its velocity and acceleration atthe end of 2 seconds.

Solution
$v = \frac{ds}{dt}= \frac{3}{2}t^2-2 = \frac{3}{2}2^2-2= 4\ {}^{m}/_{s}$, $a = \frac{dv}{dt}= 3t = 3*2 = 6\ {}^{m}/_{s}$

H .W :
Q 1 . Find f '(x) by using definition.

1 .$f(x) = \frac{x}{x-1}$ 2 . $f(x) = \sqrt{x}$

Q 2 . Find the first derivatives.

$y = 6x^2 - 10x - 5x^{-2}$	$w = (2x-7)^{-1}(x+5)$	$y = (3x^2)(x^3 - x - 1)$
$f(s) = \frac{\sqrt{s}-1}{\sqrt{s}+1}$	$r = \frac{1}{3s^2} - \frac{5}{2s}$	$u = \frac{5x+1}{2\sqrt{x}}$
$w = (\frac{1+3z}{3z})(3-z)$	$s = \frac{t^2+5t-1}{t^2}$	$w = (z+1)(z-1)(z^2+1)$
$y = (5x^3 - x^4)^7$	$y = \sqrt{3x^2 - 4x + 6}$	$y(x) = x^2(x^3 - t)^5$
$y = (1 - \frac{x}{7})^{-7}$	$y = \frac{(x+1)(x+2)}{(x-1)(x-2)}$	$y = 9\tan(\frac{x}{3})$
$y = x^2 \cos x$	$y = \sin x \tan x$	$y = \sin^5(2x)$
$y=(\sec x+\tan x)(\sec x-\tan x)$	$g(t) = \tan(5-\sin 2t)$	
$g(x) = (2-x)\tan^2 x$	$y = \sec(\tan 3x)$	$r = \sin(\theta^2)\cos(2\theta)$
$q = \cot(\frac{\sin t}{t})$	$y = x^2\sec(\frac{1}{x})$	$y = \cot(\pi - \frac{1}{x})$
$y = \frac{\tan 3x}{(x+7)^2}$	$r(\theta) = \sec\sqrt{x}\ \tan(\frac{1}{\theta})$	$q = \sin(\frac{1}{\sqrt{t-1}}$

Q 3. Find $\frac{dy}{dx}$.

1 . $y = 6u - 9$, $u = x^4/2$	2 . $y = \sin u$, $u = 3x + 1$
3 . $y = \sin u$, $u = x - \cos x$	4 . $y = u^3 - u$, $x = u^4 - u^2 + 1$

Q 4 . Find $\frac{dy}{dx}$.

1 . $y^3 + x^3 = 18xy$	2 . $y^2 = x^2 + \sin(xy)$	3. $x\cos(2x+3y) = y\sin x$
4. $x^3 = \frac{2x-y}{x+3y}$	5. $y^2 = \frac{x-1}{x+1}$	6. $y + \sin(\frac{1}{y}) = 1 - xy$

Q5 .Find y'' .

1 . $2x^3 - 3y^2 = 8$	2 . $2\sqrt{y} = x - y$
3 . $x^3 + y^3 = 16$, at the point (2 , 2)	

Q 6 .

show that the point (2,4) lies on the curve $x^3 + y^3 - 9yx = 0$. then find the tangent and normal to the curve there.