Lecture Seven
Potential due to a dipole, Electric potential energy

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## Outline

1. Potential Due to a Charged Particle
2. Potential Due to an Electric Dipole
3. Electric Potential Energy
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## 1. Potential Due to a Charged Particle

We now use Equation below to derive, for the space around a charged particle, an expression for the electric potential V relative to the zero potential at infinity. Consider a point P at distance R from a fixed particle of positive charge q .

$$
V_{f}-V_{i}=-\int_{i}^{f} \vec{E} \cdot d \vec{s} .
$$

To use Equation above, we imagine that we move a positive test charge $\mathrm{q}_{0}$ from point P to infinity. Because the path we take does not matter, let us choose the simplest one-a line that extends radially from the fixed particle through P to infinity. To use Equation above, we must evaluate the dot product:

$$
\vec{E} \cdot d \vec{s}=E \cos \theta d s
$$

The electric field is directed radially outward from the fixed particle. Thus, the differential displacement of the test particle along its path has the same direction as .That means that in Equation above, angle $\theta=0$ and $\cos \theta=1$. Because the path is radial, let us write ds as dr. Then, substituting the limits R and $\infty$, we can write as:

$$
V_{f}-V_{i}=-\int_{R}^{\infty} E d r .
$$

Next, we set $\mathrm{V}_{\mathrm{f}}=0($ at $\infty)$ and $\mathrm{V}_{\mathrm{i}}=\mathrm{V}($ at R$)$. Then, for the magnitude of the electric field at the site of the test charge, we substitute from:

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r^{2}}
$$

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r}
$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

## 2. Potential Due to an Electric Dipole

Now let us apply Equation below to an electric dipole to find the potential at an arbitrary point P in Figure 1.

$$
V=\sum_{i=1}^{n} V_{i}=\frac{1}{4 \pi \varepsilon_{0}} \sum_{i=1}^{n} \frac{q_{i}}{r_{i}} \quad(n \text { charged particles })
$$

At P , the positively charged particle (at distance $\mathrm{r}(+)$ ) sets up potential $\mathrm{V}(+)$ and the negatively charged particle (at distance $\mathrm{r}(-)$ ) sets up potential $\mathrm{V}(-)$.Then the net potential at P as:

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r^{2}} \quad \text { (electric dipole), }
$$

in which $\mathrm{p}(=\mathrm{qd})$ is the magnitude of the electric dipole moment defined in Module 223. The vector is directed along the dipole axis, from the negative to the positive charge. (Thus, $\theta$ is measured from the direction of $\vec{P}$.) We use this vector to report the orientation of an electric dipole.


Figure 1: Point P is a distance r from the midpoint O of a dipole. The line OP makes an angle u with the dipole axis.

## 3. Electric Potential Energy

In this module we are going to calculate the potential energy of a system of two charged particles and then briefly discuss how to expand the result to a system of more than two particles. Our starting point is to examine the work we must do (as an external agent) to bring together two charged particles that are initially infinitely far apart and that end up near each other and stationary. If the two particles have the same sign of charge, we must fight against their mutual repulsion. Our work is then positive and results in a positive potential energy for the final two-particle system. If, instead, the two particles have opposite signs of charge, our job is easy because of the mutual attraction of the particles. Our work is then negative and results in a negative potential energy for the system.

Let's follow this procedure to build the two-particle system, where particle 1 (with positive charge $\mathrm{q}_{1}$ ) and particle 2 (with positive charge $\mathrm{q}_{2}$ ) have separation r . Although both particles are positively charged, our result will apply also to situations where they are both negatively charged or have different signs.

We start with particle 2 fixed in place and particle 1 infinitely far away, with an initial potential energy $U_{i}$ for the two-particle system. Next we bring particle 1 to its final position, and then the system's potential energy is $U_{f}$. Our work changes the system's potential energy by $\Delta U=U_{f}-U_{i}$. With $\left(\Delta U=q\left(V_{f}-V_{i}\right)\right)$, we can relate $\Delta U$ to the change in potential through which we move particle 1:

$$
\begin{gathered}
U_{f}-U_{i}=q_{1}\left(V_{f}-V_{i}\right) . \\
U=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r} \quad \text { (two-particle system). }
\end{gathered}
$$

Equation above includes the signs of the two charges. If the two charges have the same sign, $U$ is positive. If they have opposite signs, $U$ is negative.

## 4. Reference

Walker, Jearl, Robert Resnick, and David Halliday. Halliday and resnick fundamentals of physics. Wiley, 2014.

