2/3 Plane Curvilinear Motion

We now treat the motion of a particle along a curved path which lies in a single plane. This motion is a special case of the more general three-dimensional motion introduced in Art. 2/1 and illustrated in Fig. 2/1. If we let the plane of motion be the *x-y* plane, for instance, then the coordinates z and ϕ of Fig. 2/1 are both zero, and R becomes the same as r. As mentioned previously, the vast majority of the motions of points or particles encountered in engineering practice can be represented as plane motion.

Before pursuing the description of plane curvilinear motion in any specific set of coordinates, we will first use vector analysis to describe the motion, since the results will be independent of any particular coordinate system. What follows in this article constitutes one of the most basic concepts in dynamics, namely, the *time derivative of a vector*. Much analysis in dynamics utilizes the time rates of change of vector quantities. You are therefore well advised to master this topic at the outset because you will have frequent occasion to use it.

Consider now the continuous motion of a particle along a plane curve as represented in Fig. 2/5. At time t the particle is at position A, which is located by the *position vector* $\mathbf{r}$ measured from some convenient fixed origin O. If both the magnitude and direction of $\mathbf{r}$ are known at time t, then the position of the particle is completely specified. At time $t + \Delta t$, the particle is at A', located by the position vector $\mathbf{r} + \Delta\mathbf{r}$. We note, of course, that this combination is vector addition and not scalar addition. The *displacement* of the particle during time Δt is the vector $\Delta\mathbf{r}$ which represents the vector change of position and is clearly independent of the choice of origin. If an origin were chosen at some different location, the position vector $\mathbf{r}$ would be changed, but $\Delta\mathbf{r}$ would be unchanged. The *distance* actually traveled by the particle as it moves along the path from A to A' is the scalar length Δs measured along the path. Thus, we distinguish between the vector displacement $\Delta\mathbf{r}$ and the scalar distance Δs.

Velocity

The *average velocity* of the particle between A and A' is defined as $\mathbf{v}_{\text{av}} = \Delta\mathbf{r}/\Delta t$, which is a vector whose direction is that of $\Delta\mathbf{r}$ and whose magnitude is the magnitude of $\Delta\mathbf{r}$ divided by Δt. The average speed of

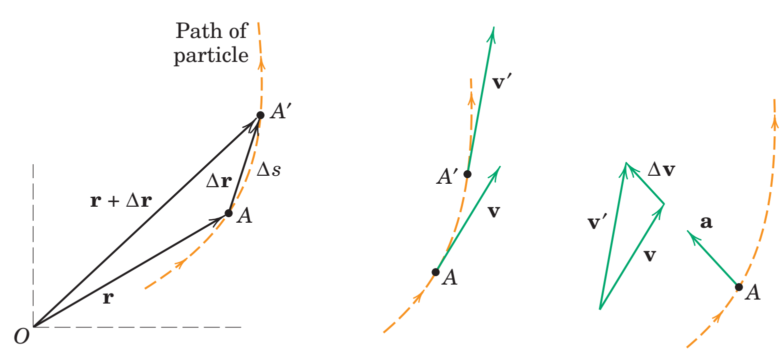


Figure 2/5

the particle between A and A' is the scalar quotient $\Delta s/\Delta t$. Clearly, the magnitude of the average velocity and the speed approach one another as the interval Δt decreases and A and A' become closer together.

The *instantaneous velocity* $\mathbf{v}$ of the particle is defined as the limiting value of the average velocity as the time interval approaches zero. Thus,

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

We observe that the direction of $\Delta\mathbf{r}$ approaches that of the tangent to the path as Δt approaches zero and, thus, the velocity $\mathbf{v}$ is always a vector tangent to the path.

We now extend the basic definition of the derivative of a scalar quantity to include a vector quantity and write

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} \tag{2/4}$$

The derivative of a vector is itself a vector having both a magnitude and a direction. The magnitude of $\mathbf{v}$ is called the *speed* and is the scalar

$$v = |\mathbf{v}| = \frac{ds}{dt} = \dot{s}$$

At this point we make a careful distinction between the *magnitude of the derivative* and the *derivative of the magnitude*. The magnitude of the derivative can be written in any one of the several ways $|d\mathbf{r}/dt| = |\dot{\mathbf{r}}| = \dot{s} = |\mathbf{v}| = v$ and represents the magnitude of the velocity, or the speed, of the particle. On the other hand, the derivative of the magnitude is written $d|\mathbf{r}|/dt = dr/dt = \dot{r}$, and represents the rate at which the length of the position vector $\mathbf{r}$ is changing. Thus, these two derivatives have two entirely different meanings, and we must be extremely careful to distinguish between them in our thinking and in our notation. For this and other reasons, you are urged to adopt a consistent notation for handwritten work for all vector quantities to distinguish them from scalar quantities. For simplicity the underline $\underline{v}$ is recommended. Other handwritten symbols such as $\vec{v}$, $\underset{\sim}{v}$, and $\hat{v}$ are sometimes used.

With the concept of velocity as a vector established, we return to Fig. 2/5 and denote the velocity of the particle at A by the tangent vector $\mathbf{v}$ and the velocity at A' by the tangent $\mathbf{v}'$. Clearly, there is a vector change in the velocity during the time Δt. The velocity $\mathbf{v}$ at A plus (vectorially) the change $\Delta\mathbf{v}$ must equal the velocity at A', so we can write $\mathbf{v}' - \mathbf{v} = \Delta\mathbf{v}$. Inspection of the vector diagram shows that $\Delta\mathbf{v}$ depends both on the change in magnitude (length) of $\mathbf{v}$ and on the change in direction of $\mathbf{v}$. These two changes are fundamental characteristics of the derivative of a vector.

Acceleration

The *average acceleration* of the particle between A and A' is defined as $\Delta\mathbf{v}/\Delta t$, which is a vector whose direction is that of $\Delta\mathbf{v}$. The magnitude of this average acceleration is the magnitude of $\Delta\mathbf{v}$ divided by Δt.

The *instantaneous acceleration* $\mathbf{a}$ of the particle is defined as the limiting value of the average acceleration as the time interval approaches zero. Thus,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

By definition of the derivative, then, we write

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}} \qquad \textbf{(2/5)}$$

As the interval Δt becomes smaller and approaches zero, the direction of the change $\Delta\mathbf{v}$ approaches that of the differential change $d\mathbf{v}$ and, thus, of $\mathbf{a}$. The acceleration $\mathbf{a}$, then, includes the effects of both the change in magnitude of $\mathbf{v}$ and the change of direction of $\mathbf{v}$. It is apparent, in general, that the direction of the acceleration of a particle in curvilinear motion is neither tangent to the path nor normal to the path. We do observe, however, that the acceleration component which is normal to the path points toward the center of curvature of the path.

Visualization of Motion

A further approach to the visualization of acceleration is shown in Fig. 2/6, where the position vectors to three arbitrary positions on the path of the particle are shown for illustrative purpose. There is a velocity vector tangent to the path corresponding to each position vector, and the relation is $\mathbf{v} = \dot{\mathbf{r}}$. If these velocity vectors are now plotted from some arbitrary point C, a curve, called the *hodograph*, is formed. The derivatives of these velocity vectors will be the acceleration vectors $\mathbf{a} = \dot{\mathbf{v}}$ which are tangent to the hodograph. We see that the acceleration has the same relation to the velocity as the velocity has to the position vector.

The geometric portrayal of the derivatives of the position vector $\mathbf{r}$ and velocity vector $\mathbf{v}$ in Fig. 2/5 can be used to describe the derivative of any vector quantity with respect to t or with respect to any other scalar variable. Now that we have used the definitions of velocity and acceleration to introduce the concept of the derivative of a vector, it is important to establish the rules for differentiating vector quantities. These rules

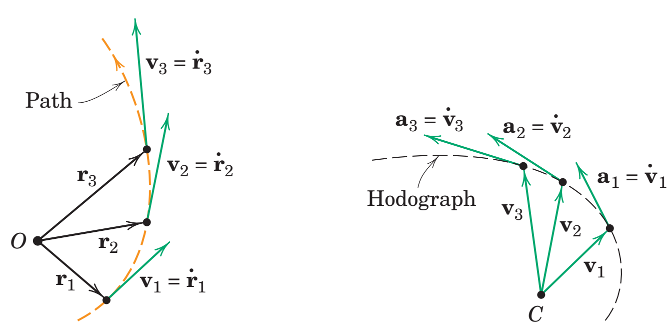


Figure 2/6

are the same as for the differentiation of scalar quantities, except for the case of the cross product where the order of the terms must be preserved. These rules are covered in Art. C/7 of Appendix C and should be reviewed at this point.

Three different coordinate systems are commonly used for describing the vector relationships for curvilinear motion of a particle in a plane: rectangular coordinates, normal and tangential coordinates, and polar coordinates. An important lesson to be learned from the study of these coordinate systems is the proper choice of a reference system for a given problem. This choice is usually revealed by the manner in which the motion is generated or by the form in which the data are specified. Each of the three coordinate systems will now be developed and illustrated.

2/4 Rectangular Coordinates (*x-y*)

This system of coordinates is particularly useful for describing motions where the x- and y-components of acceleration are independently generated or determined. The resulting curvilinear motion is then obtained by a vector combination of the x- and y-components of the position vector, the velocity, and the acceleration.

Vector Representation

The particle path of Fig. 2/5 is shown again in Fig. 2/7 along with x- and y-axes. The position vector $\mathbf{r}$, the velocity $\mathbf{v}$, and the acceleration $\mathbf{a}$ of the particle as developed in Art. 2/3 are represented in Fig. 2/7 together with their x- and y-components. With the aid of the unit vectors $\mathbf{i}$ and $\mathbf{j}$, we can write the vectors $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$ in terms of their x- and y-components. Thus,

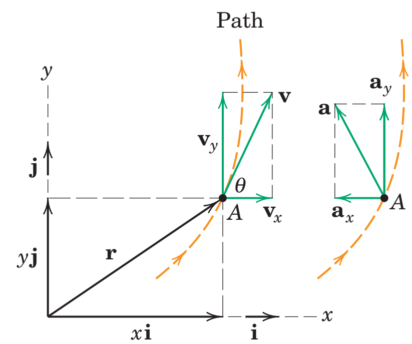


Figure 2/7

$$
\begin{aligned}
\mathbf{r} &= x\mathbf{i} + y\mathbf{j} \\
\mathbf{v} = \dot{\mathbf{r}} &= \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \\
\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} &= \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}
\end{aligned}
\qquad \textbf{(2/6)}
$$

As we differentiate with respect to time, we observe that the time derivatives of the unit vectors are zero because their magnitudes and directions remain constant. The scalar values of the components of $\mathbf{v}$ and $\mathbf{a}$ are merely $v_x = \dot{x}$, $v_y = \dot{y}$ and $a_x = \dot{v}_x = \ddot{x}$, $a_y = \dot{v}_y = \ddot{y}$. (As drawn in Fig. 2/7, a_x is in the negative x-direction, so that $\ddot{x}$ would be a negative number.)

As observed previously, the direction of the velocity is always tangent to the path, and from the figure it is clear that

$$
v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}
$$

$$
a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}
$$

If the angle θ is measured counterclockwise from the x-axis to $\mathbf{v}$ for the configuration of axes shown, then we can also observe that $dy/dx = \tan\theta = v_y/v_x$.

If the coordinates x and y are known independently as functions of time, $x = f_1(t)$ and $y = f_2(t)$, then for any value of the time we can combine them to obtain **r**. Similarly, we combine their first derivatives $\dot{x}$ and $\dot{y}$ to obtain **v** and their second derivatives $\ddot{x}$ and $\ddot{y}$ to obtain **a**. On the other hand, if the acceleration components a_x and a_y are given as functions of the time, we can integrate each one separately with respect to time, once to obtain v_x and v_y and again to obtain $x = f_1(t)$ and $y = f_2(t)$. Elimination of the time t between these last two parametric equations gives the equation of the curved path $y = f(x)$.

From the foregoing discussion we can see that the rectangular-coordinate representation of curvilinear motion is merely the superposition of the components of two simultaneous rectilinear motions in the x- and y-directions. Therefore, everything covered in Art. 2/2 on rectilinear motion can be applied separately to the x-motion and to the y-motion.

Projectile Motion

An important application of two-dimensional kinematic theory is the problem of projectile motion. For a first treatment of the subject, we neglect aerodynamic drag and the curvature and rotation of the earth, and we assume that the altitude change is small enough so that the acceleration due to gravity can be considered constant. With these assumptions, rectangular coordinates are useful for the trajectory analysis.

For the axes shown in Fig. 2/8, the acceleration components are

$$a_x = 0 \qquad a_y = -g$$

Integration of these accelerations follows the results obtained previously in Art. 2/2a for constant acceleration and yields

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)$$

In all these expressions, the subscript zero denotes initial conditions, frequently taken as those at launch where, for the case illustrated,

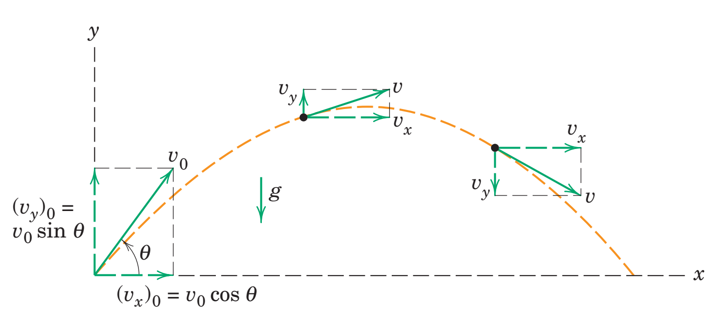


Figure 2/8

$x_0 = y_0 = 0$. Note that the quantity g is taken to be positive throughout this text.

We can see that the x- and y-motions are independent for the simple projectile conditions under consideration. Elimination of the time t between the x- and y-displacement equations shows the path to be parabolic (see Sample Problem 2/6). If we were to introduce a drag force which depends on the speed squared (for example), then the x- and y-motions would be coupled (interdependent), and the trajectory would be nonparabolic.

When the projectile motion involves large velocities and high altitudes, to obtain accurate results we must account for the shape of the projectile, the variation of g with altitude, the variation of the air density with altitude, and the rotation of the earth. These factors introduce considerable complexity into the motion equations, and numerical integration of the acceleration equations is usually necessary.

Andrew Davidhazy

This stroboscopic photograph of a bouncing ping-pong ball suggests not only the parabolic nature of the path, but also the fact that the speed is lower near the apex.

SAMPLE PROBLEM 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.

Solution. The x-coordinate is obtained by integrating the expression for v_x, and the x-component of the acceleration is obtained by differentiating v_x. Thus,

$$\left[\int dx = \int v_x\,dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\,dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

The y-components of velocity and acceleration are

$$[v_y = \dot{y}] \qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$$

$$[a_y = \dot{v}_y] \qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$$

We now calculate corresponding values of x and y for various values of t and plot x against y to obtain the path as shown.

When $y = 0$, $0 = 100 - 4t^2$, so $t = 5$ s. For this value of the time, we have

$$v_x = 50 - 16(5) = -30 \text{ m/s}$$

$$v_y = -8(5) = -40 \text{ m/s}$$

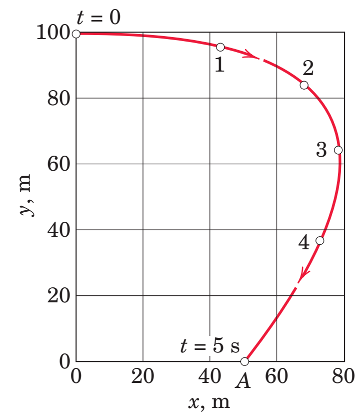
$$v = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

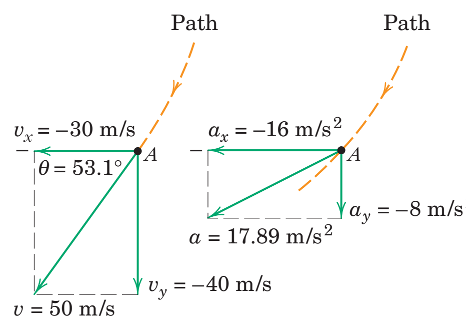
$$a = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$

The velocity and acceleration components and their resultants are shown on the separate diagrams for point A, where $y = 0$. Thus, for this condition we may write

$$\mathbf{v} = -30\mathbf{i} - 40\mathbf{j} \text{ m/s} \qquad \textit{Ans.}$$

$$\mathbf{a} = -16\mathbf{i} - 8\mathbf{j} \text{ m/s}^2 \qquad \textit{Ans.}$$

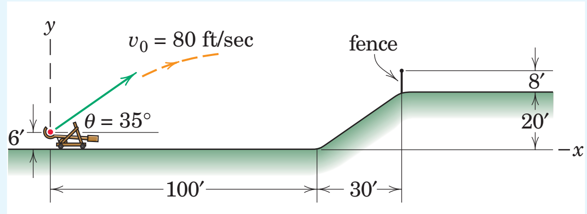




Helpful Hint

We observe that the velocity vector lies along the tangent to the path as it should, but that the acceleration vector is not tangent to the path. Note especially that the acceleration vector has a component that points toward the inside of the curved path. We concluded from our diagram in Fig. 2/5 that it is impossible for the acceleration to have a component that points toward the outside of the curve.

SAMPLE PROBLEM 2/6



A team of engineering students designs a medium-size catapult which launches 8-lb steel spheres. The launch speed is $v_0 = 80$ ft/sec, the launch angle is $\theta = 35°$ above the horizontal, and the launch position is 6 ft above ground level. The students use an athletic field with an adjoining slope topped by an 8-ft fence as shown. Determine:

(a) the *x-y* coordinates of the point of first impact

(b) the time duration t_f of the flight

(c) the maximum height h above the horizontal field attained by the ball

(d) the velocity (expressed as a vector) with which the projectile strikes the ground

Repeat part (*a*) for a launch speed of $v_0 = 75$ ft/sec.

Solution. We make the assumptions of constant gravitational acceleration and no aerodynamic drag. With the latter assumption, the 8-lb weight of the projectile is irrelevant. Using the given *x-y* coordinate system, we begin by checking the *y*-displacement at the horizontal position of the fence. ①

$$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = 0 + (80 \cos 35°)t \qquad t = 1.984 \text{ sec}$$

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \qquad y = 6 + 80 \sin 35°(1.984) - \tfrac{1}{2}(32.2)(1.984)^2 = 33.7 \text{ ft}$$

(a) Because the *y*-coordinate of the top of the fence is 20 + 8 = 28 feet, the projectile clears the fence. We now find the flight time by setting $y = 20$ ft:

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \qquad 20 = 6 + 80 \sin 35°(t_f) - \tfrac{1}{2}(32.2)t_f^{\,2} \qquad t_f = 2.50 \text{ s} \qquad \textit{Ans.}$$

$$[x = x_0 + (v_x)_0 t] \qquad x = 0 + 80 \cos 35°(2.50) = 164.0 \text{ ft}$$

(b) Thus the point of first impact is $(x, y) = (164.0, 20)$ ft. *Ans.*

(c) For the maximum height:

② $$[v_y^{\,2} = (v_y)_0^{\,2} - 2g(y - y_0)] \qquad 0^2 = (80 \sin 35°)^2 - 2(32.2)(h - 6) \qquad h = 38.7 \text{ ft} \ \textit{Ans.}$$

(d) For the impact velocity:

$$[v_x = (v_x)_0] \qquad v_x = 80 \cos 35° = 65.5 \text{ ft/sec}$$

$$[v_y = (v_y)_0 - gt] \qquad v_y = 80 \sin 35° - 32.2(2.50) = -34.7 \text{ ft/sec}$$

So the impact velocity is $\mathbf{v} = 65.5\mathbf{i} - 34.7\mathbf{j}$ ft/sec. *Ans.*

If $v_0 = 75$ ft/sec, the time from launch to the fence is found by

$$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = (75 \cos 35°)t \qquad t = 2.12 \text{ sec}$$

and the corresponding value of *y* is

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \qquad y = 6 + 80 \sin 35°(2.12) - \tfrac{1}{2}(32.2)(2.12)^2 = 24.9 \text{ ft}$$

For this launch speed, we see that the projectile hits the fence, and the point of impact is

$$(x, y) = (130, 24.9) \text{ ft} \qquad \textit{Ans.}$$

For lower launch speeds, the projectile could land on the slope or even on the level portion of the athletic field.

Helpful Hints

① Neglecting aerodynamic drag is a poor assumption for projectiles with relatively high initial velocities, large sizes, and low weights. In a vacuum, a baseball thrown with an initial speed of 100 ft/sec at 45° above the horizontal will travel about 311 feet over a horizontal range. In sea-level air, the baseball range is about 200 ft, while a typical beachball under the same conditions will travel about 10 ft.

② As an alternative approach, we could find the time at apex where $v_y = 0$, then use that time in the *y*-displacement equation. Verify that the trajectory apex occurs over the 100-ft horizontal portion of the athletic field.

PROBLEMS

(In the following problems where motion as a projectile in air is involved, neglect air resistance unless otherwise stated and use $g = 9.81$ m/s^2 or $g = 32.2$ ft/sec^2.)

Introductory Problems

2/59 At time $t = 0$, the position vector of a particle moving in the x-y plane is $\mathbf{r} = 5\mathbf{i}$ m. By time $t = 0.02$ s, its position vector has become $5.1\mathbf{i} + 0.4\mathbf{j}$ m. Determine the magnitude v_{av} of its average velocity during this interval and the angle θ made by the average velocity with the positive x-axis.

2/60 A particle moving in the x-y plane has a velocity at time $t = 6$ s given by $4\mathbf{i} + 5\mathbf{j}$ m/s, and at $t = 6.1$ s its velocity has become $4.3\mathbf{i} + 5.4\mathbf{j}$ m/s. Calculate the magnitude a_{av} of its average acceleration during the 0.1-s interval and the angle θ it makes with the x-axis.

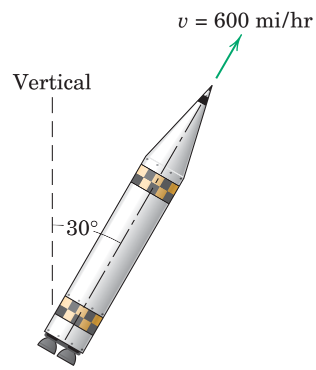
2/61 The velocity of a particle moving in the x-y plane is given by $6.12\mathbf{i} + 3.24\mathbf{j}$ m/s at time $t = 3.65$ s. Its average acceleration during the next 0.02 s is $4\mathbf{i} + 6\mathbf{j}$ m/s^2. Determine the velocity $\mathbf{v}$ of the particle at $t = 3.67$ s and the angle θ between the average-acceleration vector and the velocity vector at $t = 3.67$ s.

2/62 A particle which moves with curvilinear motion has coordinates in millimeters which vary with the time t in seconds according to $x = 2t^2 - 4t$ and $y = 3t^2 - \frac{1}{3}t^3$. Determine the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ and the angles which these vectors make with the x-axis when $t = 2$ s.

2/63 The x-coordinate of a particle in curvilinear motion is given by $x = 2t^3 - 3t$, where x is in feet and t is in seconds. The y-component of acceleration in feet per second squared is given by $a_y = 4t$. If the particle has y-components $y = 0$ and $\dot{y} = 4$ ft/sec when $t = 0$, find the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ when $t = 2$ sec. Sketch the path for the first 2 seconds of motion, and show the velocity and acceleration vectors for $t = 2$ sec.

2/64 The y-coordinate of a particle in curvilinear motion is given by $y = 4t^3 - 3t$, where y is in inches and t is in seconds. Also, the particle has an acceleration in the x-direction given by $a_x = 12t$ in./sec^2. If the velocity of the particle in the x-direction is 4 in./sec when $t = 0$, calculate the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the particle when $t = 1$ sec. Construct $\mathbf{v}$ and $\mathbf{a}$ in your solution.

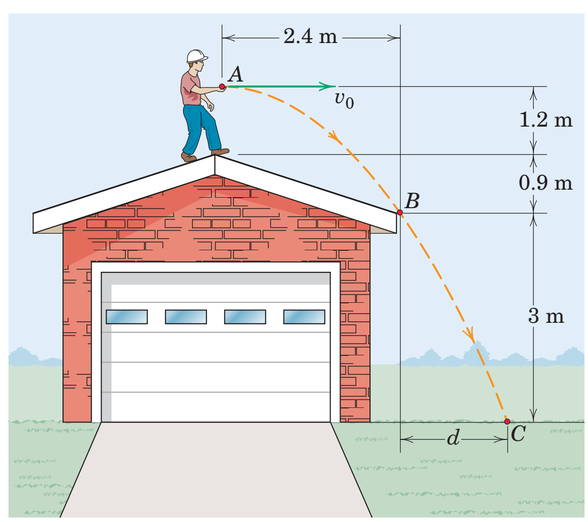
2/65 A rocket runs out of fuel in the position shown and continues in unpowered flight above the atmosphere. If its velocity in this position was 600 mi/hr, calculate the maximum additional altitude h acquired and the corresponding time t to reach it. The gravitational acceleration during this phase of its flight is 30.8 ft/sec^2.



Problem 2/65

2/66 A particle moves in the x-y plane with a y-component of velocity in feet per second given by $v_y = 8t$ with t in seconds. The acceleration of the particle in the x-direction in feet per second squared is given by $a_x = 4t$ with t in seconds. When $t = 0$, $y = 2$ ft, $x = 0$, and $v_x = 0$. Find the equation of the path of the particle and calculate the magnitude of the velocity $\mathbf{v}$ of the particle for the instant when its x-coordinate reaches 18 ft.

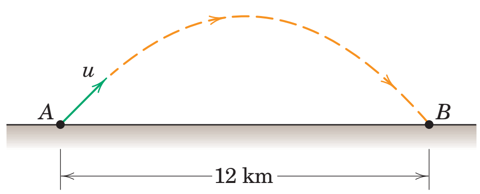
2/67 A roofer tosses a small tool to the ground. What minimum magnitude v_0 of horizontal velocity is required to just miss the roof corner B? Also determine the distance d.



Problem 2/67

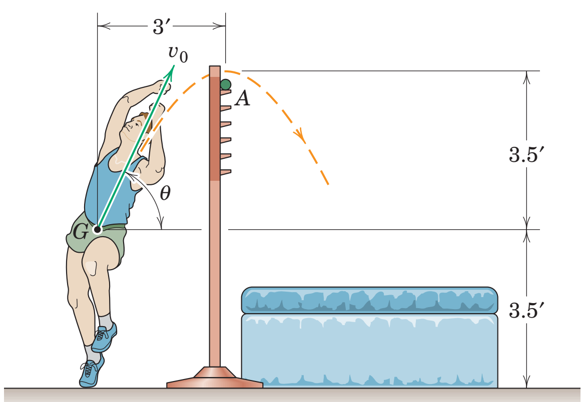
2/68 Prove the well-known result that, for a given launch speed v_0, the launch angle $\theta = 45°$ yields the maximum horizontal range R. Determine the maximum range. (Note that this result does not hold when aerodynamic drag is included in the analysis.)

2/69 Calculate the minimum possible magnitude u of the muzzle velocity which a projectile must have when fired from point A to reach a target B on the same horizontal plane 12 km away.



Problem 2/69

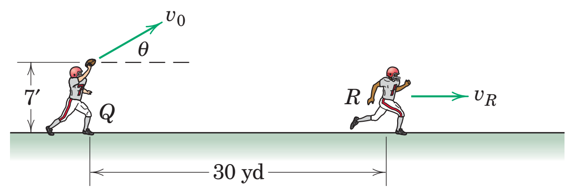
2/70 The center of mass G of a high jumper follows the trajectory shown. Determine the component v_0, measured in the vertical plane of the figure, of his takeoff velocity and angle θ if the apex of the trajectory just clears the bar at A. (In general, must the mass center G of the jumper clear the bar during a successful jump?)



Problem 2/70

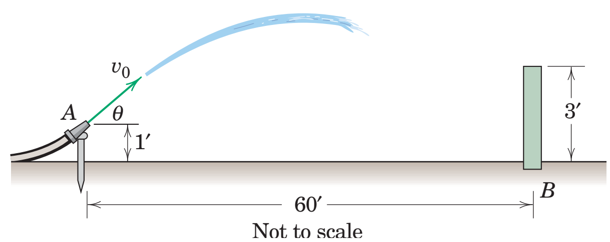
Representative Problems

2/71 The quarterback Q throws the football when the receiver R is in the position shown. The receiver's velocity is constant at 10 yd/sec, and he catches passes when the ball is 6 ft above the ground. If the quarterback desires the receiver to catch the ball 2.5 sec after the launch instant shown, determine the initial speed v_0 and angle θ required.



Problem 2/71

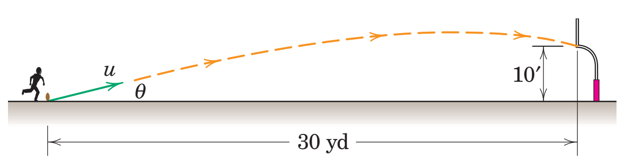
2/72 The water nozzle ejects water at a speed $v_0 = 45$ ft/sec at the angle $\theta = 40°$. Determine where, relative to the wall base point B, the water lands. Neglect the effects of the thickness of the wall.



Problem 2/72

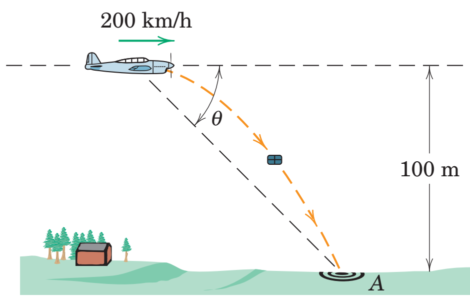
2/73 Water is ejected from the water nozzle of Prob. 2/72 with a speed $v_0 = 45$ ft/sec. For what value of the angle θ will the water land closest to the wall after clearing the top? Neglect the effects of wall thickness and air resistance. Where does the water land?

2/74 A football player attempts a 30-yd field goal. If he is able to impart a velocity u of 100 ft/sec to the ball, compute the minimum angle θ for which the ball will clear the crossbar of the goal. (*Hint:* Let $m = \tan\theta$.)



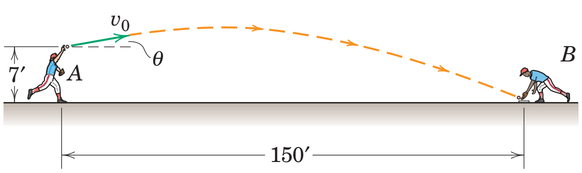
Problem 2/74

2/75 The pilot of an airplane carrying a package of mail to a remote outpost wishes to release the package at the right moment to hit the recovery location A. What angle θ with the horizontal should the pilot's line of sight to the target make at the instant of release? The airplane is flying horizontally at an altitude of 100 m with a velocity of 200 km/h.



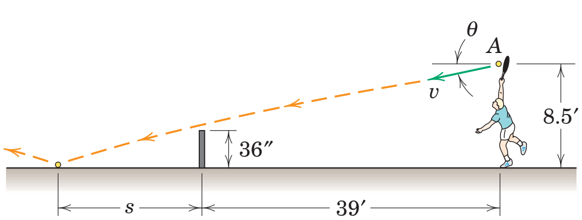
Problem 2/75

2/76 During a baseball practice session, the cutoff man A executes a throw to the third baseman B. If the initial speed of the baseball is $v_0 = 130$ ft/sec, what angle θ is best if the ball is to arrive at third base at essentially ground level?



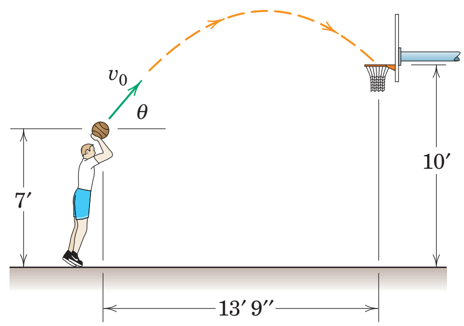
Problem 2/76

2/77 If the tennis player serves the ball horizontally ($\theta = 0$), calculate its velocity v if the center of the ball clears the 36-in. net by 6 in. Also find the distance s from the net to the point where the ball hits the court surface. Neglect air resistance and the effect of ball spin.



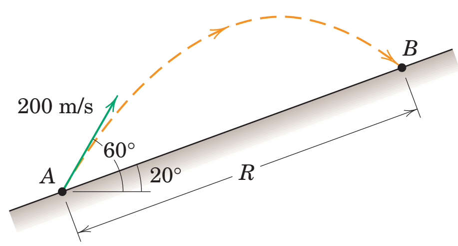
Problem 2/77

2/78 The basketball player likes to release his foul shots with an initial speed $v_0 = 23.5$ ft/sec. What value(s) of the initial angle θ will cause the ball to pass through the center of the rim? Neglect clearance considerations as the ball passes over the front portion of the rim.



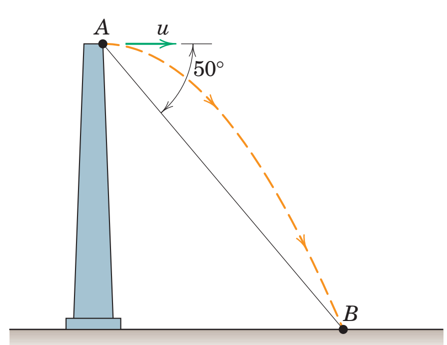
Problem 2/78

2/79 A projectile is launched with an initial speed of 200 m/s at an angle of 60° with respect to the horizontal. Compute the range R as measured up the incline.



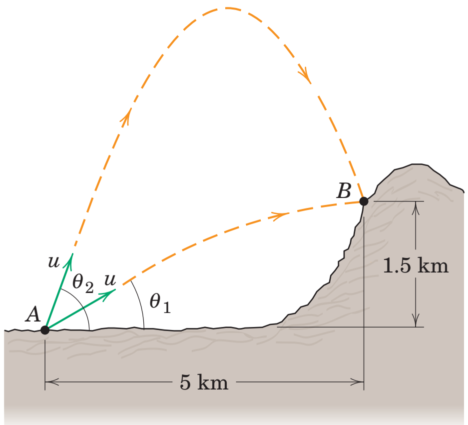
Problem 2/79

2/80 A rock is thrown horizontally from a tower at A and hits the ground 3.5 s later at B. The line of sight from A to B makes an angle of 50° with the horizontal. Compute the magnitude of the initial velocity $\mathbf{u}$ of the rock.



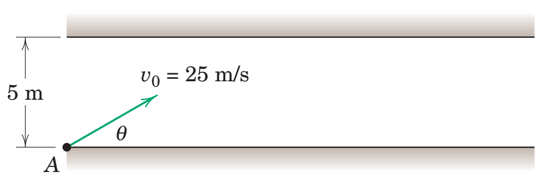
Problem 2/80

2/81 The muzzle velocity of a long-range rifle at A is $u = 400$ m/s. Determine the two angles of elevation θ which will permit the projectile to hit the mountain target B.



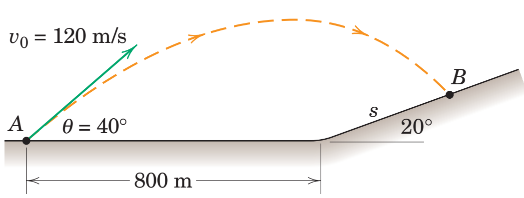
Problem 2/81

2/82 A projectile is launched with a speed $v_0 = 25$ m/s from the floor of a 5-m-high tunnel as shown. Determine the maximum horizontal range R of the projectile and the corresponding launch angle θ.



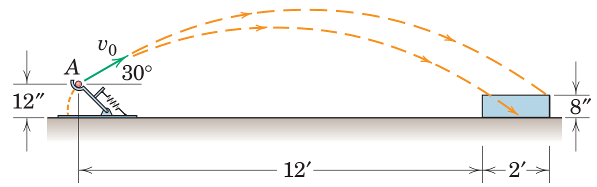
Problem 2/82

2/83 A projectile is launched from point A with the initial conditions shown in the figure. Determine the slant distance s which locates the point B of impact. Calculate the time of flight t.



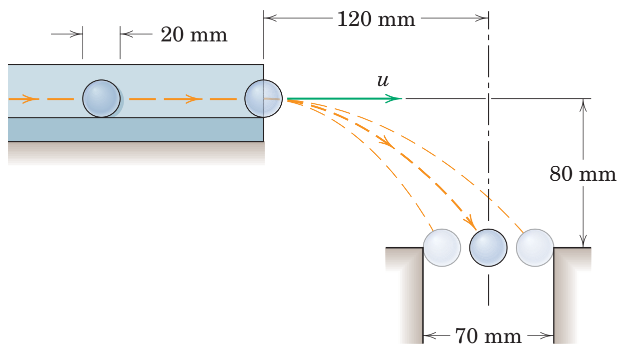
Problem 2/83

2/84 A team of engineering students is designing a catapult to launch a small ball at A so that it lands in the box. If it is known that the initial velocity vector makes a 30° angle with the horizontal, determine the range of launch speeds v_0 for which the ball will land inside the box.



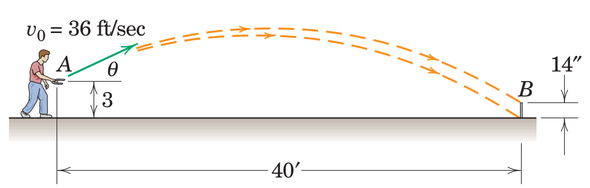
Problem 2/84

2/85 Ball bearings leave the horizontal trough with a velocity of magnitude u and fall through the 70-mm-diameter hole as shown. Calculate the permissible range of u which will enable the balls to enter the hole. Take the dashed positions to represent the limiting conditions.



Problem 2/85

2/86 A horseshoe player releases the horseshoe at A with an initial speed $v_0 = 36$ ft/sec. Determine the range for the launch angle θ for which the shoe will strike the 14-in. vertical stake.



Problem 2/86