Lecture No. 7

Kinetics of Particles

Force, Mass, and Acceleration

Rectilinear Motion

3/1 Introduction

According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces. Kinetics is the study of the relations between unbalanced forces and the resulting changes in motion. In Chapter 3 we will study the kinetics of particles. This topic requires that we combine our knowledge of the properties of forces, which we developed in statics, and the kinematics of particle motion just covered in Chapter 2. With the aid of Newton's second law, we can combine these two topics and solve engineering problems involving force, mass, and motion.

The three general approaches to the solution of kinetics problems are: (A) direct application of Newton's second law (called the force-mass-acceleration method), (B) use of work and energy principles, and (C) solution by impulse and momentum methods. Each approach has its special characteristics and advantages, and Chapter 3 is subdivided into Sections A, B, and C, according to these three methods of solution. In addition, a fourth section, Section D, treats special applications and combinations of the three basic approaches. Before proceeding, you should review carefully the definitions and concepts of Chapter 1, because they are fundamental to the developments which follow.

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SECTION A FORCE, MASS, AND ACCELERATION

3/2 Newton's Second Law

The basic relation between force and acceleration is found in Newton's second law, Eq. 1/1, the verification of which is entirely experimental. We now describe the fundamental meaning of this law by considering an ideal experiment in which force and acceleration are assumed to be measured without error. We subject a mass particle to the action of a single force $\mathbf{F}_1$, and we measure the acceleration $\mathbf{a}_1$ of the particle in the primary inertial system.* The ratio F_1/a_1 of the magnitudes of the force and the acceleration will be some number C_1 whose value depends on the units used for measurement of force and acceleration. We then repeat the experiment by subjecting the same particle to a different force $\mathbf{F}_2$ and measuring the corresponding acceleration $\mathbf{a}_2$. The ratio F_2/a_2 of the magnitudes will again produce a number C_2. The experiment is repeated as many times as desired.

We draw two important conclusions from the results of these experiments. First, the ratios of applied force to corresponding acceleration all equal the *same* number, provided the units used for measurement are not changed in the experiments. Thus,

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \cdots = \frac{F}{a} = C, \qquad \text{a constant}$$

We conclude that the constant C is a measure of some invariable property of the particle. This property is the *inertia* of the particle, which is its *resistance to rate of change of velocity*. For a particle of high inertia (large C), the acceleration will be small for a given force F. On the other hand, if the inertia is small, the acceleration will be large. The mass m is used as a quantitative measure of inertia, and therefore, we may write the expression $C = km$, where k is a constant introduced to account for the units used. Thus, we may express the relation obtained from the experiments as

$$F = kma \qquad \textbf{(3/1)}$$

where F is the magnitude of the resultant force acting on the particle of mass m, and a is the magnitude of the resulting acceleration of the particle.

The second conclusion we draw from this ideal experiment is that the acceleration is always in the direction of the applied force. Thus, Eq. 3/1 becomes a *vector* relation and may be written

$$\mathbf{F} = km\mathbf{a} \qquad \textbf{(3/2)}$$

Although an actual experiment cannot be performed in the ideal manner described, the same conclusions have been drawn from countless accurately performed experiments. One of the most accurate checks is given by the precise prediction of the motions of planets based on Eq. 3/2.

3/3 Equation of Motion and Solution of Problems

When a particle of mass m is subjected to the action of concurrent forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, . . . whose vector sum is $\Sigma\mathbf{F}$, Eq. 1/1 becomes

$$\Sigma\mathbf{F} = m\mathbf{a} \qquad \textbf{(3/3)}$$

When applying Eq. 3/3 to solve problems, we usually express it in scalar component form with the use of one of the coordinate systems developed in Chapter 2. The choice of an appropriate coordinate system depends on the type of motion involved and is a vital step in the formulation of any problem. Equation 3/3, or any one of the component forms of the force-mass-acceleration equation, is usually called the *equation of motion*. The equation of motion gives the instantaneous value of the acceleration corresponding to the instantaneous values of the forces which are acting.

Two Types of Dynamics Problems

We encounter two types of problems when applying Eq. 3/3. In the first type, the acceleration of the particle is either specified or can be determined directly from known kinematic conditions. We then determine the corresponding forces which act on the particle by direct substitution into Eq. 3/3. This problem is generally quite straightforward.

In the second type of problem, the forces acting on the particle are specified and we must determine the resulting motion. If the forces are constant, the acceleration is also constant and is easily found from Eq. 3/3. When the forces are functions of time, position, or velocity, Eq. 3/3 becomes a differential equation which must be integrated to determine the velocity and displacement.

Problems of this second type are often more formidable, as the integration may be difficult to carry out, particularly when the force is a mixed function of two or more motion variables. In practice, it is frequently necessary to resort to approximate integration techniques, either numerical or graphical, particularly when experimental data are involved. The procedures for a mathematical integration of the acceleration when it is a function of the motion variables were developed in Art. 2/2, and these same procedures apply when the force is a specified function of these same parameters, since force and acceleration differ only by the constant factor of the mass.

3/4 Rectilinear Motion

We now apply the concepts discussed in Arts. 3/2 and 3/3 to problems in particle motion, starting with rectilinear motion in this article and treating curvilinear motion in Art. 3/5. In both articles, we will analyze the motions of bodies which can be treated as particles. This simplification is possible as long as we are interested only in the motion of the mass center of the body. In this case we may treat the forces as concurrent through the mass center. We will account for the action of nonconcurrent forces on the motions of bodies when we discuss the kinetics of rigid bodies in Chapter 6.

If we choose the x-direction, for example, as the direction of the rectilinear motion of a particle of mass m, the acceleration in the y- and z-directions will be zero and the scalar components of Eq. 3/3 become

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned} \qquad \textbf{(3/4)}$$

For cases where we are not free to choose a coordinate direction along the motion, we would have in the general case all three component equations

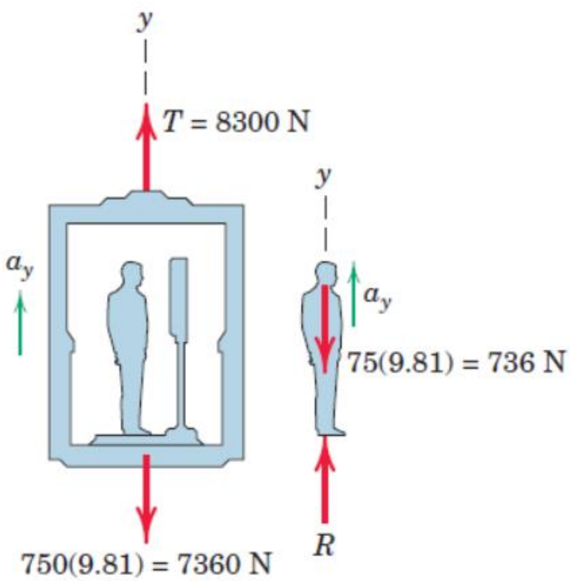
$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \\ \Sigma F_z &= ma_z \end{aligned} \qquad \textbf{(3/5)}$$

where the acceleration and resultant force are given by

$$\begin{aligned} \mathbf{a} &= a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \\ a &= \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} \\ \Sigma\mathbf{F} &= \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k} \\ |\Sigma\mathbf{F}| &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2} \end{aligned}$$

SAMPLE PROBLEM 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.



Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$[\Sigma F_y = ma_y] \qquad 8300 - 7360 = 750a_y \qquad a_y = 1.257 \text{ m/s}^2$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

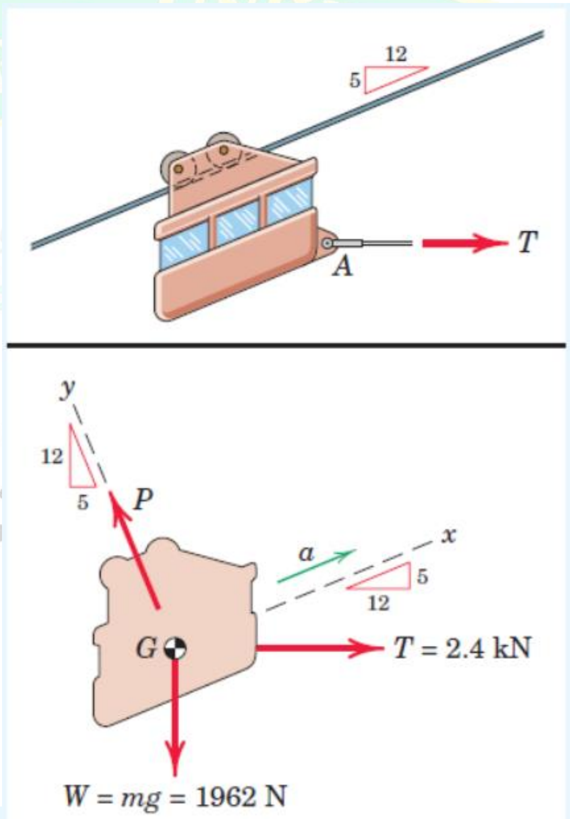
$[\Sigma F_y = ma_y] \qquad R - 736 = 75(1.257) \qquad R = 830 \text{ N}$ *Ans.*

The velocity reached at the end of the 3 seconds is

$[\Delta v = \int a\, dt] \qquad v - 0 = \int_0^3 1.257\, dt \qquad v = 3.77 \text{ m/s}$ *Ans.*

SAMPLE PROBLEM 3/2

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force P exerted by the supporting cable on the wheels.



Solution. The free-body diagram of the car and wheels taken together and treated as a particle discloses the 2.4-kN tension T, the weight $W = mg = 200(9.81) = 1962$ N, and the force P exerted on the wheel assembly by the cable.

The car is in equilibrium in the y-direction since there is no acceleration in this direction. Thus,

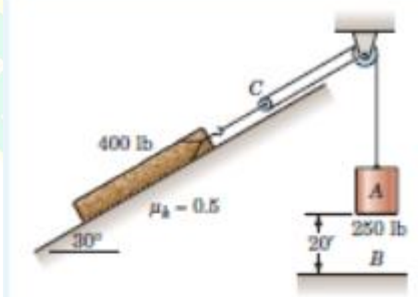
$[\Sigma F_y = 0]$ $\quad P - 2.4(\frac{5}{13}) - 1.962(\frac{12}{13}) = 0 \qquad P = 2.73 \text{ kN}$ *Ans.*

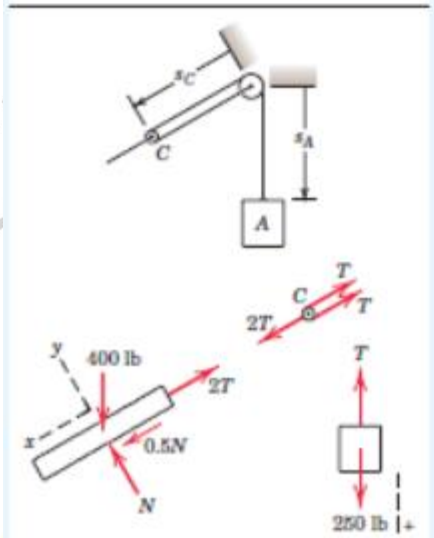
(1) In the x-direction the equation of motion gives

$[\Sigma F_x = ma_x]$ $\quad 2400(\frac{12}{13}) - 1962(\frac{5}{13}) = 200a \qquad a = 7.30 \text{ m/s}^2$ *Ans.*

SAMPLE PROBLEM 3/3

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.





Solution. The motions of the log and the block A are clearly dependent. Although by now it should be evident that the acceleration of the log up the incline is half the downward acceleration of A, we may prove it formally. The constant total length of the cable is $L = 2s_C + s_A$ + constant, where the constant accounts ① for the cable portions wrapped around the pulleys. Differentiating twice with respect to time gives $0 = 2\ddot{s}_C + \ddot{s}_A$, or

$$0 = 2a_C + a_A$$

We assume here that the masses of the pulleys are negligible and that they turn with negligible friction. With these assumptions the free-body diagram of the pulley C discloses force and moment equilibrium. Thus, the tension in the cable attached to the log is twice that applied to the block. Note that the accelerations of the log and the center of pulley C are identical.

The free-body diagram of the log shows the friction force $\mu_k N$ for motion up the plane. Equilibrium of the log in the y-direction gives

② $[\Sigma F_y = 0]$ $\qquad N - 400 \cos 30° = 0 \qquad N = 346$ lb

and its equation of motion in the x-direction gives

$[\Sigma F_x = ma_x]$ $\qquad 0.5(346) - 2T + 400 \sin 30° = \dfrac{400}{32.2} a_C$

For the block in the positive downward direction, we have

③ $[+\downarrow \Sigma F = ma]$ $\qquad 250 - T = \dfrac{250}{32.2} a_A$

Solving the three equations in a_C, a_A, and T gives us

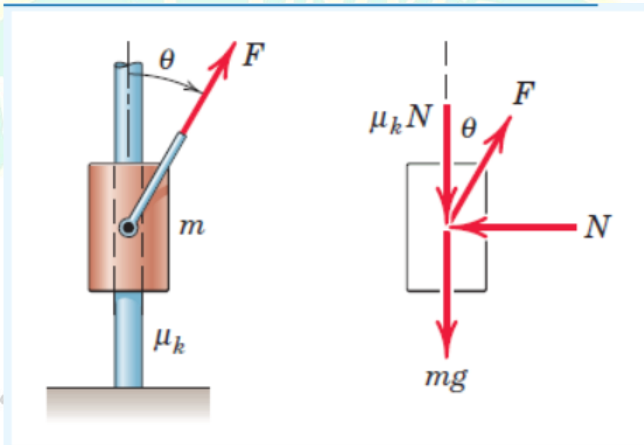
$$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$$

④ For the 20-ft drop with constant acceleration, the block acquires a velocity

$[v^2 = 2ax]$ $\qquad v_A = \sqrt{2(5.83)(20)} = 15.27$ ft/sec $\qquad$ *Ans.*

SAMPLE PROBLEM 3/5

The collar of mass m slides up the vertical shaft under the action of a force F of constant magnitude but variable direction. If $\theta = kt$ where k is a constant and if the collar starts from rest with $\theta = 0$, determine the magnitude F of the force which will result in the collar coming to rest as θ reaches $\pi/2$. The coefficient of kinetic friction between the collar and shaft is μ_k.



Solution. After drawing the free-body diagram, we apply the equation of motion in the y-direction to get

1 $[\Sigma F_y = ma_y]$ $$F\cos\theta - \mu_k N - mg = m\frac{dv}{dt}$$

where equilibrium in the horizontal direction requires $N = F\sin\theta$. Substituting $\theta = kt$ and integrating first between general limits give

$$\int_0^t (F\cos kt - \mu_k F\sin kt - mg)\,dt = m\int_0^v dv$$

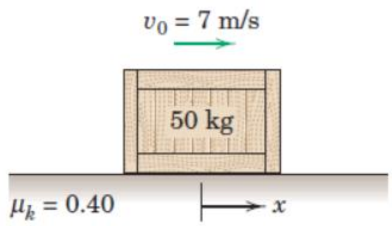
which becomes

$$\frac{F}{k}[\sin kt + \mu_k(\cos kt - 1)] - mgt = mv$$

For $\theta = \pi/2$ the time becomes $t = \pi/2k$, and $v = 0$ so that

2 $$\frac{F}{k}[1 + \mu_k(0 - 1)] - \frac{mg\pi}{2k} = 0 \quad \text{and} \quad F = \frac{mg\pi}{2(1 - \mu_k)}$$ *Ans.*

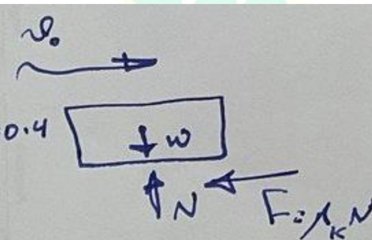
3/1 The 50-kg crate is projected along the floor with an initial speed of 7 m/s at $x = 0$. The coefficient of kinetic friction is 0.40. Calculate the time required for the crate to come to rest and the corresponding distance x traveled.



① Time required to the rest = ?

$\Rightarrow v_0 = v_1$

$v_2 = v_2$



② travelled displacement = ?

$S_1 = 0$

$S_2 = ?$; $v_2 = 0$

since the acceleration is constant,

$v_2 = v_1 + at$ —— ①

$v_2^2 = v_1^2 + 2a(x_2 - x_1)$ —— ②

To find the accleration :

$\Sigma F_x = m a_x \Rightarrow -F = m a_x$; The normal force $\Sigma F_y = 0 \Rightarrow$ $N = mg$

$\therefore -mg\mu_k = m a_x$

$a_x = -\mu_k * g = 0.4 * 9.81 = -3.92 \ m/s^2$

From ① $v_2^{\,0} = v_1 + (-3.92) * t \Rightarrow 7 = 3.92t$

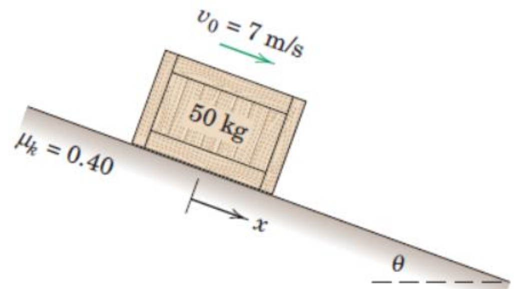
$t = 1.785 \ s$ → Ans.

from ② $v_2^2{}^{\,0} = v_1^2 + 2a(x_2 - x_1^{\,0})$

$\Rightarrow 7^2 = 2(3.92)(x_2)$

$x_2 = 6.25 \ m$ → Ans.

3/2 The 50-kg crate of Prob. 3/1 is now projected down an incline as shown with an initial speed of 7 m/s. Investigate the time t required for the crate to come to rest and the corresponding distance x traveled if (*a*) $\theta = 15°$ and (*b*) $\theta = 30°$.

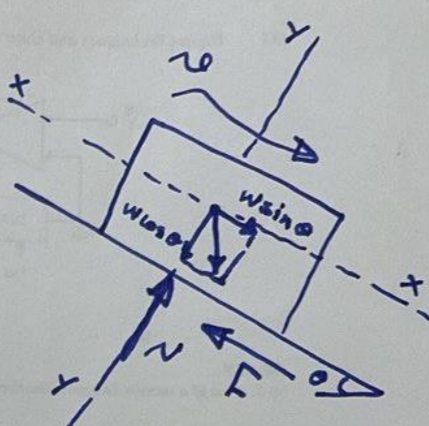


$$\Sigma F_x = m a_x$$

$$-F + W \sin\theta = m a_x$$

$$\Sigma F_y = 0$$

$$N = W \cos\theta$$



$$\Rightarrow -W \mu_k \cos\theta + W \sin\theta = m a_x$$

$$-mg \cdot \mu_k \cos\theta + mg \sin\theta = mg\, a_x$$

$$\boxed{a_x = -g \mu_k \cos\theta + g \sin\theta}$$

at $\theta = 15°$: $a_x = -9.81 * 0.4 (\cos 15) + 9.81 \sin 15$

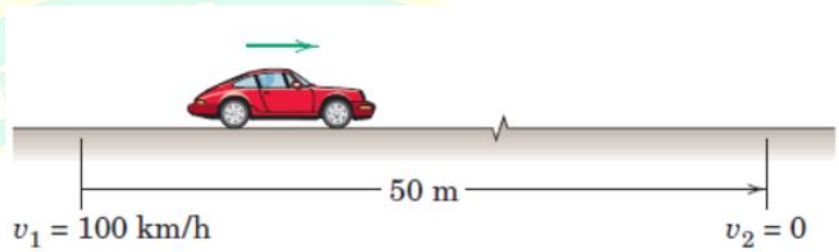
$= -1.25 \ m/s^2$

$v_2 = v_1 + at \Rightarrow 7 = 1.25 * t \Rightarrow \boxed{t = 5.6 \text{ sec}}$ → Ans

$v_2^2 = v_1^2 + 2a(x_2 - x_1) \Rightarrow 7^2 = 2 * 1.25\, x$

$\boxed{x = 19.6 \ m}$ → Ans.

3/5 During a brake test, the rear-engine car is stopped from an initial speed of 100 km/h in a distance of 50 m. If it is known that all four wheels contribute equally to the braking force, determine the braking force F at each wheel. Assume a constant deceleration for the 1500-kg car.



$v_1 = 100 \frac{km}{h} = 100 * \frac{1000}{3600} = 27.8 \frac{m}{s}$

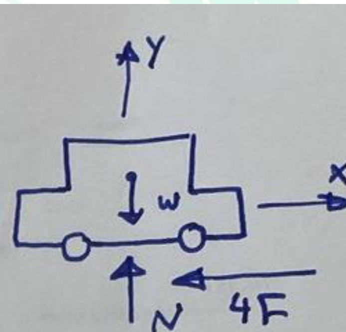
$v_2 = 0$; $x_2 = 50\ m$; $x_1 = 0$

$F = ?$; constant accleration

$v_2^2 = v_1^2 + 2a(x_2 - x_1)$

$0 = (27.8)^2 + 2a(50 - 0)$

$\Rightarrow a = -7.72\ m/s^2$



* (4F) لانه يوجد brake ع الاطارات الاربعة وبالتالي تكون قوة الاحتكاك هي مجموع قوى الاحتكاك للاربع اطارات

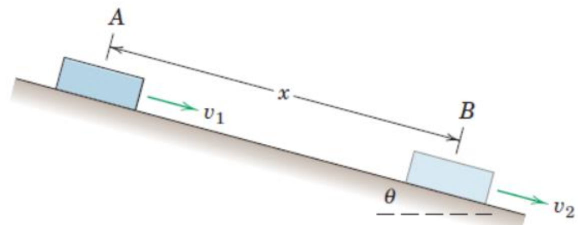
$\Sigma F_x = 0$

$-4F = ma$;

$-4F = 1500 * -7.72$

$F = 2895\ N$ → Ans.

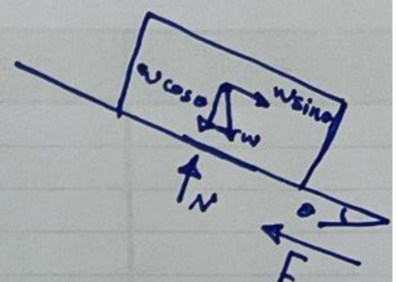
3/20 The block shown is observed to have a velocity $v_1 = 20$ ft/sec as it passes point A and a velocity $v_2 = 10$ ft/sec as it passes point B on the incline. Calculate the coefficient of kinetic friction μ_k between the block and the incline if $x = 30$ ft and $\theta = 15°$.



$\Sigma F_x = m a_x$

$-F + w \sin\theta = m a_x$; $\Sigma F_y = 0$

$N = w \cos\theta$



$\Rightarrow -\mu_k W \cos\theta + W \sin\theta = m a_x$

$-\mu_k mg \cos\theta + w \sin\theta = m a_x$, $w = mg$

$$\Rightarrow \boxed{\mu_k = \frac{g \sin\theta - a_x}{g \cos\theta}}$$

المجهول هو a_x

$v_1 = 20$ ft/sec ; $v_2 = 10$ ft/sec ; $x_1 = 30$ ft

$v_2^2 = v_1^2 + 2a(x_2 - x_1)$, $x_1 \to 0$

$10^2 = 20^2 + 2a * 30 \Rightarrow a = -5$ ft/s^2

$$\therefore \mu_k = \frac{32.3 \sin(15) + 5}{32.3 \cos 15} = 0.428 \longrightarrow \text{Ans.}$$