Class: 2 nd class
Subject: Strength of Materials
Lecturer: Mujtaba Al Khafaji
E-mail: mujtabaabdulkadhim@mustaqbal-college.edu.iq

LECTURE NO. 9

BEAM DEFLECTION

Introduction

When we consider designing beams based on rigidity consideration the deflection of the beam must be known at specific or critical location. Several methods are avaible for determining beam deflection. Although based on the same principles, they differ in technique and in their immediate objective.

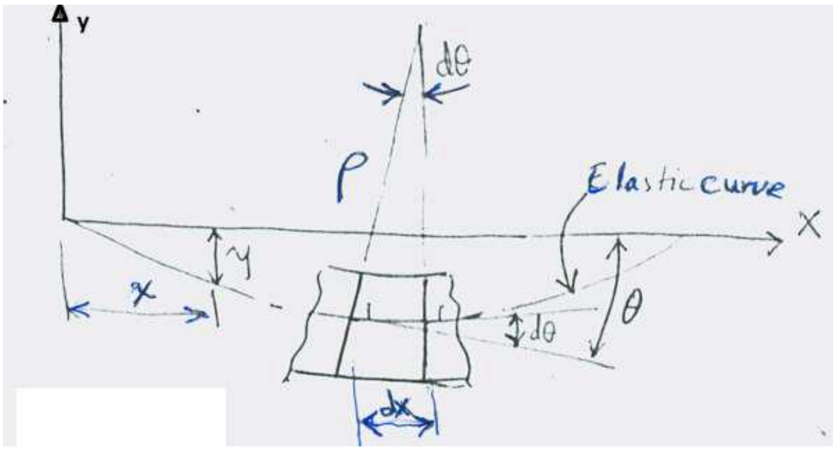
Methods of Determining Beam Deflections

Numerous methods are available for the determination of beam deflections. These methods include:

- Double-integration method
- Area-moment method
- Strain-energy method (Castigliano's Theorem)
- Conjugate-beam method
- Method of superposition

Double-integration method

The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve. *The double integration method can be used if the moment (M) has a single expression for the whole beam.*



Slope $=\tan\theta = \frac{dy}{dx}$

Since θ is small, then $\tan\theta \approx \theta$

$\theta = \frac{dy}{dx}$ and $\frac{d\theta}{dx} = \frac{d^2y}{dx^2}$ **(1)**

The defferential length ds can be expressed in terms of radius of curvature ρ as:-

$$ds = \rho\, d\theta$$

Or $\frac{1}{\rho} = \frac{d\theta}{ds} \approx \frac{d\theta}{dx}$ **(2)**

Subtitube eq.(1) and eq.(2)

$\frac{1}{\rho} = \frac{d^2y}{dx^2}$ **(3)**

In deriving the flexure formula (Bending Stress Lecture), we obtained the relation:

$\frac{1}{\rho} = \frac{M}{EI}$ **(4)**

Equating eq.(3) and (4), we have

$$EI\frac{d^2y}{dx^2} = M \qquad \textbf{(5)}$$

This is known as the differential equation of the elastic curve of the beam.

Integrating equation (5), assuming *EI* constant, we obtain

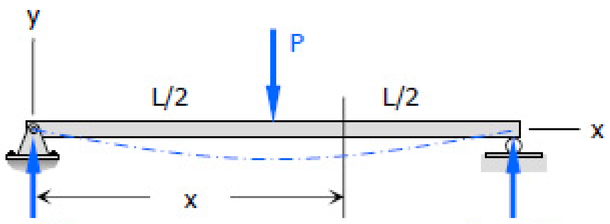
$$EI\frac{dy}{dx} = \int M dx + c_1 \qquad (slope\ equation)$$

And

$$EIy = \int\left(\int M dx\right)dx + c_1 x + c_2 \qquad (deflection\ equation)$$

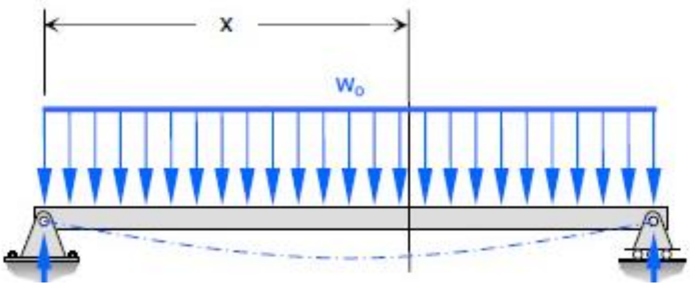
Example 1

Determine the maximum deflection δ in a simply supported beam of length L carrying a concentrated load P at midspan.



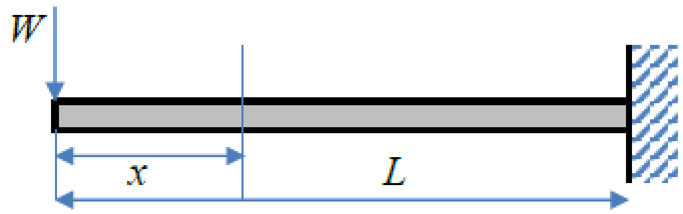
Example 2

Determine the maximum deflection δ in a simply supported beam of length L carrying a uniformly distributed load of intensity wo applied over its entire length.



Example 3

Determine the maximum deflection δ in a cantilever beam of length L carrying a concentrated load P at midspan.



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Example 3

Determine the maximum deflection δ in a cantilever beam of length L carrying a concentrated load P at midspan.