Experiment no. 15:Mathmatical Models of System

Case(1)

Consider the simple mechanical system of Figure 2.1. Three forces influence the motion of the mass, namely, the applied force, the frictional force, and the spring force.

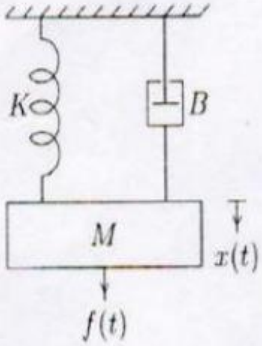

FIGURE 2.1
Mechanical translational system.

Applying Newton's law of motion, the force equation of the system is

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)$$

Let $x_1 = x$ and $x_2 = \frac{dx}{dt}$, then

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = \frac{1}{M}[f(t) - Bx_2 - Kx_1]$$

With the system initially at rest, a force of 25 Newton is applied at time $t = 0$. Assume that the mass $M = 1$ Kg, frictional coefficient $B = 5$ N/m/sec., and the spring constant $K = 25$ N/m. The above equations are defined in an M-file **mechsys.m** as follows:

```
function xdot = mechsys(t,x);% returns the state derivatives
F = 25;                          % Step input
M =1; B = 5; K = 25;
xdot = [x(2) ; 1/M*( F - B*x(2) - K*x(1) ) ];
```

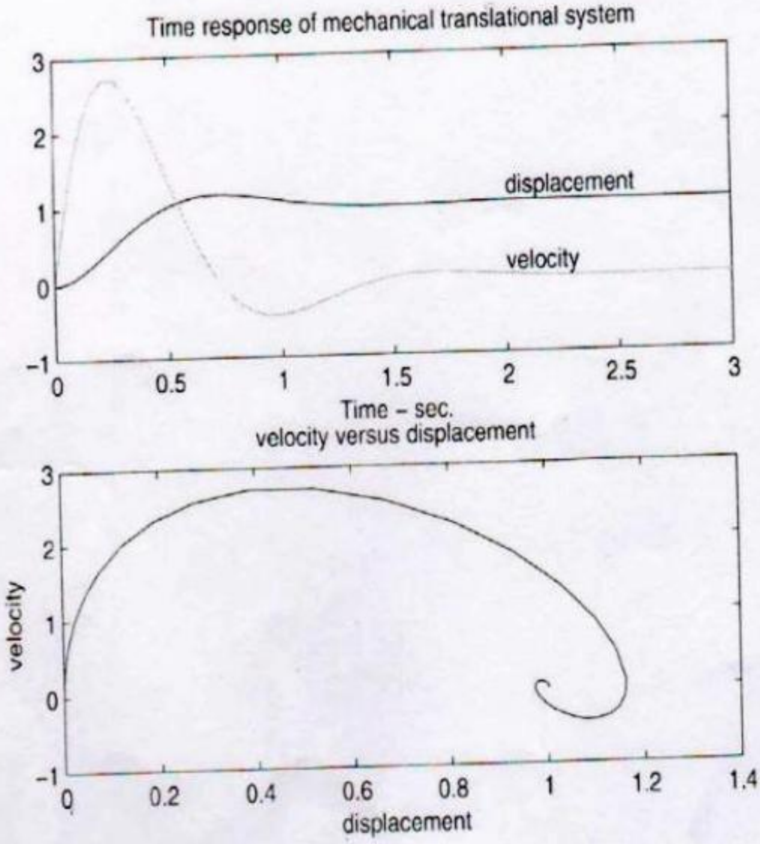

FIGURE 2.2
Response of the mechanical system of Example 2.1.

```
tspan = [0, 3]      ;                    % time interval
x0 = [0, 0];                             % initial conditions
[t,x] = ode23('mechsys', tspan, x0);
subplot(2,1,1),plot(t,x)
title('Time response of mechanical translational system')
xlabel('Time - sec.')
text(2,1.2,'displacement')
text(2,.2,'velocity')

d= x(:,1);   v = x(:,2);
subplot(2,1,2), plot(d, v)
title('velocity versus displacement ')
xlabel('displacement')
ylabel('velocity')
```

Case(2)

The circuit elements in Figure 2.3 are $R = 1.4\Omega$, $L = 2H$, and $C = 0.32F$. the initial inductor current is zero, and the initial capacitor voltage is .5 volts. A step voltage of 1 volt is applied at time $t = 0$. Determine $i(t)$ and $v(t)$ over the range $0 < t < 15$ sec. Also, obtain a plot of current versus capacitor voltage.

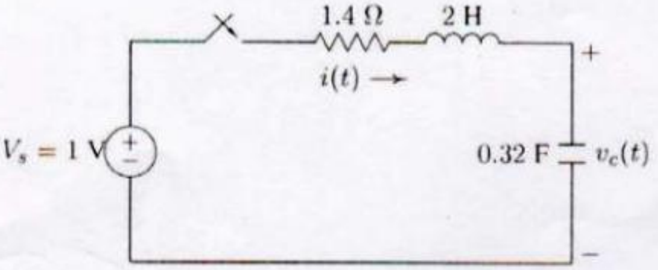

FIGURE 2.3
RLC circuit for time-domain solution example.

Applying KVL

$$Ri + L\frac{di}{dt} + v_c = V_s$$

and

$$i = C\frac{dv_c}{dt}$$

Let

$$x_1 = v_c$$

and

$$x_2 = i$$

then

$$\dot{x}_1 = \frac{1}{C}x_2$$

and

$$\dot{x}_2 = \frac{1}{L}(V_s - x_1 - Rx_2)$$

The above equations are defined in an M-file **electsys.m** as follows:

```
function xdot = electsys(t,x);
                                 % returns the state derivatives
V = 1;                           % Step input
R =1.4; L = 2; C = 0.32;
xdot = [x(2)/C ; 1/L*( V - x(1) - R*x(2) ) ];
```

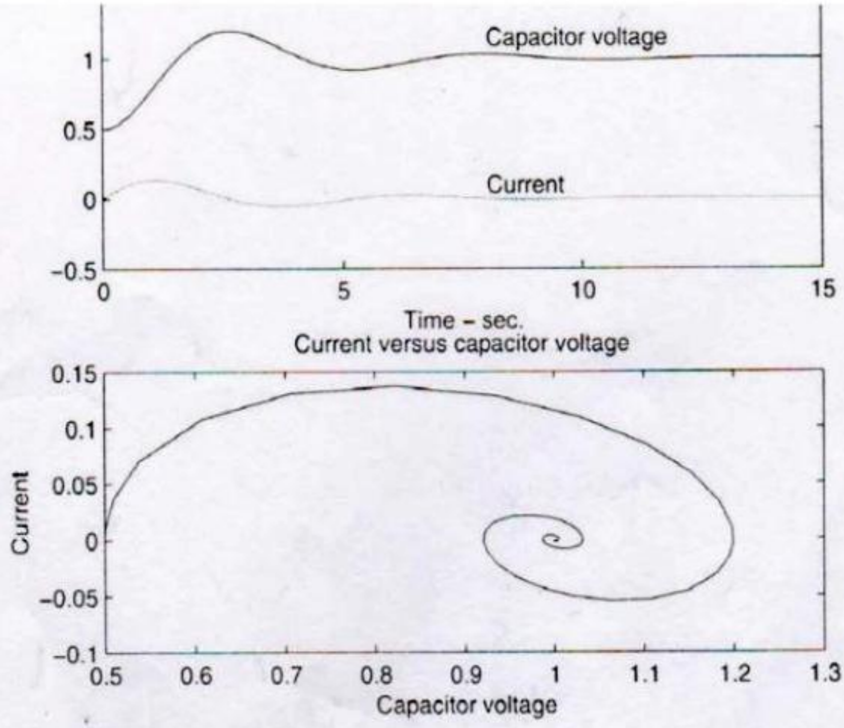

FIGURE 2.4
Response of the series RLC circuit of Example 2.2.

The following M-file, **ch2ex02.m**, uses **ode23** to simulate the system over an interval of 0 to 15 sec.

```
x0 = [0.5, 0];          % initial conditions
tspan=[0, 15];          % time interval
[t,x] = ode23('electsys',tspan, x0);
subplot(2, 1, 1),plot(t,x)
title('Time response of an RLC series circuit')
xlabel('Time - sec.')
text(8,1.15, 'Capacitor voltage')
text(8, .1, 'Current')
vc= x(:,1);   i = x(:,2);
subplot(2, 1, 2),plot(vc, i)
title('Current versus capacitor voltage ')
xlabel('Capacitor voltage')
ylabel('Current'), subplot(111)
```

A great majority of physical systems are linear within some range of the variables. However, all systems ultimately become nonlinear as the ranges are increased without limit. For the nonlinear systems, the principle of superposition does not apply. **ode23** and **ode45** simplify the task of solving a set of nonlinear differential equations as demonstrated in Example 2.3.

Example 2.3

Consider the simple pendulum illustrated in Figure 2.5 where a weight of $W = mg$ kg is hung from a support by a weightless rod of length L meters. While usually approximated by a linear differential equation, the system really is nonlinear and includes viscous damping with a damping coefficient of B kg/m/sec.

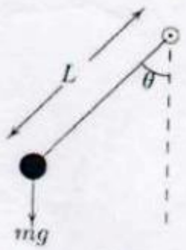

FIGURE 2.5
Pendulum oscillator.

If θ in radians is the angle of deflection of the rod, the velocity of the weight at the end will be $L\dot{\theta}$ and the tangential force acting to increase the angle θ can be written:

$$F_T = -W \sin\theta - BL\dot{\theta}$$

From Newton's law

$$F_T = mL\ddot{\theta}$$

Combining the two equations for the force, we get:

$$mL\ddot{\theta} + BL\dot{\theta} + W\sin\theta = 0$$

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$ (angular velocity), then

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{B}{m}x_2 - \frac{W}{mL}\sin x_1$$

The above equations are defined in an M-file **pendulum.m** as follows:

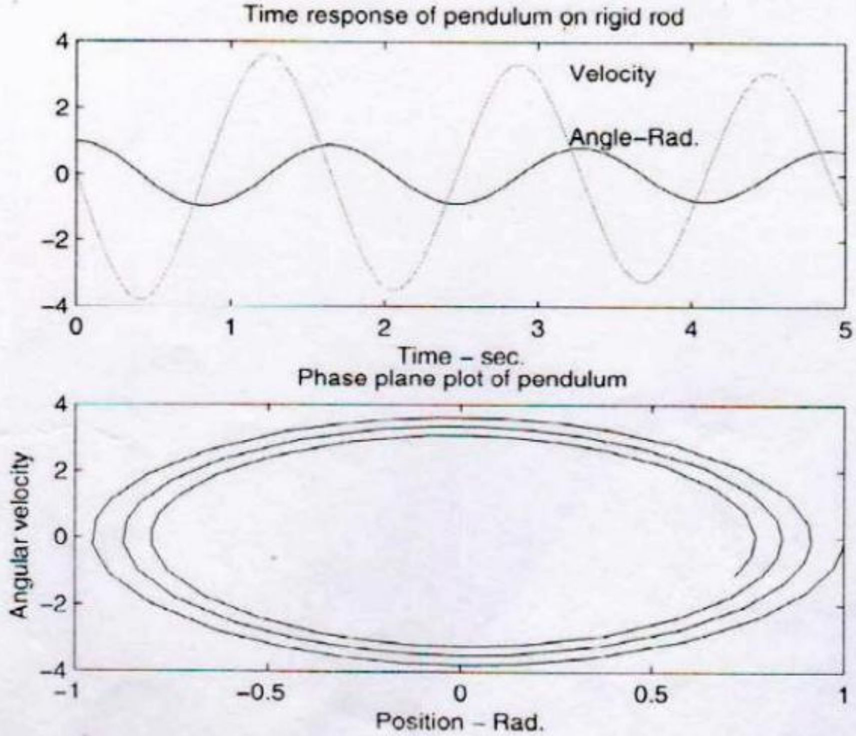

FIGURE 2.6
Response of the pendulum described in Example 2.3.

```
function xdot = pendulum(t,x);%returns the state derivatives
W = 2; L = .6; B = 0.02; g = 9.81;  m = W/g;
xdot = [x(2) ; -B/m*x(2)-W/(m*L)*sin(x(1)) ];
```

The following M-file, **ch2ex03.m**, uses **ode23** to simulate the system over an interval of 0 to 5 sec.

```
tspan = [0, 5];                          % time interval
x0 = [1, 0];                             % initial conditions
[t,x] = ode23('pendulum', tspan, x0);
subplot(2,1,1),plot(t,x)
title('Time response of pendulum on rigid rod')
xlabel('Time - sec.')
text(3.2,3.1,'Velocity'), text(3.2,1.2,'Angle-Rad.')
th= x(:,1);   w = x(:,2);
subplot(2,1,2),plot(th, w)
title('Phase plane plot of pendulum')
xlabel('Position - Rad.'), ylabel('Angular velocity')
```

Results of the simulation are shown in Figure 2.6.