



**Al-Mustaqbal University College**  
**Department of Computer**  
**Engineering Techniques**



**Information Theory and coding**  
**Fourth stage**

**Lecture 8**  
**Channel Capacity (Discrete channel)**

**By:**  
***MSC. Ridhab Sami***



### 1. Channel Capacity (Discrete channel):

This is defined as the maximum of  $I(X,Y)$ :

$$C = \text{channel capacity} = \max[I(X,Y)] \quad \text{bits/symbol}$$

Physically it is the maximum amount of information each symbol can carry to the receiver. Sometimes this capacity is also expressed in bits/sec if related to the rate of producing symbols  $r$ :

$$R(X,Y) = r \times I(X,Y) \quad \text{bits/sec} \quad \text{or} \quad R(X,Y) = I(X,Y)/\bar{\tau}$$

### Channel capacity of Symmetric channels:

The channel capacity is defined as  $\max [I(X,Y)]$ :

$$I(X,Y) = H(Y) - H(Y | X)$$

$$I(X,Y) = H(Y) + \sum_{j=1}^m \sum_{i=1}^n P(x_i, y_j) \log_2 P(y_j | x_i)$$

But we have

$$P(x_i, y_j) = P(x_i)P(y_j | x_i) \quad \text{put in above equation yields:}$$

$$I(X,Y) = H(Y) + \sum_{j=1}^m \sum_{i=1}^n P(x_i)P(y_j | x_i) \log_2 P(y_j | x_i)$$

If the channel is symmetric the quantity:

$$\sum_{j=1}^m P(y_j | x_i) \log_2 P(y_j | x_i) = K$$

Where  $K$  is constant and independent of the row number  $i$  so that the equation becomes:

$$I(X,Y) = H(Y) + K \sum_{i=1}^n P(x_i)$$



Hence  $I(X,Y) = H(Y) + K$  for symmetric channels

Max of  $I(X,Y) = \max[H(Y) + K] = \max[H(Y)] + K$

When Y has equiprobable symbols then  $\max[H(Y)] = \log_2 m$

Then

$$I(X,Y) = \log_2 m + K$$

Or

$$C = \log_2 m + K$$

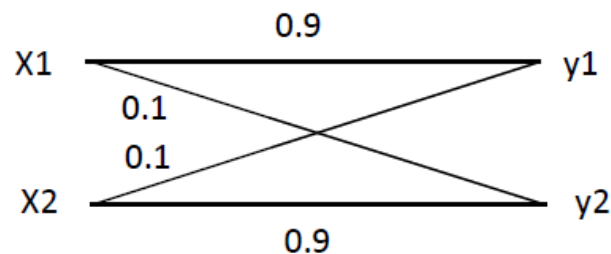
$$efficiency(\eta) = \frac{I(x,y)}{c} * 100\%$$

$$redundancy(R) = 100\% - efficiency$$

**Example:** Draw the channel model from the transmission matrix

$$P(y/x) = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

**Solution:**

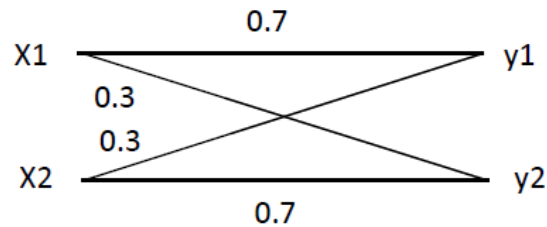


**Example:** A binary symmetric channel with error probability of 0.3 and marginal entropy  $H(y) = 0.97095$  bits/symbol.

- i- Draw the channel model.
- ii- Calculate the channel capacity.
- iii- Find the efficiency and redundancy.

### Solution:

i-The channel model is as following:



ii- Then the channel capacity is calculated as follow:

$$C = \log_2 m + K \quad \text{and } n = m$$

$$K = 0.7 \log_2 0.7 + 0.3 \log_2 0.3 = -0.88129$$

$$C = 1 - 0.88129 = 0.1187 \frac{\text{bits}}{\text{symbol}}$$

iii- The channel efficiency  $\eta = \frac{I(X,Y)}{C}$

$$I(X,Y) = H(Y) + K$$

We have the marginal entropy  $H(Y) = 0.97095 \text{ bits/symbol}$

$$\therefore I(X,Y) = 0.97095 + (-0.88129) = 0.0896 \text{ bits/symbol}$$

$$\text{Then } \eta = \frac{0.0896}{0.1187} = 75.6\%$$

To find the channel redundancy:

$$R = 1 - \eta = 1 - 0.756 = 0.244 \text{ or } 24.4\%$$



**Example:** Given a binary symmetric channel with transmission matrix shown below;  
 Determine the channel capacity, channel efficiency and redundancy. If  $p(x_1)=0.4$ .

$$P(y/x) = \begin{Bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{Bmatrix}$$

**Solution:**

$$P(x_1)=0.4, \quad P(x_2)=1-0.4=0.6$$

$$k = \sum_{j=1}^m P(y_j/x_i) \log_2 P(y_j/x_i) = \frac{0.8 \ln 0.8 + 0.2 \ln 0.2}{\ln 2} = -0.721$$

$$C = \log_2 m + k = \log_2 2 - 0.721 = 1 - 0.721 = 0.279 \text{ bit/symbol}$$

$$P(x, y) = P(y/x) * P(x) = \begin{pmatrix} 0.8 * 0.4 & 0.2 * 0.4 \\ 0.2 * 0.6 & 0.8 * 0.6 \end{pmatrix} = \begin{pmatrix} 0.32 & 0.08 \\ 0.12 & 0.48 \end{pmatrix}$$

$$P(y_1)=0.32+0.12=0.44, \quad P(y_2)=0.08+0.48=0.56$$

$$H(y) = - \sum_{j=1}^m P(y_j) \log_2 P(y_j) = - \left[ \frac{0.44 \ln 0.44 + 0.56 \ln 0.56}{\ln 2} \right] = 0.989 \text{ bit/symbol}$$

$$I(x, y) = H(y) + k = 0.989 - 0.721 = 0.268$$

$$\text{efficiency} = \frac{I(x, y)}{c} * 100\% = \frac{0.268}{0.279} * 100\% = 96.057\%$$

$$\text{redundancy} = 100\% - \text{efficiency} = 100\% - 96.057\% = 3.943\%$$