



**Al-Mustaqbal University College**  
**Department of Computer**  
**Engineering Techniques**



**Information Theory and coding**  
**Fourth stage**

**Lecture 9**  
**Cascading of Channels**

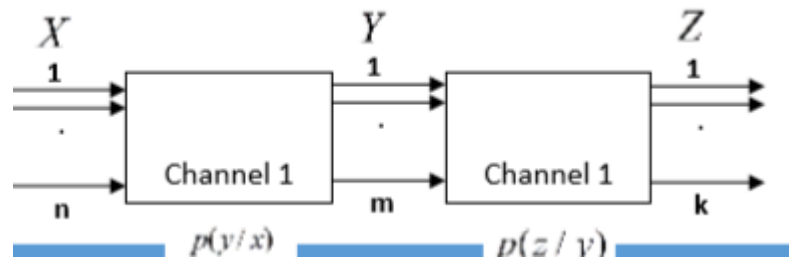
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### 1. Cascading of Channels

If two channels are cascaded, then the overall transition matrix is the product of the two transition matrices.

$$p(z / x) = p(y / x) \cdot p(z / y)$$

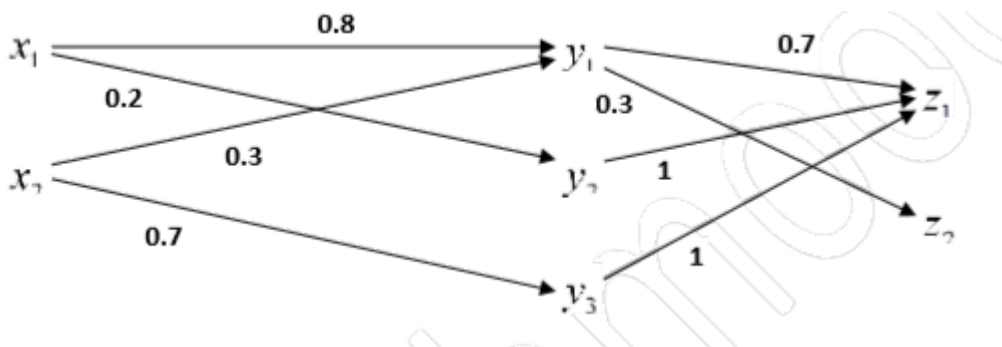
$$(n \times k) \quad (n \times m) \quad (m \times k)$$



For the series information channel, the overall channel capacity is not exceed any of each channel individually.

$$I(X,Z) \leq I(X,Y) \quad \& \quad I(X,Z) \leq I(Y,Z)$$

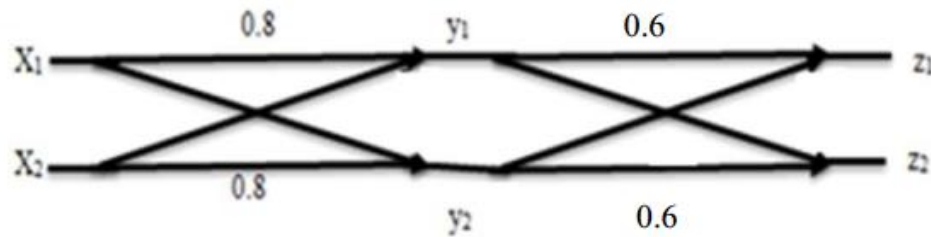
**Example:** Find the transition matrix  $p(z / x)$  for the cascaded channel shown:



$$p(Y|X) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0 & 0.7 \end{bmatrix}, \quad p(Z|Y) = \begin{bmatrix} 0.7 & 0.3 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$p(Z|X) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0.76 & 0.24 \\ 0.91 & 0.09 \end{bmatrix}$$

**Example:** Two BSC is cascaded as shown:



- i) Find the resultant channel matrix; then plot the final model
- ii) Find the channel capacity

**Solution**

$$P(Y/X) = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

$$P(Z/Y) = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

$$P(Z/X) = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \times \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

$$P(Z/X) = \begin{bmatrix} 0.56 & 0.44 \\ 0.44 & 0.56 \end{bmatrix}$$

$$C = \log_2 2 + K$$

$$k = \sum_{j=1}^m P(z_j/x_i) \log_2 P(z_j/x_i) = \frac{0.56 \ln 0.56 + 0.44 \ln 0.44}{\ln 2} = -0.989$$

$$C = 1 - 0.989 = 0.011 \text{ bits / symbol}$$

