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اسم المادة :الميكانيك

عنوان المحاضرة: **Center of mass and centroid**

رقم المحاضرة: 8

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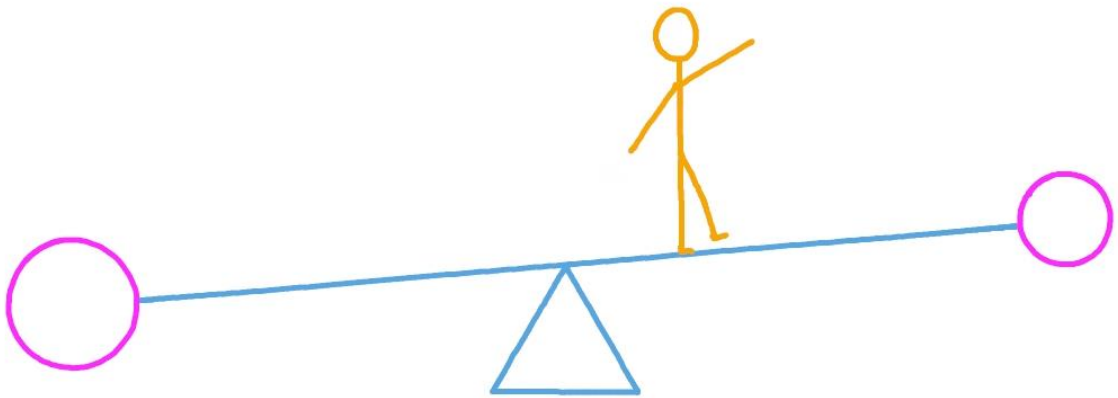
Center of mass and centroid

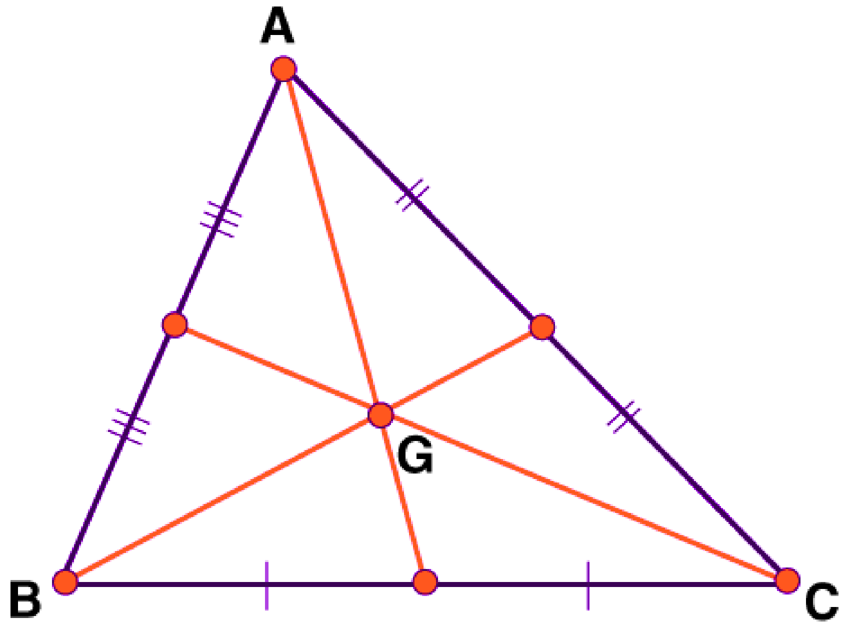
What is the center of mass?

- The *center of mass* is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses.



What is centroid ?

- The centroid or geometric center of a plane figure is the arithmetic mean position of all the points in the figure. Informally, it is the point at which a cutout of the shape could be perfectly balanced on the tip of a pin.

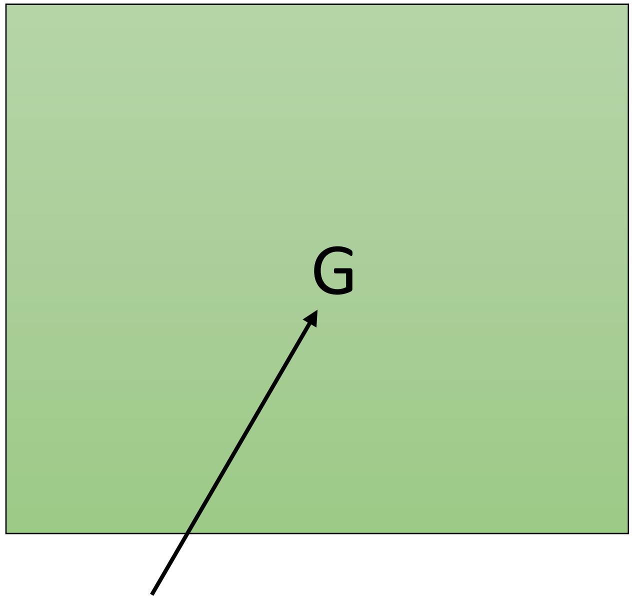


- The center of mass equal to centroid if the body is **homogeneous**.

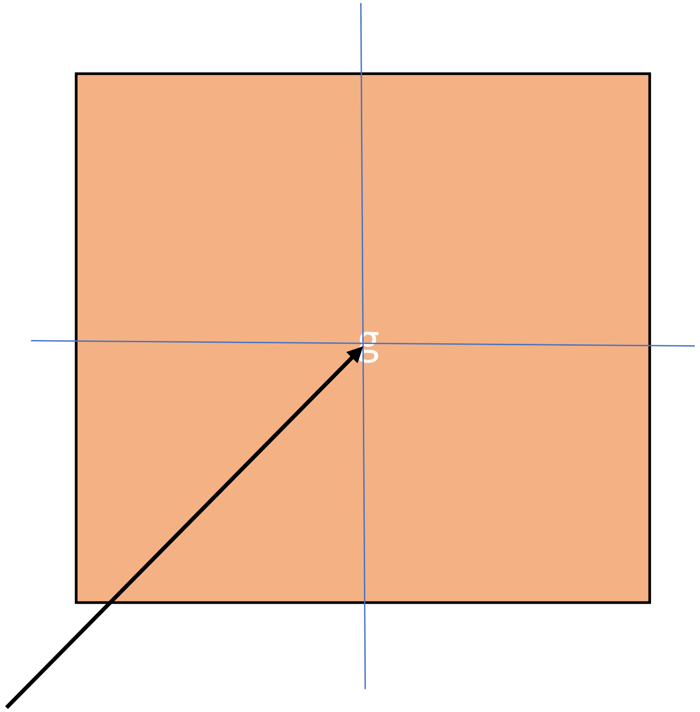
- A body is said to be **homogeneous** if all the material points are **materially uniform** with respect to a single placement. A body that is not homogeneous is said to be inhomogeneous

Examples

Center of mass (homogeneous)



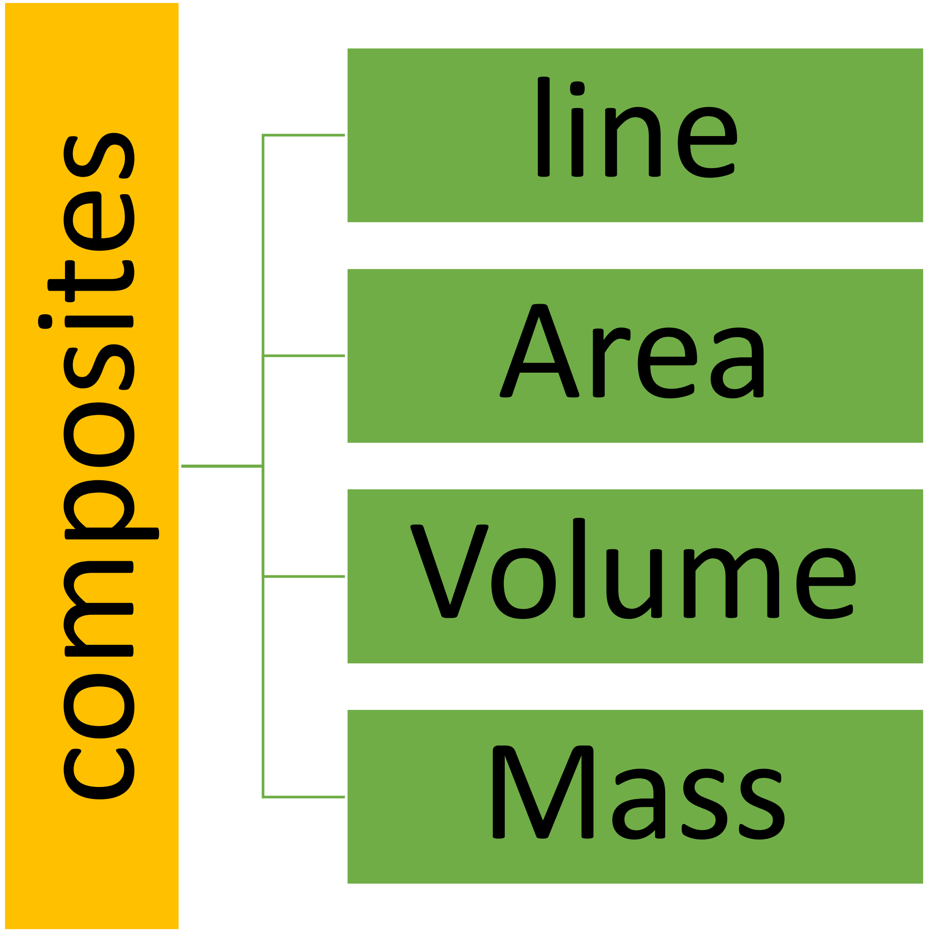
centroid (homogeneous)



Composites

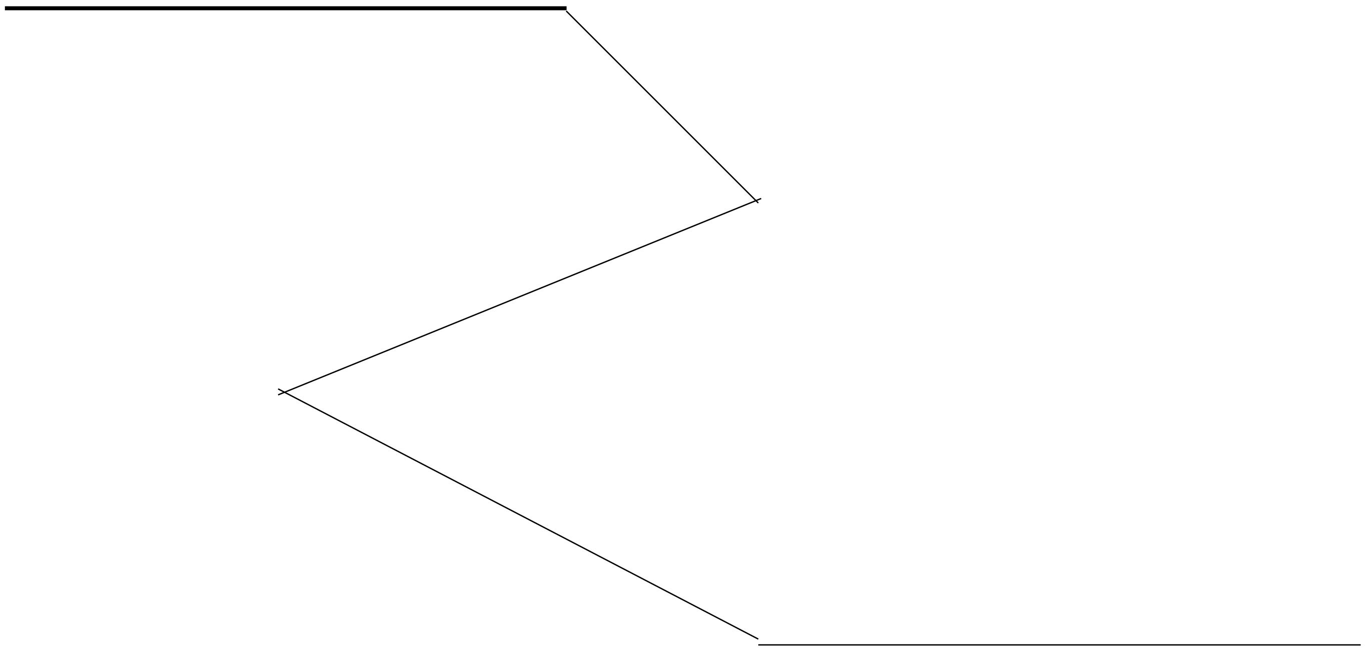
- Often ,many bodies with complex geometries can be broken into simple shapes , of which the centroid are easy to locate.

- Composites bodies can be divided into four types:



1- composite line

- Composite line contains group of lines connected together

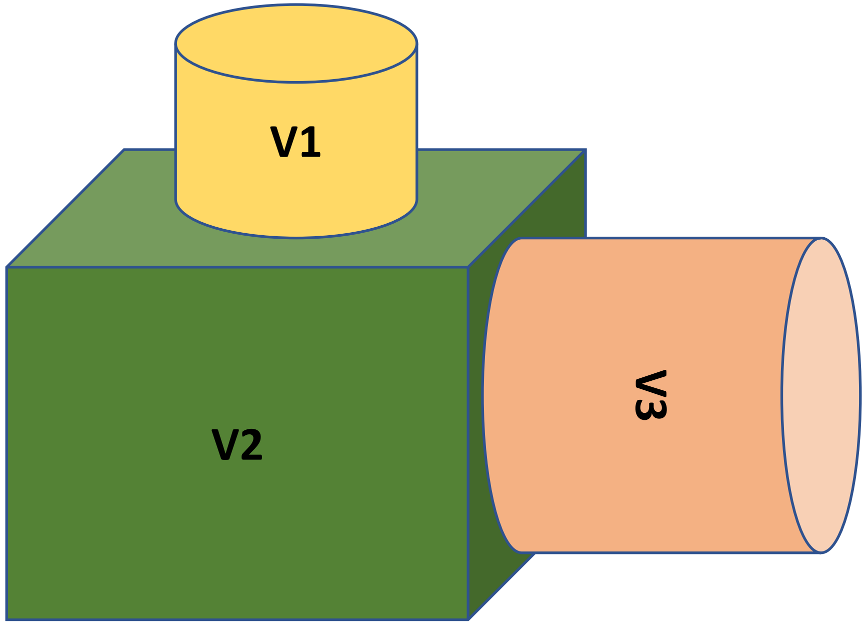


2- composite area

- Contains group of different shapes with different areas

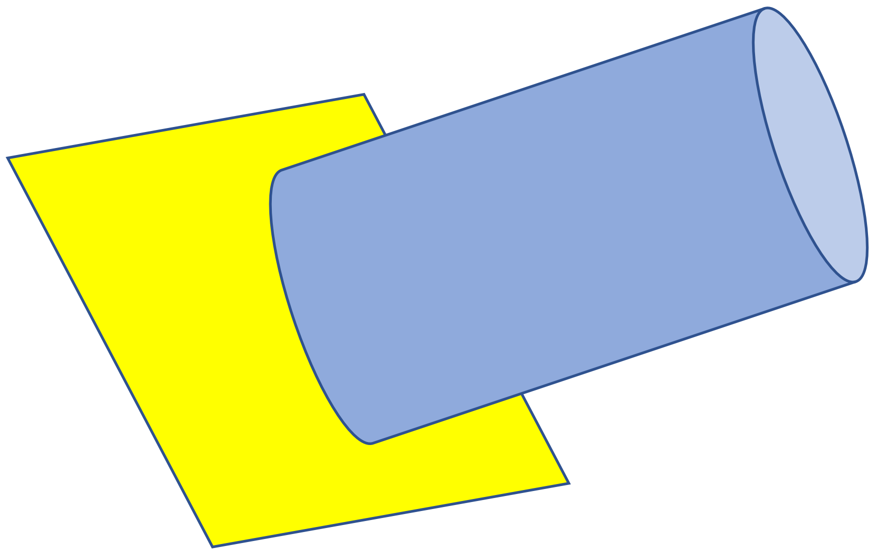
3- Composite volume

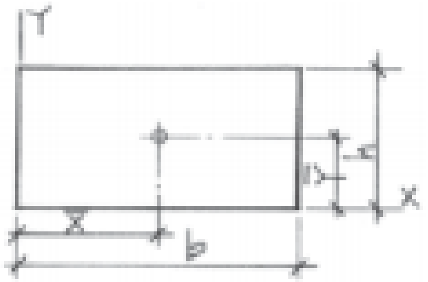
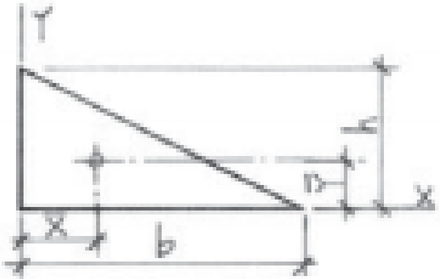
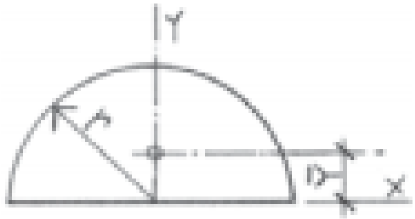
- Contains different shapes with different volumes

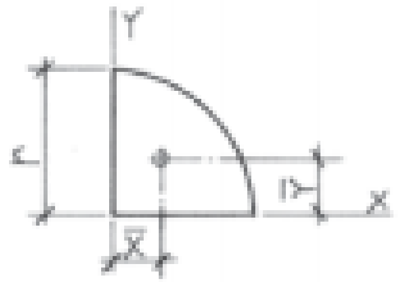
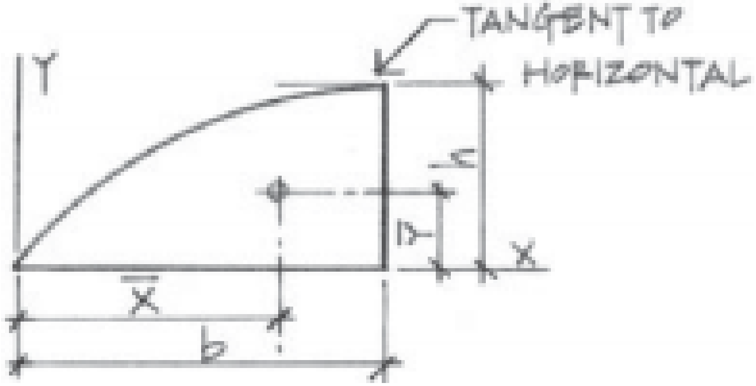
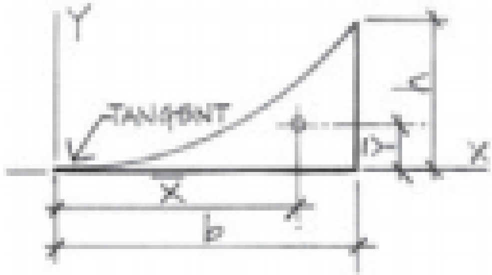


4- Composite mass

- Contains a mix of areas , lines , volumes



Shape	Drawing	$\bar{x}$	$\bar{y}$	Area
Rectangle		$b/2$	$h/2$	bh
Triangle		$b/3$	$h/3$	$bh/2$
Semicircle		0	$4r/3\pi$	$\pi r^2/2$

Quarter Circle		$4r/3\pi$	$4r/3\pi$	$\pi r^2/4$	
Parabolic Segment		$5b/8$	$2h/5$	$2bh/3$	
Complement of a Parabolic Segment		$3b/4$	$3h/10$	$bh/3$	

CENTROID LOCATIONS FOR A FEW COMMON VOLUMES

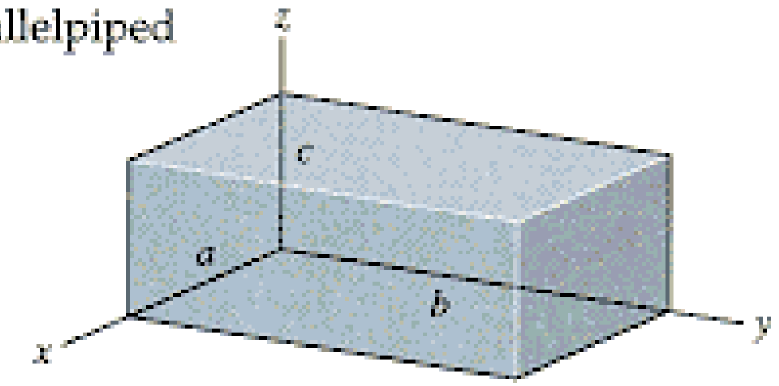
Rectangular parallelpiped

$V = abc$

$x_C = \frac{a}{2}$

$y_C = \frac{b}{2}$

$z_C = \frac{c}{2}$



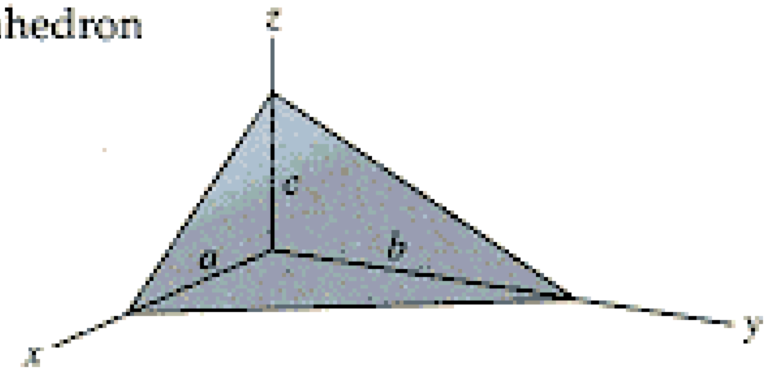
Rectangular tetrahedron

$V = \frac{abc}{6}$

$x_C = \frac{a}{4}$

$y_C = \frac{b}{4}$

$z_C = \frac{c}{4}$



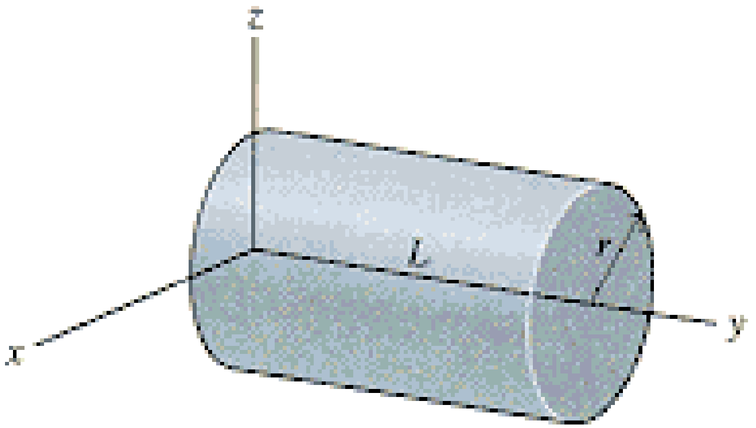
Circular cylinder

$V = \pi r^2 L$

$x_C = 0$

$y_C = \frac{L}{2}$

$z_C = 0$



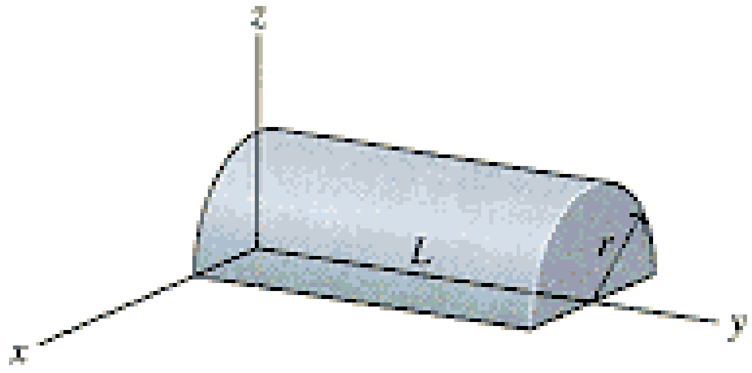
Semicylinder

$V = \frac{\pi r^2 L}{2}$

$x_C = 0$

$y_C = \frac{L}{2}$

$z_C = \frac{4r}{3\pi}$



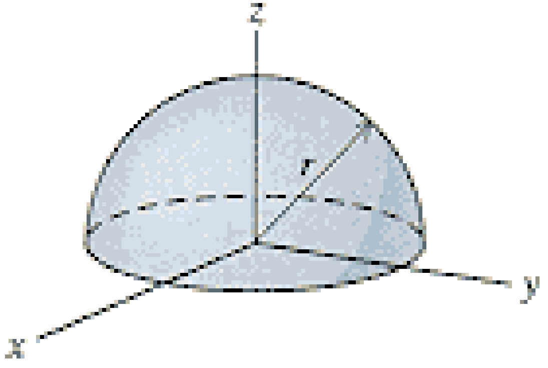
Hemisphere

$$V = \frac{2\pi r^3}{3}$$

$$x_C = 0$$

$$y_C = 0$$

$$z_C = \frac{3r}{8}$$



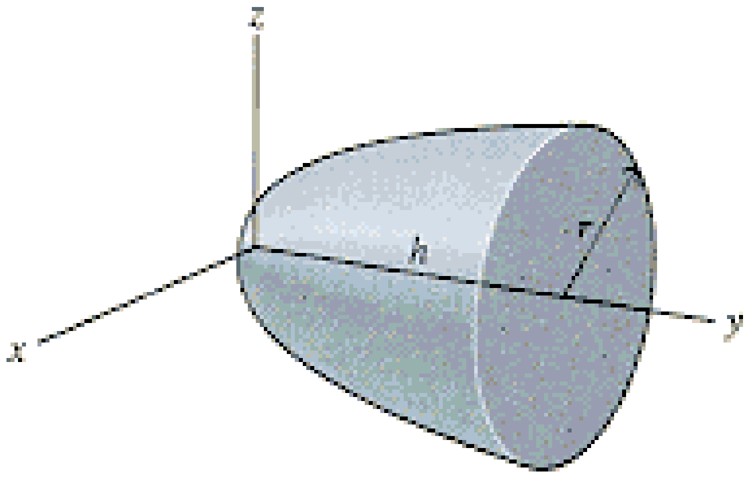
Paraboloid

$$V = \frac{\pi r^2 h}{2}$$

$$x_C = 0$$

$$y_C = \frac{2h}{3}$$

$$z_C = 0$$



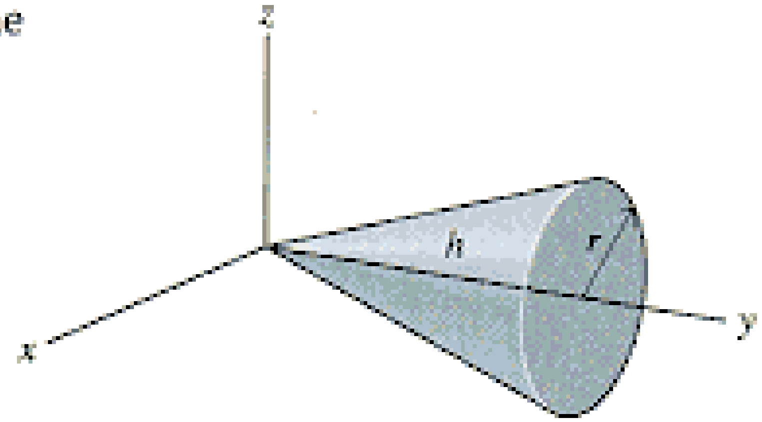
Right circular cone

$$V = \frac{\pi r^2 h}{3}$$

$$x_C = 0$$

$$y_C = \frac{3h}{4}$$

$$z_C = 0$$



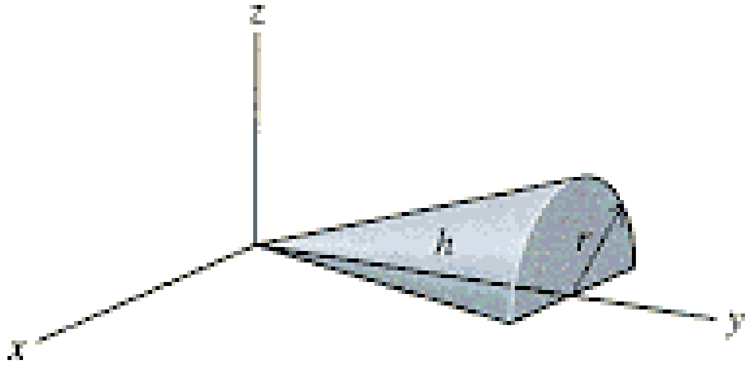
Half cone

$$V = \frac{\pi r^2 h}{6}$$

$$x_C = 0$$

$$y_C = \frac{3h}{4}$$

$$z_C = \frac{r}{\pi}$$



How to solve centroid questions ?

- First ,we will create a table to fill it with the suitable information.
- for example , if we have a composite line system with four lines. Then, the table will be :

	X	Y	Z	L	X*L	Y*L	Z*L
1							
2							
3							
4							
				ΣL	ΣXL	ΣYL	

- if we have a composite area system with four shapes. Then, the table will be :

	X	Y	Z	A	X*A	Y*A	Z*A
1							
2							
3							
4							
				ΣA	ΣXA	ΣYA	

if we have a composite volume system with four shapes. Then, the table will be :

	X	Y	Z	V	X*V	Y*V	Z*V
1							
2							
3							
4							
				ΣV	ΣXV	ΣYV	

if we have a composite mass system with four shapes. Then, the table will be :

	X	Y	Z	m	X*m	Y*m	Z*m
1							
2							
3							
4							
				Σm	ΣXm	ΣYm	

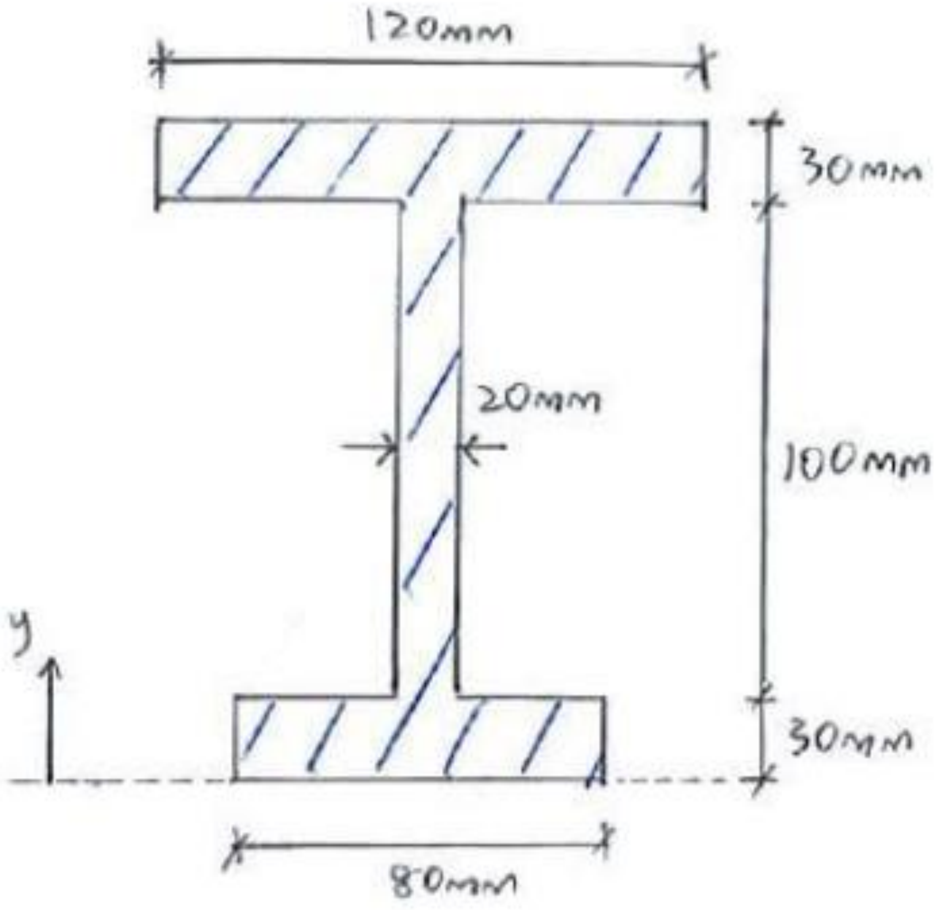
The final formula for the centroid

- X= $\frac{\Sigma(XL, XA, XV, XM)}{\Sigma(L, A, V, M)}$

- Y= $\frac{\Sigma(YL, YA, YV, YM)}{\Sigma(L, A, V, M)}$

- Z= $\frac{\Sigma(ZL, ZA, ZV, ZM)}{\Sigma(L, A, V, M)}$

Question 1

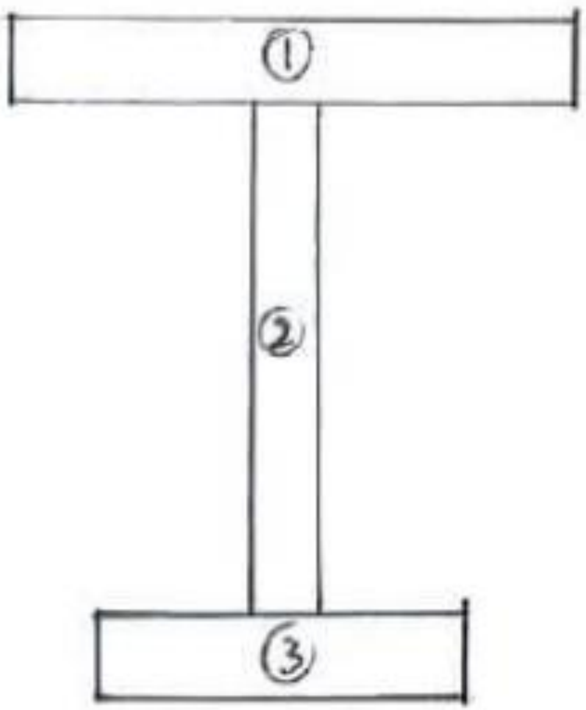


Find the centroid $\bar{y}$ of the unsymmetrical I-section with respect to its base.

SHAPE	A' (mm²)	$\bar{y}'$ (mm)	$A'\bar{y}'$ (mm³)
①	$120 \times 30 = 3{,}600$	$130 + \frac{30}{2} = 145$	522,000
②	$20 \times 100 = 2{,}000$	$30 + \frac{100}{2} = 80$	160,000
③	$80 \times 30 = 2{,}400$	$\frac{30}{2} = 15$	36,000

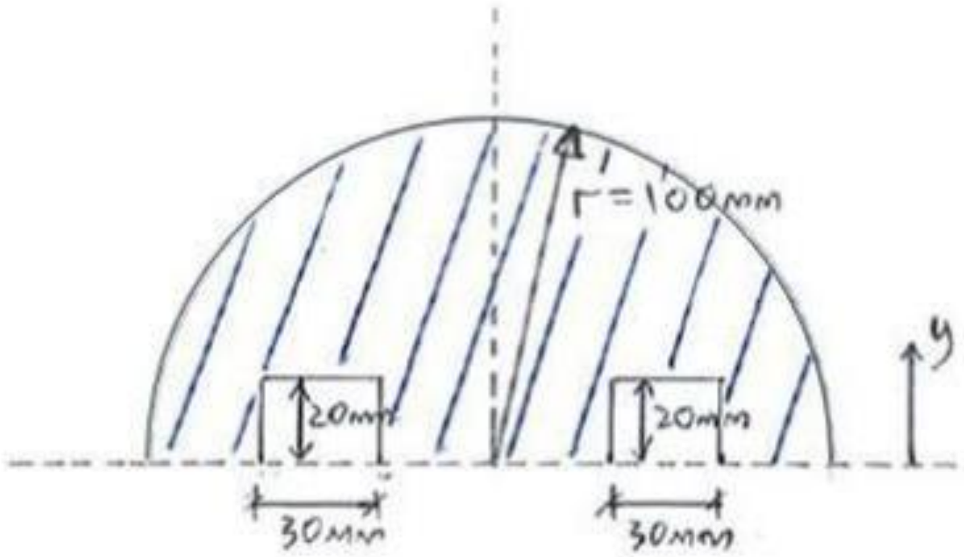
$\Sigma A' = 8{,}000 \text{ mm}^2$ //

$\Sigma A'\bar{y}' = 718{,}000 \text{ mm}^3$ //



$$\bar{y} = \frac{\Sigma A'\bar{y}'}{\Sigma A'} = \frac{718{,}000 \text{ mm}^3}{8{,}000 \text{ mm}^2} = 89.75 \text{ mm}$$ //

Question 2



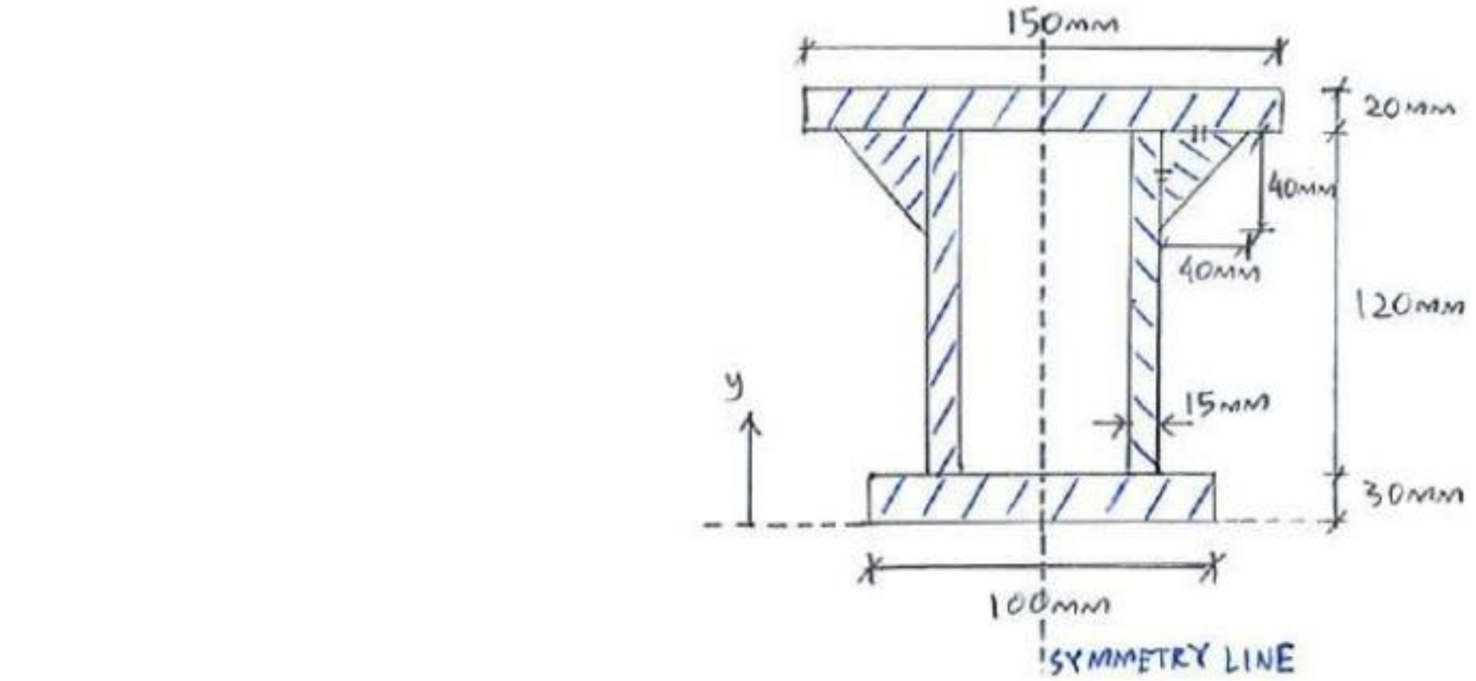
Calculate the centroid $\bar{y}$ of the geometry with respect to its base.

SHAPE	A' (mm^2)	$\bar{y}'$ (mm)	$A'\bar{y}'$ (mm^3)
①	$\frac{\pi(100)^2}{2} = 15{,}707.96$	$\frac{4r}{3\pi} = \frac{4(100)}{3\pi} = 42.44$	$666{,}666.67$
②	$-30 \times 20 = -600$	$\frac{20}{2} = 10$	$-6{,}000$
③	$-30 \times 20 = -600$	$\frac{20}{2} = 10$	$-6{,}000$

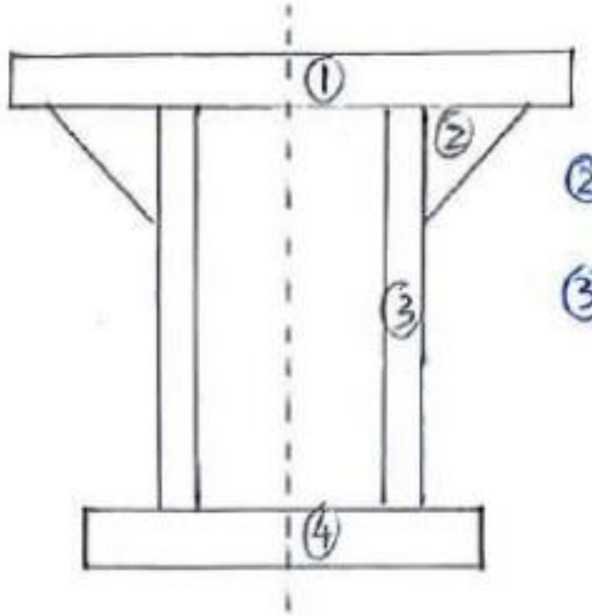
$\Sigma A' = \underline{14{,}507.96 \, mm^2}$ //

$\Sigma A'\bar{y}' = \underline{654{,}666.67 \, mm^3}$ //

Question 3



Calculate the centroid $\bar{y}$ of the geometry with respect to its base.



2 = ▽ ▽
3 = ▯▯

GROUPING COMMON GEOMETRIES WITH SAME $\bar{y}$ CENTROIDS.

SHAPE	A' (mm^2)	$\bar{y}'$ (mm)	$A'\bar{y}'$ (mm^3)
1	$150 \times 20 = 3{,}000$	$30 + 120 + \frac{20}{2} = 160$	480,000
2	$2 \times \left(\frac{40 \times 40}{2}\right) = 1{,}600$	$30 + 120 - \frac{40}{3} = 136.67$	218,666.67
3	$2 \times (15 \times 120) = 3{,}600$	$30 + \frac{120}{2} = 90$	324,000
4	$100 \times 30 = 3{,}000$	$\frac{30}{2} = 15$	45,000

$\Sigma A' = \underline{11{,}200 \; mm^2}$

$\Sigma A'\bar{y}' = \underline{1{,}067{,}666.67 \; mm^3}$