Implicit differentiation

The Implicit differentiation is used to differentiate the equations that do not have the value of y in terms of (x)

Ex.1/ find $\frac{dy}{dx}$ for $x^2+xy+2y^2=1$

Sol:- $2x+\left[x\frac{dy}{dx}+y(1)\right]+4y\frac{dy}{dx}=0$

$$\frac{dy}{dx}(x+4y)=-y-2x$$

$$\frac{dy}{dx}=\frac{-y-2x}{x+4y}$$

Ex2/ find $\frac{d^2y}{dx^2}$ if $2x^3-3y^2=7$

Sol:- $6x^2-6y\frac{dy}{dx}=0$

$$6y\frac{dy}{dx}=6x^2$$

$$\frac{dy}{dx}=\frac{6x^2}{6y}=\frac{x^2}{y}$$

$$\frac{d^2y}{dx^2}=\frac{y(2x)-x^2\left(\frac{dy}{dx}\right)}{y^2}$$

$$= \frac{2xy}{y^2} - \frac{x^2\left(\frac{dy}{dx}\right)}{y^2}$$

$$= \frac{2x}{y^2} - \frac{x^2}{y^2}\left(\frac{dy}{dx}\right)$$

$$= \frac{2x}{y} - \frac{x^2}{y^2}\left(\frac{x^2}{y}\right)$$

$$= \frac{2x}{y} - \frac{x^4}{y^3}$$

$Ex_3/$ $x^2y^2 = x^2 + y^2$

$$\left[x^2\, 2y\frac{dy}{dx} + y^2(2x)\right] = 2x + 2y\frac{dy}{dx}$$

$$2yx^2\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - 2xy^2$$

$$\frac{dy}{dx}(2yx^2 - 2y) = 2x - 2xy^2$$

$$\frac{dy}{dx} = \frac{2x - 2xy^2}{2x^2y - 2y} = \frac{2(x - xy^2)}{2(yx^2 - y)}$$

$$\frac{dy}{dx} = \frac{x - xy}{x^2y - y}$$

Ex / find $\frac{dy}{dx}$ for $(x+y)^3+(x-y)^3=x^4+y^4$ at $A(2,1)$

Sol:- $3(x+y)(1+\frac{dy}{dx})+3(x-y)(1-\frac{dy}{dx})$

$=4x^3+4y^3\frac{dy}{dx}$

$3(x+y)^2+3(x+y)^2\frac{dy}{dx}+3(x-y)-3(x-y)^2\frac{dy}{dx}$

$=4x^3+4y^3\frac{dy}{dx}$

$3(x+y)^2\frac{dy}{dx}-3(x-y)\frac{dy}{dx}-4y^3\frac{dy}{dx}$

$=4x^3-3(x+y)^2-3(x-y)^2$

$\frac{dy}{dx}\left[3(x+y)^2-3(x-y)^2-4y^3\right]$

$=4x^3-3(x+y)^2-3(x-y)^2$

$\therefore \frac{dy}{dx}=\frac{4x^3-3(x+y)^2-3(x-y)^2}{3(x+y)^2-3(x-y)^2-4y^3}$

$\frac{dy}{dx}=\frac{4(2)^2-3(2+1)-3(2-1)}{3(2+1)^2-3(2-1)^2-4(1)}$

$\frac{dy}{dx}=\frac{1}{10}$

Ex/ $\sin xy = \tan^2 x^2 - \sin(x+y) + 3\pi$

$\cos xy\left[x\frac{dy}{dx} + y(1)\right] = 2\tan x^2 \sec^2 . 2x$

$\cos(x+y)\left(1 + \frac{dy}{dx}\right)$

$x\cos xy\frac{dy}{dx} + y\cos xy = 4x\tan^2 x^2 \sec^2 x^2$

$-\cos(x+y) - \cos(x+y)\frac{dy}{dx}$

$\frac{dy}{dx}\left(x\cos xy + \cos(x+y)\right) = 4x\tan^2 x^2 \sec^2 x^2$

$+ y\cos xy + \cos(x+y)$

$$\frac{dy}{dx} = \frac{4x\tan^2 x^2 \sec^2 x^2 + y\cos xy + \cos(x+y)}{x\cos xy + \cos(x+y)}$$

Ex/ find $\sqrt{x.y} + 1 = y$

Sol:- $(xy)^{\frac{1}{2}} + 1 = y$

$\frac{1}{2}(x.y)^{-\frac{1}{2}}\left(x \cdot \frac{dy}{dx} + y(1)\right) + 0 = \frac{dy}{dx}$

$\frac{1}{2}x(xy)^{-\frac{1}{2}}\frac{dy}{dx} + \frac{1}{2}y(xy)^{-\frac{1}{2}} = \frac{dy}{dx}$

$\frac{dy}{dx}\left(\frac{1}{2}x(xy)^{-\frac{1}{2}} - 1\right) = -\frac{1}{2}y(xy)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{-\frac{1}{2} y (xy)^{-\frac{1}{2}}}{\frac{1}{2} x (xy)^{-\frac{1}{2}} - 1}.$$

$$= \frac{-0.5y / \sqrt{xy}}{0.5x / \sqrt{xy} - 1}.$$