قوانين مشتقات الدوال المثلثية

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Ex3/ $y = \cos x \tan 3x$

$$\frac{dy}{dx} = \cos x\,(\sec^2 3x \cdot 3) + \tan 3x\,(-\sin x)$$

$$= 3\cos x \sec^2 3x - \sin x \tan 3x$$

$$= \lim_{\Delta x \to 0} \frac{-\sin^2 \Delta x}{\Delta x(\cos \Delta x + 1)} = -\lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \cdot \frac{\sin \Delta x}{\cos \Delta x - 1}$$

$$= -1 \left(\frac{0}{1+1}\right) = 0$$

$$\frac{dy}{dx} = \sin(0) + \cos x(1)$$

$$= 0 + \cos x = \cos x.$$

$$\therefore \frac{d}{dx} \sin x = \cos x$$ prove.

Ex. / find $\frac{dy}{dx}$ for the function $y = \sin 3x$

Sol:- $y = \sin 3x$

$$y' = \cos 3x \cdot 3$$

∴ مشتقة الدالة الأصلية = مشتقة الدالة * مشتقة الأس.

$3x \to 3$; $\sin 3x \to \cos 3x$

Ex 2/ $y = 2\sin(x^2 - 1)^2$

$$\frac{dy}{dx} = 2\cos(x^2-1)^2 \left(2(x^2-1)2x\right)$$

$$= 8x(x^2-1) * \cos(x^2-1)^2$$

Derivatives of trigonometric functions:

1. $\frac{d}{dx} \sin x$

$$y = \sin x \rightarrow f(x) = \sin x$$

$$f(x+\Delta x) = \sin(x+\Delta x)$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin x(\cos \Delta x - 1) + \cos x \sin \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \sin x \left(\frac{\cos \Delta x - 1}{\Delta x}\right) + \lim_{\Delta x \to 0} \cos x \frac{\sin \Delta x}{\Delta x}$$

Now: $\lim_{\Delta x \to 0} \frac{\cos \Delta x - 1}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cos \Delta x - 1}{\Delta x} \times \frac{\cos \Delta x + 1}{\cos \Delta x + 1}$

$$= \lim_{\Delta x \to 0} \frac{\cos^2 \Delta x - 1}{\Delta x (\cos \Delta x + 1)}$$

Ex4 / $y = \tan\sqrt{3x}$

$$\frac{dy}{dx} = \sec^2(3x)^{\frac{1}{2}} \cdot \frac{1}{2}(3x)^{\frac{-1}{2}} \cdot 3$$

$$= \frac{3}{2\sqrt{3x}} \cdot \sec^2\sqrt{3x}$$

Ex 5 / $y = \sin^2\left(\frac{1}{x}\right)$

$$y = \left(\sin\left(\frac{1}{x}\right)\right)^2$$

$$= 2\left(\sin\left(\frac{1}{x}\right)\right)\left(\cos\frac{1}{x}\right) \cdot \frac{-1}{x^2}$$

$$= \frac{-2}{x^2} \sin\frac{1}{x} \cos\frac{1}{x}$$

Ex 6 / $y = \sin$

$$\frac{dy}{dx} = \cos(7x^2 + 4x + 1) * (14x + 4)$$

Rule $\boxed{\frac{\sin x}{x} = 1}$