Al-Mustaqbal University College

فيزياء طبية-مرحلة اولى-ميكانيك

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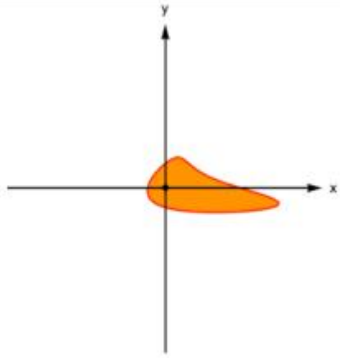
Torque and rotational motion

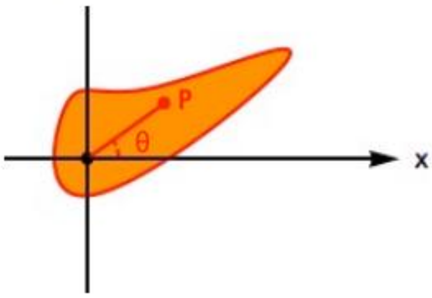
rotations of a rigid body

➜ suppose we have a body which rotates about some axis



➜ we can define its orientation at any moment by an angle, θ

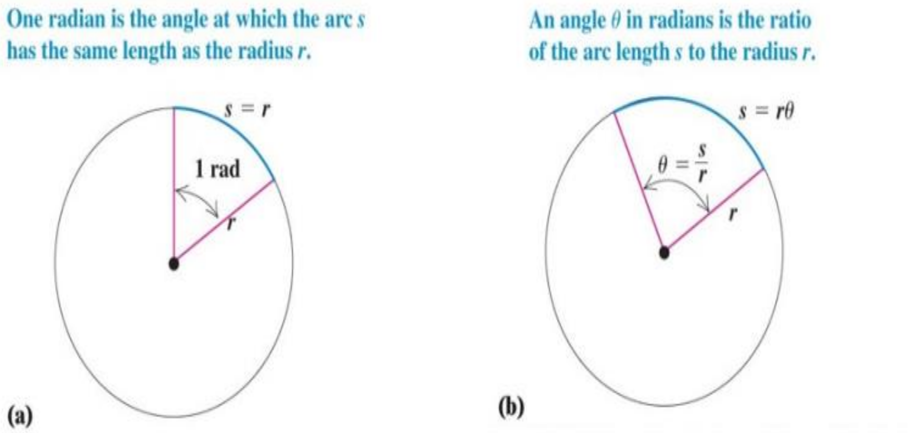
(any point P will do)



radians

➜ measuring θ in degrees turns out to be a poor choice

➜ **radians** are a more natural choice of angular unit



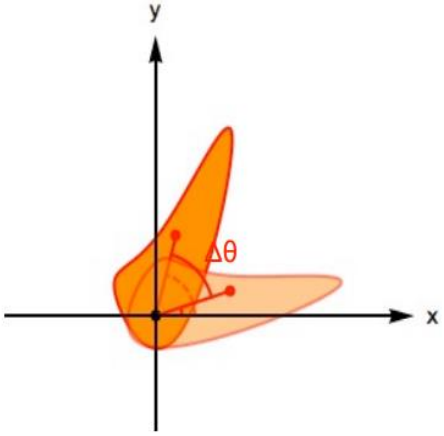
$$1\,\text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

angular velocity

➜ describe the rate of rotation by the change in angle in a given time

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

(notice, just like linear motion but with x→θ)



angular acceleration

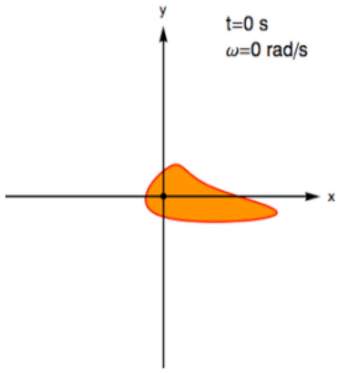
➜ suppose the rate of rotation changes - we need angular acceleration

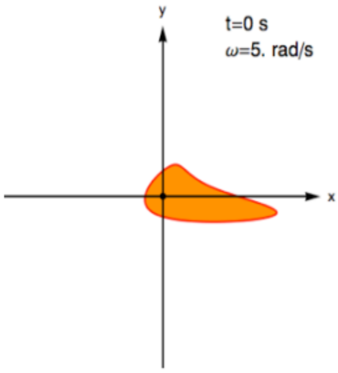
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

(notice, just like linear motion but with v→ω)

positive constant α

negative constant α
begins with positive ω





angular motion vs. linear motion

➜ the analogy between angular motion & linear motion is strong

➜ for constant acceleration we have

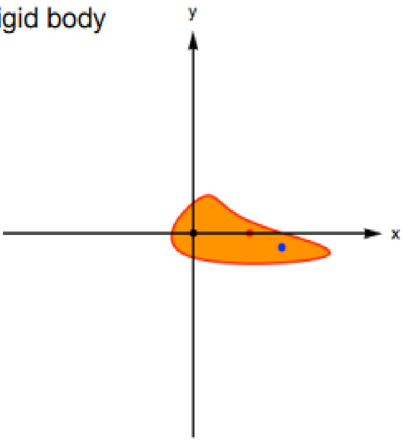
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

➜ for constant angular acceleration we have

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

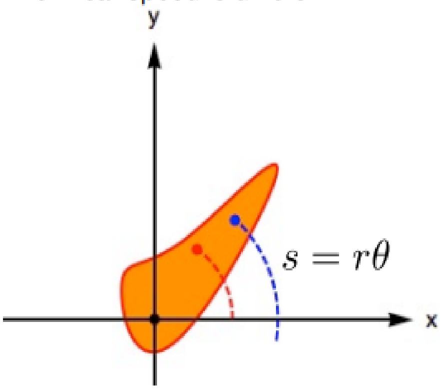
motion of points in a rigid body

➜ consider the motion of a couple of points within the rigid body



the blue point at a large radius travels further in the same time than the red point

so although the angular speed is the same, the linear speed is different



$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$$

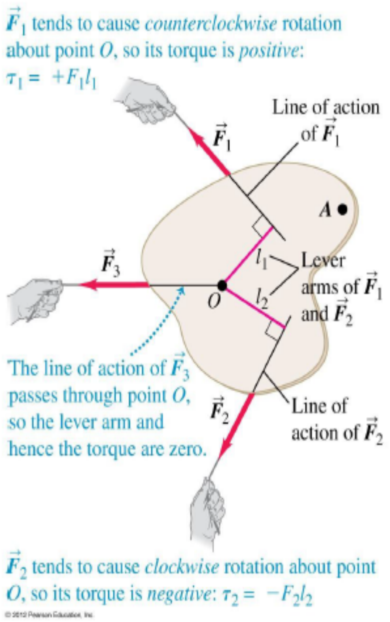
Force vs. Torque

- Forces cause accelerations
- What cause angular accelerations?
- There are three factors that determine the effectiveness of the force:
  - The *magnitude* of the force
  - The *position* of the application of the force
  - The *angle* at which the force is applied

Torque Definition

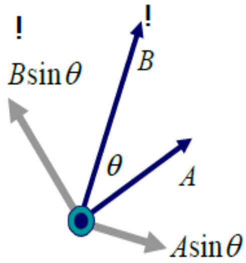
- Torque, τ, is the tendency of a force to rotate an object about some axis
- Let **F** be a force acting on an object, and let **r** be a position vector from a rotational center to the point of application of the force, with **F** perpendicular to **r**. The magnitude of the torque is given by

$$\tau = rF$$



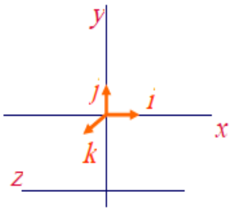
Cross Product

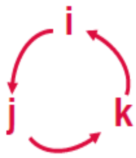
$$\vec{C} = \vec{A} \times \vec{B}$$



- The cross product of two vectors says something about how perpendicular they are.
- Magnitude: $|\vec{C}| = |\vec{A} \times \vec{B}| = AB\sin\theta$
  - θ is smaller angle between the vectors
  - Cross product of any parallel vectors = zero
  - Cross product is maximum for perpendicular vectors
  - Cross products of Cartesian unit vectors:

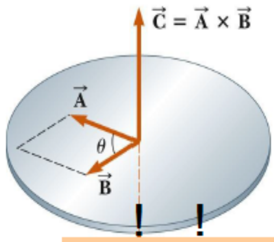
$$\hat{i} \times \hat{j} = \hat{k};\ \hat{i} \times \hat{k} = -\hat{j};\ \hat{j} \times \hat{k} = \hat{i}$$
$$\hat{i} \times \hat{i} = 0;\ \hat{j} \times \hat{j} = 0;\ \hat{k} \times \hat{k} = 0$$





Cross Product

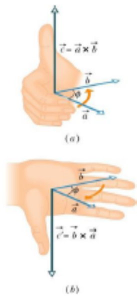
- Direction: C perpendicular to both A and B (right-hand rule)
  - Place A and B tail to tail
  - Right hand, not left hand
  - Four fingers are pointed along the first vector A
  - "sweep" from first vector A into second vector B through the smaller angle between them
  - Your outstretched thumb points the direction of C
- First practice $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$?



$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$?

Right-hand rule

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$



Torque Units and Direction

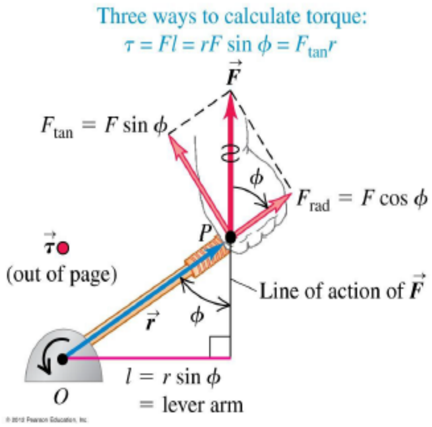
- The SI units of torque are N·m
- Torque is a vector quantity

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Torque **magnitude** is given by

$$\tau = rF\sin\phi = Fl$$

- Torque will have **direction**
  - If the turning tendency of the force is counterclockwise, the torque will be positive
  - If the turning tendency is clockwise, the torque will be negative



Net Torque

- The force $\mathbf{F}_1$ will tend to cause a counterclockwise rotation about O
- The force $\mathbf{F}_2$ will tend to cause a clockwise rotation about O
- $\Sigma\tau = \tau_1 + \tau_2 + \tau_3 = F_1 l_1 - F_2 l_2$
- If $\Sigma\tau \neq 0$, starts rotating
- If $\Sigma\tau = 0$, rotation rate does not change