Al-Mustaqbal University College

Department of Medical Instrumentation Technologies

Mathematics II / Second Stage

By

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Multiple Integrals

Double integrals:

Example: Evaluate the integral

$$\int_0^3 \int_1^2 (1 + 8xy)\,dy\,dx$$

Solution:

$$= \int_0^3 \int_1^2 (1 + 8xy)\,dy\,dx = \int_0^3 \left[y + \frac{8xy^2}{2} \right]_{y=1}^{y=2} dx$$

$$= \int_0^3 [y + 4xy^2]_{y=1}^{y=2}\, dx$$

$$= \int_0^3 [(2 + (4)(x)(2)^2) - (1 + (4)(x)(1)^2)]\, dx$$

$$= \int_0^3 [(2+16x)-(1+4x)]\,dx = \int_0^3 (2+16x-1-4x)dx$$

$$= \int_0^3 (1+12x)dx = \left[x+\frac{12x^2}{2}\right]_0^3$$

$$= [x+6x^2]_0^3 = (3+(6)(3)^2-0)$$

$$= 3+54 = 57$$

Example: Evaluate the integral

$$\int_0^3 \int_0^2 (4-y^2)dydx$$

Solution:

$$\int_0^3 \int_0^2 (4-y^2)dydx = \int_0^3 \left[4y-\frac{y^3}{3}\right]_{y=0}^{y=2} dx$$

$$= \int_0^3 \left((4)(2)-\frac{(2)^3}{3}\right)dx$$

$$= \int_0^3 \left(8-\frac{8}{3}\right)dx = \int_0^3 \left(\frac{24-8}{3}\right)dx$$

$$= \int_0^3 \left(\frac{16}{3}\right)dx = \frac{16}{3}\int_0^3 dx$$

$$= \frac{16}{3}[x]_0^3 = \left(\frac{16}{3}\right)(3) = 16$$

Areas of bounded regions in the plane:

The area of a closed, bounded plane region R is:

$$A = \iint_R dA$$

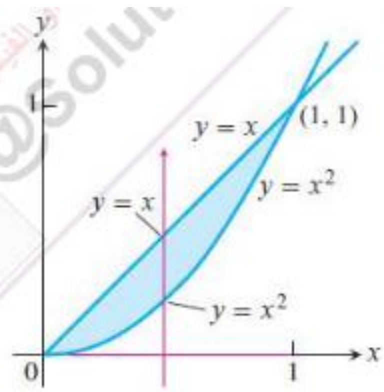
Example: Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

Solution: we sketch the region, nothing where the two curves intersect and calculate the area as:

$$A = \int_0^1 \int_{x^2}^{x} dy dx = \int_0^1 [y]_{x^2}^{x} dx$$

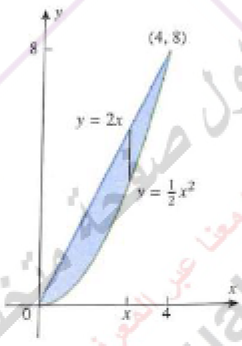
$$= \int_0^1 (x - x^2)\, dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$



Example: Use a double integral to find the area of the region R enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$

Solution:



(a)

$$area = \int_0^4 \int_{x^2/2}^{2x} dydx = \int_0^4 [y]_{x^2/2}^{2x}\ dx$$

$$= \int_0^4 \left(2x - \frac{x^2}{2}\right) dx$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{6}\right]_0^4 = \left[x^2 - \frac{x^3}{6}\right]_0^4$$

$$= \left[(4)^2 - \frac{(4)^3}{6}\right] = 16 - \frac{64}{6}$$

$$= \frac{96 - 64}{6} = \frac{32}{6} = \frac{16}{3}$$