Electricity and Magnetism

Lecture Four

Flux of the Electric Field and Gauss's Law

Dr. Mohammed Hashim Abbas

first stage

Department of medical physics

Al-Mustaqbal University-College

2021-2022

Outline

1. **Flux of the Electric Field**
2. **Gauss' Law**
3. **References**

1. Flux of the Electric Field

Electric flux **is the rate of flow of the electric field through a given area** (Fig. 1). Electric flux is proportional to the number of electric field lines going through a virtual surface.

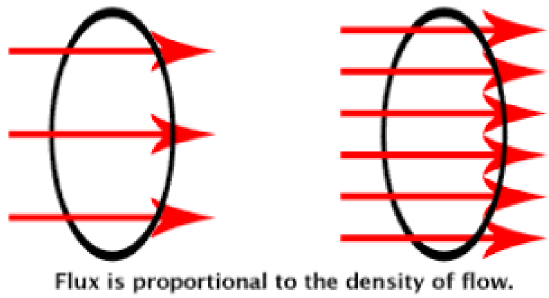


Figure 1: Electric Flux: Electric flux visualized. The ring shows the surface boundaries. The red arrows for the electric field lines.

Flat Surface, Uniform Field: We begin with a flat surface (Fig. 2) with area A in a uniform electric field $\vec{E}$. The total flux Φ is then:

$$\Phi = \int \vec{E}.d\vec{A} \quad (total\ flux)$$

$$\Phi = \int (E\ cos\theta)\ d\vec{A} \quad (total\ flux)$$

When the electric field is uniform and the surface is flat:

$$\Phi = (E\ cos\theta)A \quad (\text{uniform field, flat surface})$$

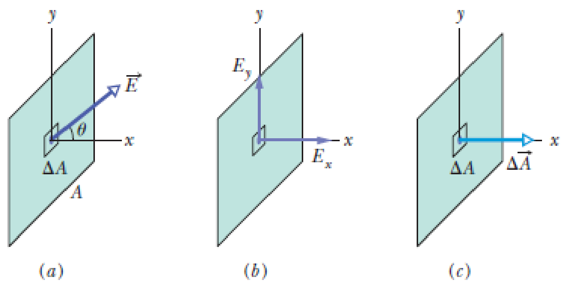


Figure 2: An electric field vector pierces a small square patch on a flat surface.

Example: A nonuniform electric field given by $\vec{E} = 3x\hat{\imath} + 4\hat{\jmath}$ pierces the Gaussian Square area $d\vec{A}$ with length 2 m. what is the flux throw the surface when x= 3m point in the positive direction of the x axis.?

Solution:

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}) \cdot (dA\hat{\mathrm{i}})$$

$$= \int [(3.0x)(dA)\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} + (4.0)(dA)\hat{\mathrm{j}} \cdot \hat{\mathrm{i}}]$$

$$= \int (3.0x\, dA + 0) = 3.0 \int x\, dA.$$

The integral $\int dA$ gives us the area A = 4 m^2 of the surface:

$$\Phi_r = 3.0 \int (3.0)\, dA = 9.0 \int dA.$$

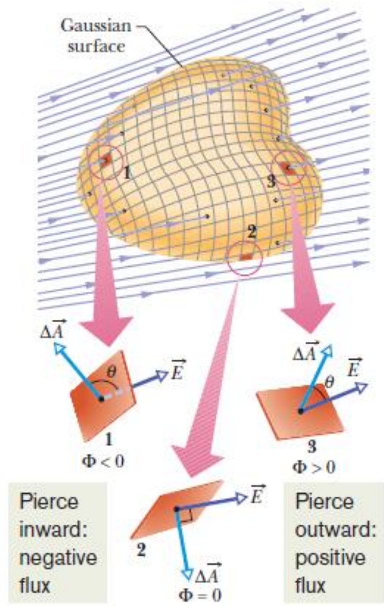
$$\Phi_r = (9.0\ \mathrm{N/C})(4.0\ \mathrm{m}^2) = 36\ \mathrm{N \cdot m^2/C}.$$

Closed Surface. Let's use the closed surface in (Fig.3) that sits in a nonuniform electric field. To use Gauss' law to relate flux and charge, we need a closed surface.

An inward field is negative flux. An outward field is positive flux

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux).}$$

The loop on the integral sign indicates that we must integrate over the entire closed surface, to get the net flux through the surface (flux might enter on one side and leave on another side).



Figur 3: A Gaussian surface of arbitrary shape immersed in an electric field.

2. Gauss' Law

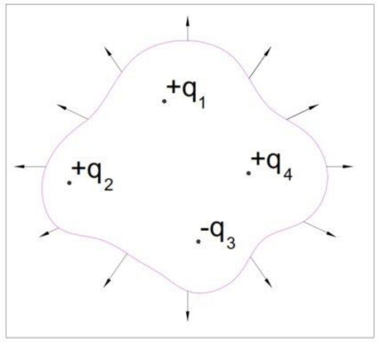
Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface. It tells us that:

$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss' law).}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{(Gauss' law).}$$

The net charge q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero.

If q_{enc} is positive, the net flux is outward; if q_{enc} is negative, the net flux is inward.



q_{enc} in figure above given below:

$$q_{enc} = q_1 + q_2 - q_3 + q_4$$

3. Refrences

Walker, Jearl, Robert Resnick, and David Halliday. Halliday and resnick fundamentals of physics. Wiley, 2014.