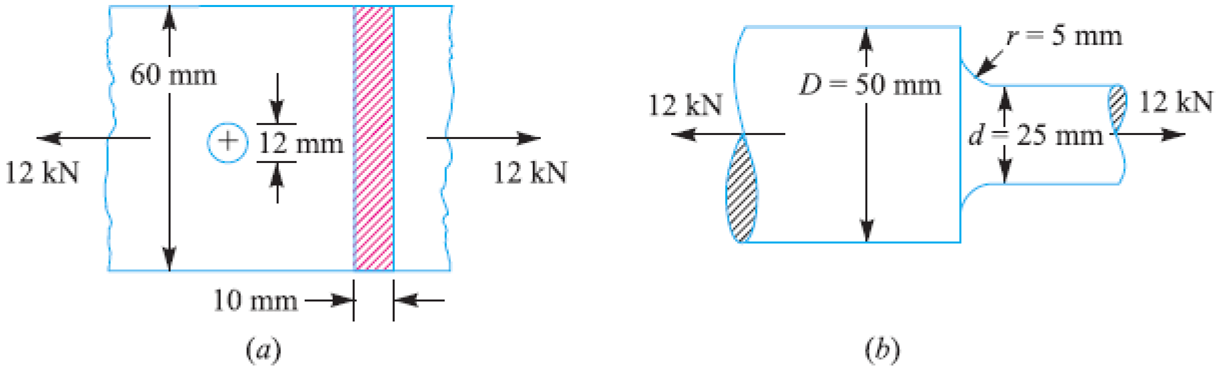
**Problem 2**

Find the maximum stress induced in the following cases taking stress concentration into account: 1. A rectangular plate 60 mm ×10 mm with a hole 12 diameter as shown in Figure (a) and subjected to a tensile load of 12 kN. 2. A stepped shaft as shown in Figure (b) and carrying a tensile load of 12 kN.




Solution

**Case 1.** $b$ = 60 mm ; $t$ = 10 mm ; $d$ = 12 mm ; $W$ = 12 kN = 12 × 103 N We know that cross-sectional area of the plate,

$$A = (b - d)\,t = (60 - 12)\,10 = 480 \text{ mm}^2$$

$$\text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{480} = 25 \text{ N/mm}^2 = 25 \text{ MPa}$$

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0.2$$

From Table 1, we find that for $d$ / $b$ = 0.2, theoretical stress concentration factor, $K_t$ = 2.5

∴ Maximum stress = $K_t$ × Nominal stress = 2.5 × 25 = 62.5 MPa

**Case 2.**

$D$ = 50 mm ; $d$ = 25 mm ; $r$ = 5 mm ; $W$ = 12 kN = 12 × 103 N We know that cross-sectional area for the stepped shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(25)^2 = 491 \text{ mm}^2$$

$$\text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{491} = 24.4 \text{ N/mm}^2 = 24.4 \text{ MPa}$$

Ratio of maximum diameter to minimum diameter,

$$D/d = 50/25 = 2$$

Ratio of radius of fillet to minimum diameter,

$$r/d = 5/25 = 0.2$$

From Table 3, we find that for $D/d = 2$ and $r/d = 0.2$, theoretical stress concentration factor,

$$K_t = 1.64.$$

$\therefore$ Maximum stress = $K_t$ × Nominal stress = 1.64 × 24.4 = 40 MPa

**Fatigue Stress Concentration Factor**

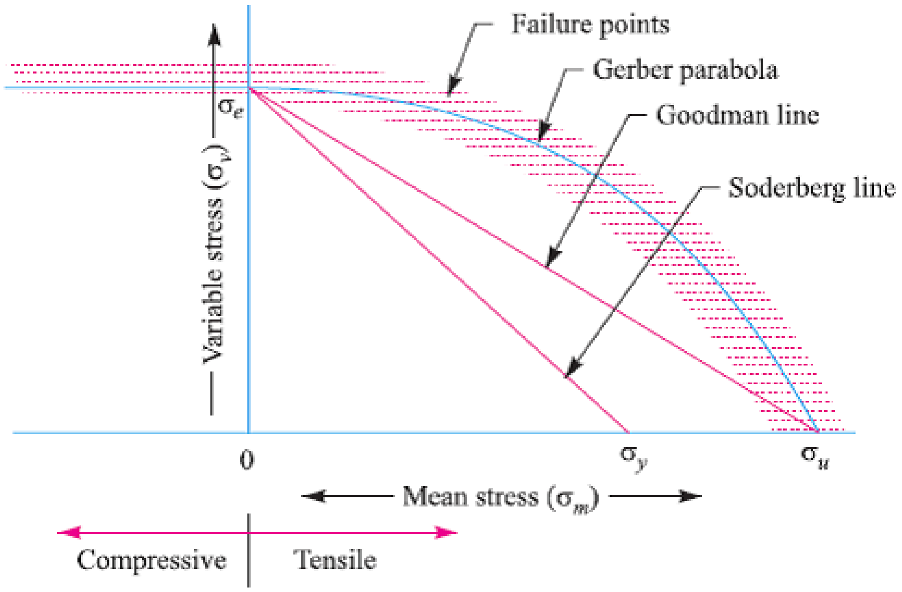
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

**Combined Steady and Variable Stress**

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Figure as functions of variable stress ($\sigma_v$) and mean stress ($\sigma_m$).

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

**1.** Gerber method, **2.** Goodman method, and **3.** Soderberg method.




## Gerber Method for Combination of Stresses

The relationship between variable stress ($\sigma_v$) and mean stress ($\sigma_m$) for axial and bending loading for ductile materials are shown in Figure. The point $\sigma_e$ represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point $\sigma_u$ represents the static ultimate strength corresponding to $\sigma_v = 0$.

$$\sigma_v = \sigma_e \left[ \frac{1}{F.S.} - \left( \frac{\sigma_m}{\sigma_u} \right)^2 F.S. \right]$$

or
$$\frac{1}{F.S.} = \left( \frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

Where $F.S.$ = Factor of safety,

$\sigma_m$ = Mean stress (tensile or compressive),

$\sigma_u$ = Ultimate stress (tensile or compressive), and

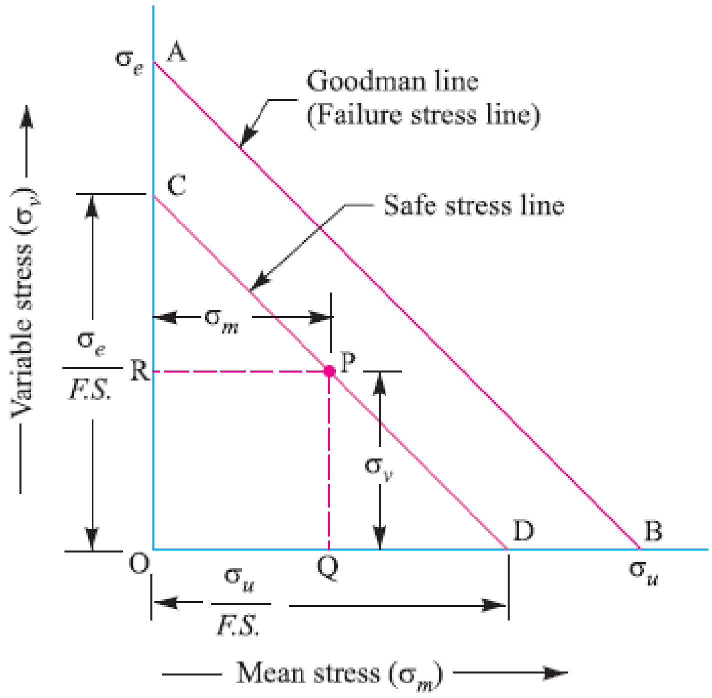
$\sigma_e$ = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor ($K_f$), the equation may be written as

$$\frac{1}{F.S.} = \left( \frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

## Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ*e*) and the ultimate strength (σ*u*), as shown by line *AB* in Figure, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.




Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad \ldots(\because\ QD = OD - OQ)$$

$$\therefore \qquad \frac{*\,\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Where $F.S.$ = Factor of safety,

$\sigma_m$ = Mean stress,

$\sigma_u$ = Ultimate stress,

$\sigma_v$ = Variable stress,

$\sigma_e$ = Endurance limit for reversed loading, and

$K_f$ = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}}$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad \ldots(\because\ \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

$K_b$ = Load factor for reversed bending load,

$K_{sur}$ = Surface finish factor, and

$K_{sz}$ = Size factor.

**Note:** Here we have assumed the same factor of safety ($F.S.$) for the ultimate tensile strength ($\sigma_u$) and endurance limit ($\sigma_e$). In case the factor of safety relating to both these stresses is different, then the following relation may be used

$$\frac{\sigma_v}{\sigma_e/(F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u/(F.S.)_u}$$

$(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Thus for brittle materials, the equation **may** be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

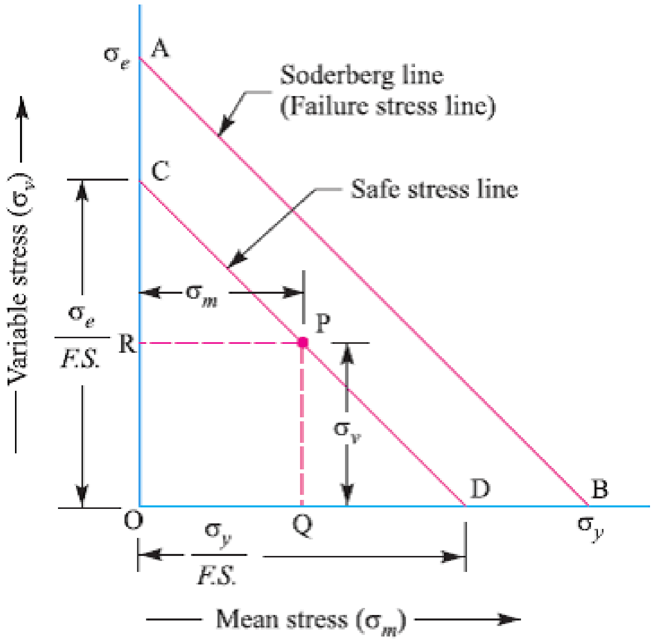
$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

Where suffix '*s*' denotes for shear. For reversed torsional or shear loading, the values of ultimate shear strength ($\tau_u$) and endurance shear strength ($\tau_e$) may be taken as follows:

$\tau_u = 0.8\ \sigma_u$; and $\tau_e = 0.8\ \sigma_e$

## Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit ($\sigma_e$) and the yield strength ($\sigma_y$), as shown by the line *AB* in Figure, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.




Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD}$$

$$\quad ...(\because\ QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor ($K_f$) should be applied to only variable stress ($\sigma_v$).

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Considering the load factor, surface finish factor and size factor

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

When a machine component is subjected to reversed axial loading

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

When a machine component is subjected to reversed shear loading

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

## Problem 3

A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m2 and – 150 MN/m2. Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation. Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

## Solution

$\sigma_1 = 300$ MN/m2 ; $\sigma_2 = -150$ MN/m2 ; $\sigma y = 0.55\ \sigma u$ ; $\sigma e = 0.5\ \sigma u$ ; $F.S. = 2$

Let $\sigma_u$ = Minimum ultimate strength in MN/m2.

We know that the mean or average stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

1. **According to Gerber relation**

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\,\sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\,\sigma_u$$

or $$(\sigma_u)^2 - 900\,\sigma_u - 22\,500 = 0$$

$$\therefore \quad \sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$= 924.35$ MN/m$^2$ **Ans.** ...(Taking +ve sign)

## 2. According to modified Goodman relation

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\,\sigma_u} = \frac{525}{\sigma_u}$$

$$\sigma_u = 2 \times 525 = 1050 \text{ MN/m}^2$$

## 3. According to Soderberg relation

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{0.55\ \sigma_u} + \frac{255}{0.5\ \sigma_u} = \frac{586.36}{\sigma_u}$$

$$\sigma_u = 2 \times 586.36 = 1172.72\ \text{MN/m}^2$$