Shaft

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses

Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(*a*) Shafts subjected to twisting moment or torque only,

(*b*) Shafts subjected to bending moment only,

(*c*) Shafts subjected to combined twisting and bending moments, and

(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad (1)$$

T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fiber

= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation **(1)** may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad (2)$$

From this equation, we may determine the diameter of round solid shaft (d). We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$. Substituting these values in equation **(1)**, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad (3)$$

k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o Now the equation **(3)** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \qquad (4)$$

From the equations **(3)** or **(*4*)**, the outside and inside diameter of a hollow shaft may be determined.

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation:

   We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

   T = Twisting moment in N-m, and

   N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Problem 1

A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Solution

P = 1 MW = 1 × 10^6 W ; N = 240 r.p.m. ; $T_{max} = 1.2\, T_{mean}$; τ = 60 MPa = 60 N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784 \text{ N-m} = 39\,784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

$$d = 159.4 \text{ say } 160 \text{ mm}$$

Problem 2

Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution

P = 20 kW = 20 × 10^3 W ; N = 200 r.p.m. ; τ_u = 360 MPa = 360 N/mm^2 ;$F.S.$ = 8 , $k = di / do = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$$d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm}$$

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4)$$

$$= \frac{\pi}{16} \times 45 \, (d_o)^3 \, [1 - (0.5)^4] = 8.3 \, (d_o)^3$$

$$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm}$$

$$d_i = 0.5 \, d_o = 0.5 \times 50 = 25 \text{ mm}$$