Measures of Central Tendency

It is the metrics that attempt to describe the point of data collection (observations) and its idea goes back to the English scholar Francis Galton. Central tendency measures are used to summarize data numerically since they are considered typical or ideal values for data. Also, these metrics are used to describe a data set or to compare it with other data sets. There are many types of measures of central tendency as follows:

The Mean, The Median and The Mode

1- The Mean

The mean represents the average value of the dataset. It can be calculated as the sum of all the values in the dataset divided by the number of values. In general, it is considered as the arithmetic mean. Some other measures of mean used to find the central tendency are as follows:

A-The Arithmetic Mean
B- The Geometric Mean
C-The Harmonic Mean

A- The Arithmetic Mean

It is a value which a set of data are collected around it. It is denoted by the symbol $(\bar{X})$ and calculated as follows:

a) Arithmetic Mean for not tabulated Data

If there is a set of data $(x_1, x_2, x_3, x_4, \dots\dots\dots, x_n)$, The Arithmetic Mean is calculated as :

$$\bar{X} = \frac{x_1 + x_2 + x_3 + x_4 + \cdots \dots \dots \dots + x_n}{n}$$

$$\bar{X} = \frac{\sum x_i}{n}$$

Example 1: Find the arithmetic mean for this data:500, 20, 40, 60, 100, 200, 50

Solution:

$$\bar{X} = \frac{\sum x_i}{n}$$

$$\bar{X} = \frac{500 + 20 + 40 + 60 + 100 + 200 + 50}{7} = \frac{970}{7} = 138.571$$

b) Arithmetic Mean for Tabulated Data

Tabulated data are the data set in frequency distribution table, and for each class there is an upper and lower limit:

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$x_i = \frac{upper\ limit + lower\ limit}{2}$$

f_i: $frequency\ of\ class$

Example: Find the arithmetic mean for this data:

Class	frequency
20-25	4
25-30	8
30-35	16
35-40	8
40-45	4

Solution:

$$x_i = \frac{upper\ limit + lower\ limit}{2}$$

$$x_1 = \frac{25 + 20}{2} = \frac{45}{2} = 22,5$$

$$x_2 = \frac{30 + 25}{2} = \frac{55}{2} = 27.5$$

class	f_i	x_i	$f_i\ x_i$
20-25	4	22.5	90
25-30	8	27.5	220
30-35	16	32.5	520
35-40	8	37.5	300
40-45	4	42.5	170
summation	$\sum f_i = 40$		$\sum f_i x_i = 1300$

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1300}{40} = 32.5$$

B- The Geometric Mean

The geometric mean is used to calculate average values in case data are ratios, as is the case with population growth rates, and it is calculated in two cases:

a) The Geometric Mean for not tabulated Data

The geometric mean of a set of values $(x_1, x_2, x_3, x_4, \dots\dots\dots, x_n)$ is defined as the square root of the product of these values, and is denoted by the symbol $\bar{X}_g$

$$\boldsymbol{log\ \bar{X}_g = \frac{1}{n} \sum log\ x_i}$$

Example: Find the value of the geometric mean for these values:
6, 4.2, 6, 4.2, 3.7, 4.8

Solution:

$$\text{Log } \bar{X}_g = \frac{1}{6}\Sigma\ (Log\ 6 + Log\ 4.2 + Log 6 + Log\ 4.2 + Log\ 3.7 + Log\ 4.8)$$

$$\text{Log } \bar{X}_g = \frac{1}{6}(0.77 + 0.62 + 0.77 + 0.62 + 0.56 + 0.68)$$

$$\text{Log } \bar{X}_g = 0.67$$

$$\bar{X}_g = 10^{0.67} = 4.677$$

b) The Geometric Mean for Tabulated Data

To calculate the value of the geometric mean for tabulated data used this equation:

$$\boldsymbol{log\ \bar{X}_g = \frac{1}{\sum f_i} \sum f_i\ log\ x_i}$$

x_i: central of class , f_i: frequency of class

Example: Find the geometric mean for data:

class	frequency
60-80	22
80-100	38
100-120	45
120-140	35
140-160	20
Total	160

Sol.

class	frequency	x_i	$log\ x_i$	$f_i\ log\ x_i$
60-80	22	70	1.845	40.59
80-100	38	90	1.954	74.25
100-120	45	110	2.041	91.85
120-140	35	130	2.114	73.99
140-160	20	150	2.176	43.52
Total	160			324.2

$$log\ \bar{X}_g = \frac{1}{\sum f_i} \sum f_i\ log\ x_i$$

$$log\ \bar{X}_g = \frac{1}{160}\ 324.2$$

$$log\ \bar{X}_g = 2.02625$$

$$\bar{X}_g = 106.23$$

C-The Harmonic Mean

The harmonic mean is calculated in two cases:

a) The Harmonic Mean for not tabulated Data

Its calculated by using this equation:

$$\overline{X_h} = \frac{1}{\frac{1}{n}\sum\frac{1}{x_i}} = \frac{n}{\sum\frac{1}{x_i}} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\cdots\ldots\ldots+\frac{1}{x_n}}$$

Example: Find the Harmonic Mean for data:18, 37, 25, 46, 77, 20

Solution:

$$\overline{X_h} = = \frac{n}{\sum\frac{1}{x_i}} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\cdots\ldots\ldots+\frac{1}{x_n}}$$

$$= \frac{7}{\frac{1}{18}+\frac{1}{37}+\frac{1}{25}+\frac{1}{46}+\frac{1}{77}+\frac{1}{20}} = 31.1315$$

b) The Harmonic Mean for tabulated Data

Its calculated by using this equation:

$$\overline{X_h} = = \frac{\sum f_i}{\sum \frac{f_i}{x_i}}$$

Example: Calculate the harmonic mean for the following data:

class	1	3	5	7	9	11
frequency	2	4	6	8	10	12

Solution:

class	frequency	f/x
1	2	2
3	4	1.332
5	6	1.2
7	8	1.144
9	10	1.111
11	12	1.092
sum	42	Σ f/x = 7.879

$$\overline{X_h} = = \frac{\sum f_i}{\sum \frac{f_i}{x_i}} = \frac{42}{7.879} = 5.330$$

2- The Median

It is the value in which a set of values is mediated after ascending or descending order and symbolized by the symbol (M). It is calculated in two cases:

- **Case of not tabulated data**

  The group values are arranged in ascending or descending order, and the median is the value that falls exactly in the middle.

  - If n is odd number then order of median $\frac{n+1}{2}$
  - If n is even number then order of median $\frac{value\ \frac{n}{2} + (\frac{n}{2}+1)}{2}$

Example: Find the value of the Median from the data:13, 50, 7, 15, 47, 12, 5

Solution: The data is arranged in ascending or descending order:

In descending order: 50, 47, 15, 13, 12, 7, 5

In ascending order: 5, 7, 12, 13, 15, 47, 50

median	5	7	12	13	15	47	50
n	1	2	3	4	5	6	7

$n = 7$, order of median $\frac{n+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$

The median is (13)

Example: Find the value of the Median from the data: 20, 12, 42, 15, 65, 30

Solution: In ascending order: 12, 15, 20, 30, 42, 65

$n = 6$, $\frac{n}{2} = \frac{6}{2} = 3$

$\frac{n}{2} = 3$

$\left(\frac{n}{2} + 1\right) = 3 + 1 = 4$

median	12	15	20	30	42	65
n	1	2	3	4	5	6

$$M = \frac{30 + 20}{2} = 25$$

- Case of tabulated data

The median is calculated in this method according to the following steps:

- ❖ Calculated the cumulative ascending or descending repetitive distribution.
- ❖ Find the median order by dividing the sum of frequencies by (2).
- ❖ Selected the median class of distribution, which is the class that corresponds to the median iteration ($\frac{\sum f_i}{2}$)which was calculated in the previous step. So if the median order between two cumulative frequencies, then the median class will be the next class of median order.
- ❖ Calculate the value of median by using the following equation

$$M = A + \frac{\frac{\sum f_i}{2} - f_1}{f_2 - f_1} * L$$

M: Median

A: Lower limit for median class

$\frac{\sum f_i}{2}$ = Median location

f_1: Previous accumulative frequency of the median value.

L : The length of median class

f_2: Next accumulative frequency of the median value.

Example: Find the Median for the data

Class	Frequency
20-	4
25-	8
30-	16
35-	8
40	4
	$\sum f_i = 40$

Sol.

Length of the frequency	The frequency of cumulative ascending
Less than 20	0
Less than 25	4
Less than 30	12
Less than 35	28
Less than 40	36
Less than 45	40

A → Less than 30 → 12 (f_1)

28 (f_2)

$$\frac{\sum f_i}{2} = \frac{40}{2} = 20$$

$L = 25 - 20 = 5$ or $L = 30 - 25 = 5$ or $L = 35 - 30 = 5$

$$M = A + \frac{\frac{\sum f_i}{2} - f_1}{f_2 - f_1} * L$$

$$M = 30 + \frac{20 - 12}{28 - 12} * 5$$

$$M = 30 + 2.5$$

$$M = 32.5$$