Lecture No. 1
Dry Friction

Introduction

Consider a solid block of mass m resting on a horizontal surface, as shown in Figure (1-1 a). We assume that the contacting surfaces have some roughness. The experiment involves the application of a horizontal force P which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity. The free-body diagram of the block for any value of P is shown in Figure (1-1 b), where the tangential ***friction force*** exerted by the plane on the block is labeled F. ***This friction force acting on the body will always be in a direction to oppose motion or the tendency toward motion of the body.*** There is also a ***normal force*** N which in this case equals $m * g$, and the total force R exerted by the supporting surface on the block is the resultant of N and F. A magnified view of the irregularities of the mating surfaces, Figure (1-1 c), helps us to visualize the mechanical action of friction. Support is necessarily intermittent and exists at the mating humps.

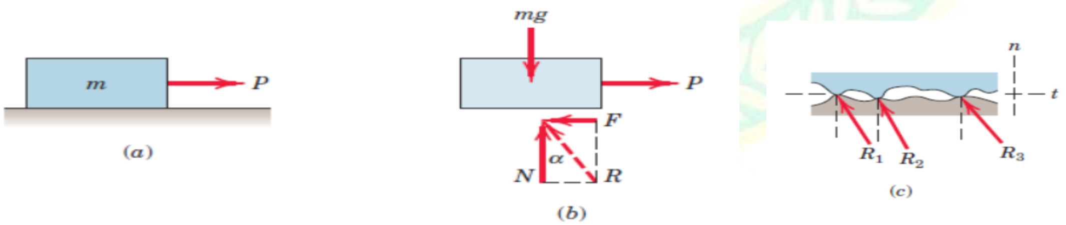


Figure 1-1

The direction of each of the reactions on the block, R_1, R_2, R_3, etc. depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point.

- *The total normal force N is the sum of the n-components of the R's,* and
- *the total frictional force F is the sum of the t-components of the R's.*

When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R's are smaller than when the surfaces are at rest relative to one another. ***This observation helps to explain the well-known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.*** If we perform the experiment and record the friction force F as a function of P, we obtain the relation shown in Figure (1-2), When P is zero, equilibrium requires that there be no friction force. As P is increased, the friction force must be equal and opposite to P as long as the block does not slip. During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations. Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force. At this same time the friction force decreases slightly and abruptly. It then remains essentially constant for a time but then decreases still more as the velocity increases.

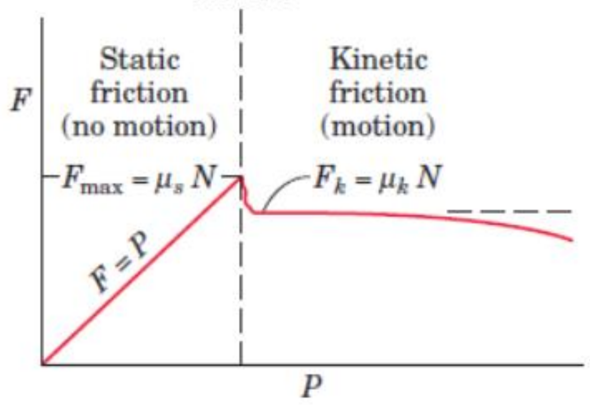


Figure 2-2

Static Friction

The region in Figure 1-2 up to the point of slippage or impending motion is called the ***range of static friction***, and in this range the value of the friction force is determined by **the equations of equilibrium**. This friction force may have any value from zero up to and including the maximum value. For a given pair of mating surfaces, the experiment shows that this maximum value of static friction $F_{\max}$ is proportional to the normal force N. Thus, we may write:

$$F_{max} = \mu_s * N \qquad (1\text{-}1)$$

where μ_s is the proportionality constant, called the coefficient of static friction. Be aware that Eq. (1-1) describes only the limiting or maximum value of the static friction force and not any lesser value. Thus, the equation applies only to cases where motion is impending with the friction force at its peak value. For a condition of static equilibrium when motion is not impending, the static friction force is

$$F_m < \mu_s * N \tag{1-2}$$

Kinetic Friction

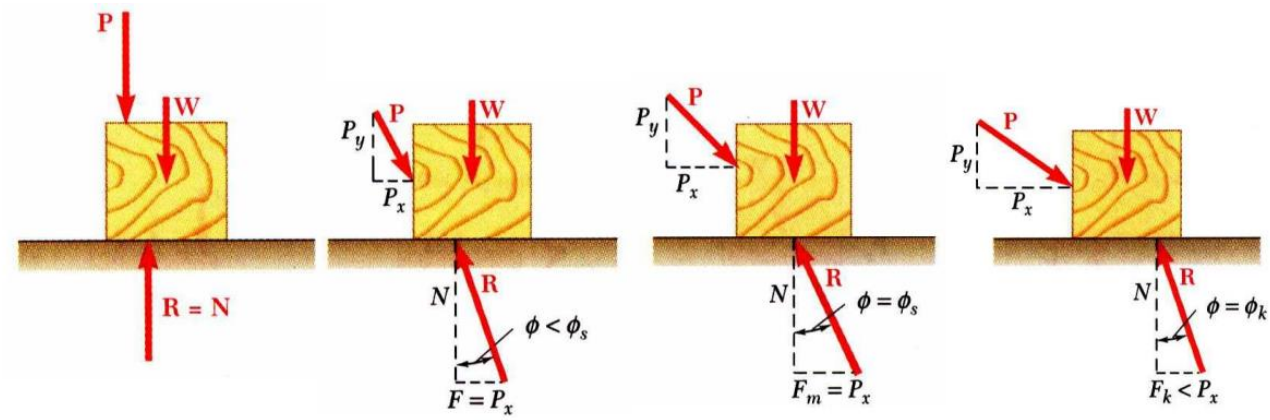
After slippage occurs, a condition of kinetic friction accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k is also proportional to the normal force. Thus,

$$F_k = \mu_k * N \tag{1-3}$$

where μ_k is the coefficient of kinetic friction. **It follows that μ_s is generally less than μ_s**. As the velocity of the block increases, **the kinetic friction decreases somewhat**, and at high velocities, this decrease may be significant. **Coefficients of friction depend greatly on the exact condition of the surfaces, as well as on the relative velocity, and are subject to considerable uncertainty.**

Angles of Friction

It is sometimes convenient to replace normal force N and friction force F by their resultant R:



- No friction
- No motion
- Motion impending
- Motion

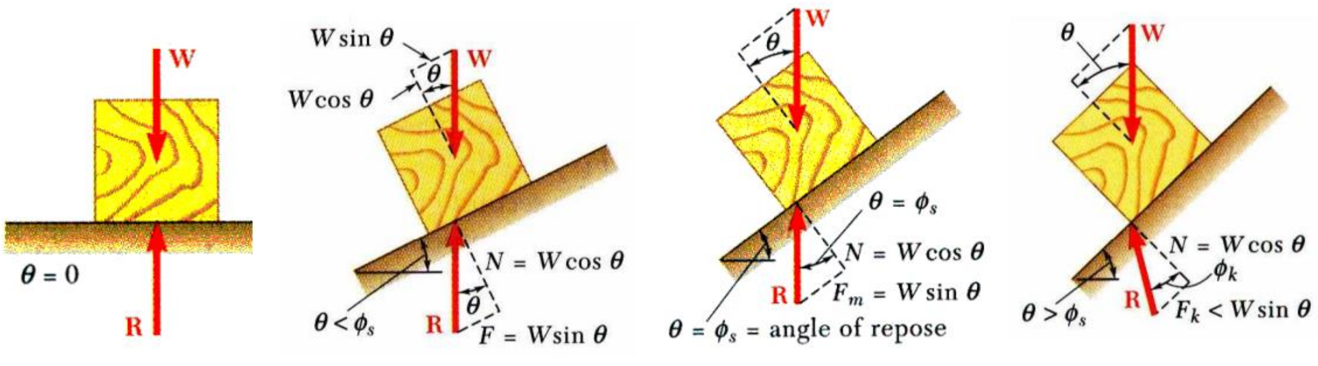
$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

Consider block of weight W resting on board with variable inclination angle θ.



- No friction
- No motion
- Motion impending
- Motion

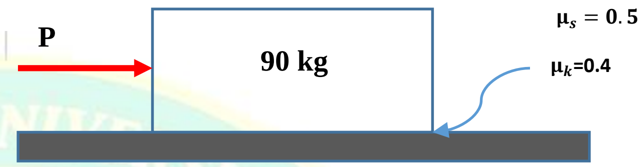
Types of Friction Problems

We can now recognize the following three types of problems encountered in applications involving dry friction. The first step in solving a friction problem is to identify its type.

- In the first type of problem, the condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping, and the friction force equals the limiting static friction $F_{max} = \mu_s * N$. The equations of equilibrium will, of course, also hold.
- In the second type of problem, neither the condition of impending motion nor the condition of motion is known to exist. To determine the actual friction conditions, we first assume static equilibrium and then solve for the friction force F necessary for equilibrium. Three outcomes are possible:
  - $\boldsymbol{F < (F_{max} = \mu_s * N)}$ Here the friction force necessary for equilibrium can be supported, and therefore the body is in static equilibrium as assumed. We emphasize that the actual friction force F is less than the limiting value F_{max} given by Eq.1-1 and that F is determined solely by the equations of equilibrium.
  - $\boldsymbol{F = (F_{max} = \mu_s * N)}$ Since the friction force F is at its maximum value F_{max}, motion impends, as discussed in problem type (1). The assumption of static equilibrium is valid.
  - $\boldsymbol{F > (F_{max} = \mu_s * N)}$: Clearly this condition is impossible, because the surfaces cannot support more force than the maximum $\boldsymbol{\mu_s * N}$. The assumption of equilibrium is therefore invalid, and motion occurs. The friction force F is equal to $\boldsymbol{\mu_k * N}$ from Eq.1-3
- In the third type of problem, relative motion is known to exist between the contacting surfaces, and thus the kinetic coefficient of friction clearly applies. For this problem type, Eq. 1-3 always gives the kinetic friction force directly.

Example 1: the force P is applied to the 90-kg crate, which is stationary before the force is applied determine the magnitude and direction of the friction force F exerted by the horizontal surface on the crate if: $P = 300\ N$, $P = 400\ N$, $P = 500\ N$.

Solution:



Given:

- ✓ Weight of the crate (**W**) = **90 kg**;
- ✓ Coefficient of friction $\mu_s = 0.5$
- ✓ Coefficient of friction μ_k=0.4
- ✓ Three cases of applied loads.

For $P = 300\ N$

$F_m = \mu_s * R$

In order to determine the F_m, it should be determine the normal force:

$$\sum F_y = 0$$

$$R - 90 * 9.81 = 0$$

$$R = 882.9\ N$$

$$\Rightarrow F_m = 0.5 * 882.9$$

$$= 441.45\ N$$

Resolving the forces horizontally, to determine the magnitude of F

$$\sum F_x = 0$$

$$-F + P = 0$$

$$\Rightarrow \mathrm{F = P = 300\ N}$$

By comparing between F_m & F

$$\mathbf{(F = 300\ N) < (F_m = 441.45\ N)}$$

Since the F_m is greater than the F

⇒ No motion occurs

and the friction force equal to

$$\mathrm{F} = \mu_s * R = 0.5 * 300$$

$F = 150\ N$ Ans. (1)

For $P = 400\ N$

$F_m = \mu_s * R$

In order to determine the F_m, it should be determine the normal force:

$$\sum F_y = 0$$

$R - 90 * 9.81 = 0$

$R = 882.9\ N$

$\Rightarrow F_m = 0.5 * 882.9$

$= 441.45\ N$

Resolving the forces horizontally, to determine the magnitude of F

$\sum F_x = 0$

$-F + P = 0$

$\Rightarrow \mathrm{F = P = 400\ N}$

By comparing between F_m & F

$$\mathbf{(F = 400\ N) < (F_m = 441.45\ N)}$$

Since the F_m is greater than the F

$\Rightarrow$ No motion occurs

and the friction force equal to

$F = 400\ N$ Ans. (2)

For $P = 500\ N$

$F_m = \mu_s * R$

In order to determine the F_m, it should be determine the normal force:

$$\sum F_y = 0$$

$R - 90 * 9.81 = 0$

$R = 882.9\ N$

$\Rightarrow F_m = 0.5 * 882.9$

$= 441.45\ N$

Resolving the forces horizontally, to determine the magnitude of F

$\sum F_x = 0$

$-F + P = 0$

$\Rightarrow$ F = P = 500 N

By comparing between F_m & F

$$(\mathbf{F = 300\ N}) > (\boldsymbol{F_m = 441.45\ N})$$

Since the F_m is smaller than the F

$\Rightarrow$ motion occurs

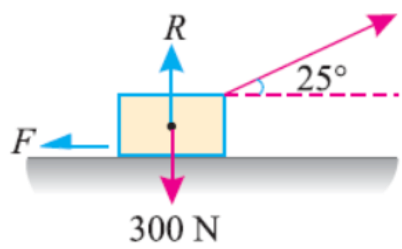
and the friction force equal to

$F = \mu_k * R = 0.4 * 882.9$

$F = 353.16\ N$ Ans. (3)

Example 2: A body of weight **300 N** is lying on a rough horizontal plane having a coefficient of friction as **0.3**. Find the magnitude of the force, which can move the body, while acting at an angle of **25°** with the horizontal.

Solution:



Given:

- ✓ Weight of the body (**W**) = **300 N**;
- ✓ Coefficient of friction (**μ**) = **0.3**
- ✓ angle made by the force with the horizontal (**α**) = **25°**

let:

- ➢ P = Magnitude of the force, which can move the body,
- ➢ F = Force of friction.

Resolving the forces horizontally,

$\sum F_x = 0$

$F = P\cos\boldsymbol{\alpha} = P\cos 25°$

$= P \times 0.9063$ -- (1)

and now resolving the forces vertically,

$$\sum F_y = 0$$

$$R = W - P\sin\alpha$$

$$= 300 - P\,sin\,25°$$

$$= 300 - P * 0.4226 \text{---(2)}$$

We know that the force of friction (F), equations (1) & (2) in **Friction Law**

$$F_m = \mu_s * R$$

$$0.9063\,P = \mu_s * R$$

$$= 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

$$\Rightarrow 90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$\therefore\ P = \frac{90}{1.0331} = 87.1\text{ N}$ *Ans.*

Example 3: A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

Solution:

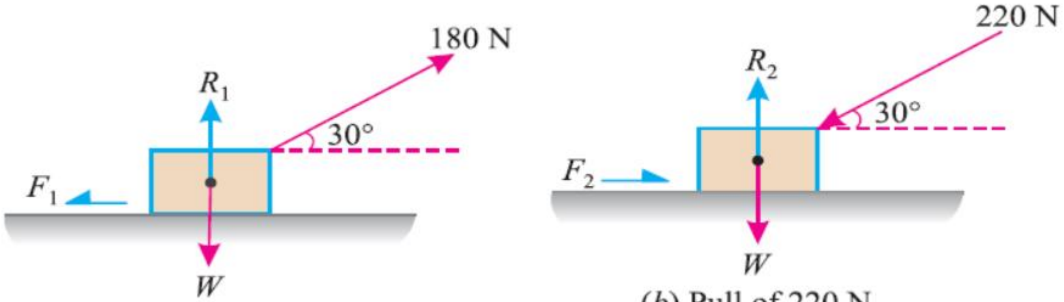
Given:

- ✓ Pull = 180 N
- ✓ Push = 220 N
- ✓ angle at which force is inclined with horizontal plane (α) = 30°

Let

- ➢ W = Weight of the body
- ➢ R = Normal reaction,
- ➢ μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F1) will act towards left as shown in **Figure (1-2)**



(*a*) Pull of 180 N (*b*) Pull of 220 N

Resolving the forces horizontally,

$$\sum F_x = 0$$

$F1 = 180 \cos 30° = 180 \times 0.866 = 155.9\ N$ ---(1)

and now resolving the forces vertically,

$$\sum F_y = 0$$

$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90\ N$ ----------------------------------(2)

We know that the force of friction (F1), equations (1) & (2) in **Friction Law**

$F_1 = \mu_s * R$

$155.9 = \mu R1 = \mu (W - 90)$ --- ***(i)***

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F2) will act towards right as shown in **Figure (1-2)**

Resolving the forces horizontally,

$\sum F_x = 0$

$F_1 = 220 \cos 30° = 220 \times 0.866 = 190.5\ N$ ---(3)

and now resolving the forces vertically,

$\sum F_y = 0$

$R_2 = W - 220 \sin 30° = W - 220 \times 0.5 = W + 110\ N$------------------------------(4)

We know that the force of friction (F1), equations (3) & (4) in **Friction Law**

$F_2 = \mu_s * R_2$

$190.5 = \mu * R_2 = \mu (W + 110)$ --- ----***(ii)***

Dividing equation ***(i)*** by ***(ii)***

$$\frac{155.9}{190.5} = \frac{\mu\,(W - 90)}{\mu\,(W + 110)} = \frac{(W - 90)}{(W + 110)}$$

$$155.9\,W + 17\,149 = 190.5\,W - 17\,145$$

$$34.6\,W = 34\,294$$

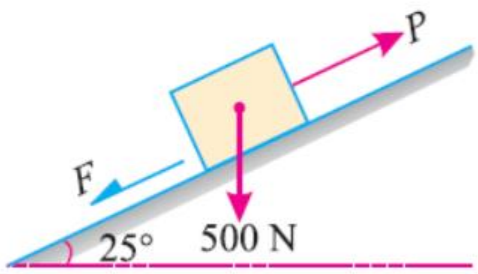
$\therefore w = \frac{34\,294}{34.6} = 991.2\text{ N}$ ***Ans.(1)***

Now substituting the value of W in equation ***(i)***

$$155.9 = \mu\,(991.2 - 90) = 901.2\,\mu$$

$\therefore \mu = \frac{155.9}{901.2} = 991.2\text{ N}$ ***Ans.(2)***

Example 4: A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in Fig. 8.9. Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°.



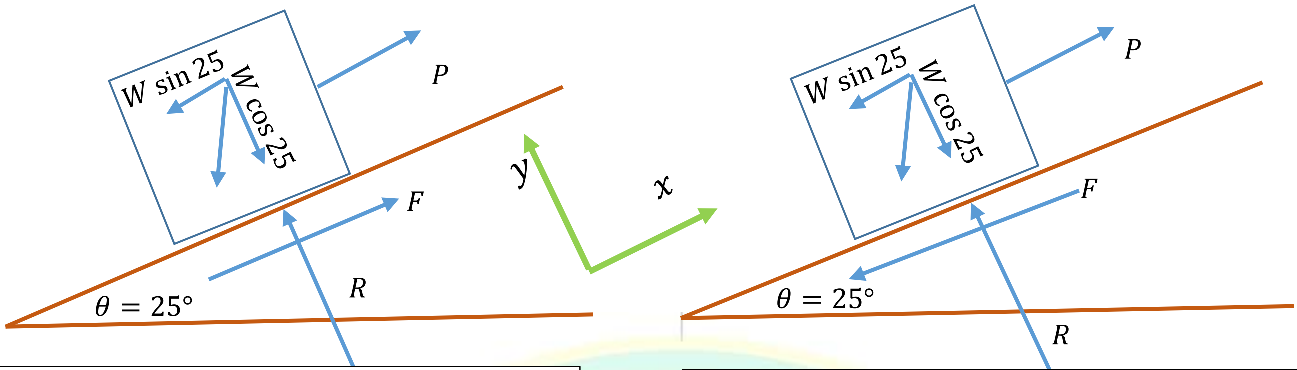
Solution:

Given:

- ✓ Weight of the body (W) = 500 N;
- ✓ Angle at which plane is inclined (α) = 25°
- ✓ angle of friction (φ) = 20°.

Minimum value of P

We know that for the minimum value of *P*, the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force:



Minimum value of P

Maximum Value of P

To determine the normal force (R)

$\sum F_y = 0$

$R - Wcos(25) = 0$

$R = 500 * \cos(25)$

$= \quad 453.1539\ N$

To determine the force P:

$\sum F_x = 0$

$P + F - W * \sin(25) = 0$

P +(tan(20)*R)-W*sin(25)=0 ⇒

$P = -$(tan (20) *R) +W*sin (25)

$P = 46.4\ N$ ***Ans.(1)***

Maximum value of P

To determine the normal force (R)

$\sum F_y = 0$

$R - Wcos(25) = 0$

$R = 500 * \cos(25)$

$= \quad 453.1539\ N$

To determine the force P:

$\sum F_x = 0$

$P - F - W * \sin(25) = 0$

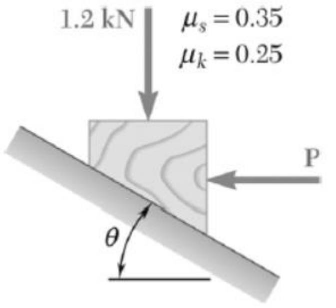
P −(tan(20)*R)-W*sin(25)=0 ⇒

P =(tan (20) *R) +W*sin (25)

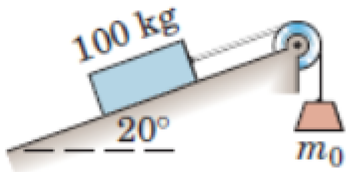
$P = 376.2437\ N$ Ans. (2)

Homework

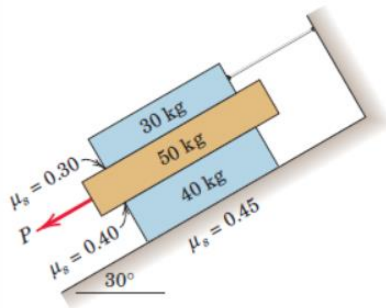
1. Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when θ = 25° and *P* = 750 N.



2. Determine the range of values which the mass m0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



3. The three flat blocks are positioned on the 30_ incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping takes place.



4. Determine the magnitude and direction of the friction force which the vertical wall exerts on the 100-lb block if (a) θ=15° and (b) θ=30°