Lecture No. 2
Centroid

Introduction

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The center of area of such figures is known as *centroid*. The method of finding out the centroid of a figure is the same as that of finding out the center of gravity of a body. In many books, the authors also write center of gravity for centroid and vice versa.

METHODS FOR CENTRE OF GRAVITY

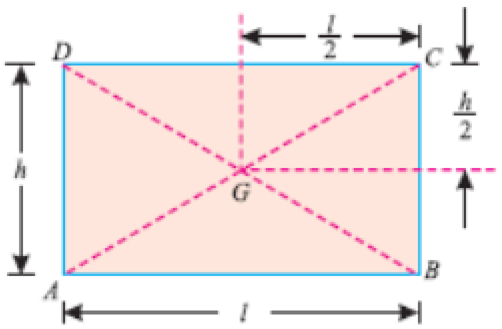
Any one of the following two methods may find out the center of gravity (or centroid):

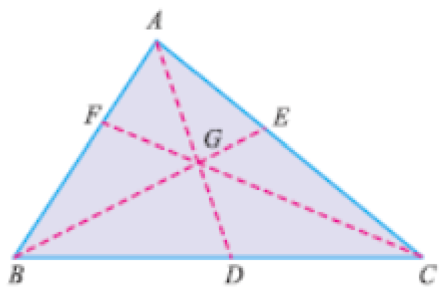
- By geometrical considerations
- By moments
- By graphical method

CENTRE OF GRAVITY BY GEOMETRICAL CONSIDERATIONS

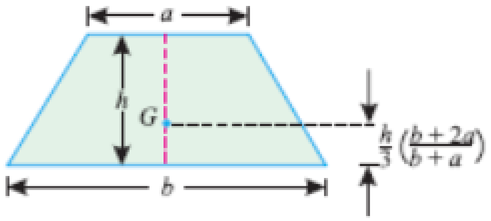
The center of gravity of simple figures may be found out from the geometry of the figure as given below.

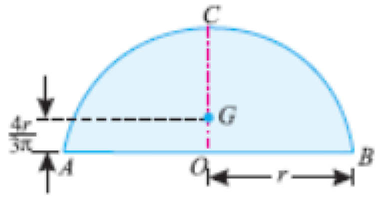
- The center of gravity of uniform rod is at its middle point.





- The center of gravity of a semicircle is at a distance $\frac{4*r}{3*\pi}$ of from its base measured along the vertical radius





- The center of gravity of a circular sector making semi-vertical angle α is at a distance of $\frac{2*r*\sin\propto}{3*\propto}$ from the centre of the sector measured along the central

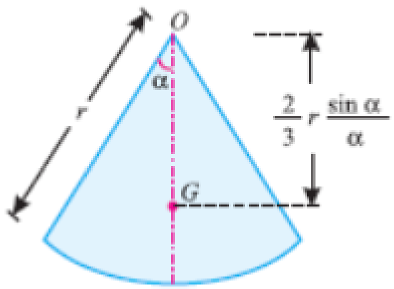
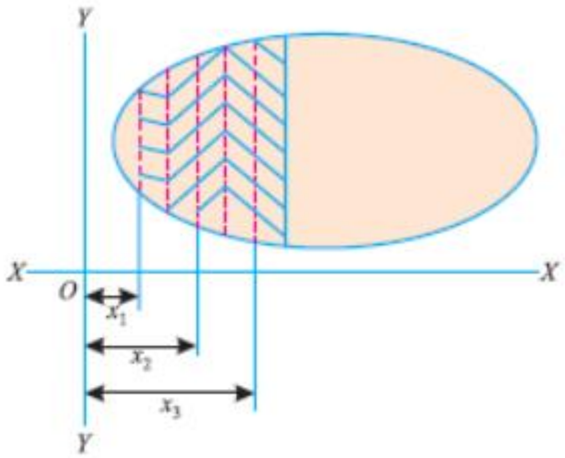


Fig. 6.5. Circular sector

Centre of Gravity by Moments

The center of gravity of a body may also be found out by moments as discussed below:



Consider a body of mass *M* whose centre of gravity is required to be found out. Divide the body into small masses, whose centres of gravity are known as shown in Fig. 6.9. Let $m1$, $m2$, $m3$....; etc. be the masses of the particles and ($x1$, $y1$), ($x2$, $y2$), ($x3$, $y3$), ... be the co-ordinates of the centers of gravity from a fixed point *O* as shown in Fig.

Let x and y be the co-ordinates of the center of gravity of the body. From the principle of moments, we know that

$$M\,\bar{x} = m_1\,x_1 + m_2\,x_2 + m_3\,x_3 \ldots..$$
$$M = m_1 + m_2 + m_3 + \cdots.$$

Or
$$\bar{x} = \frac{\sum m * x}{M}$$

Similarly

$$\bar{y} = \frac{\sum m * y}{M}$$

$$M = m_1 + m_2 + m_3 + \cdots.$$

AXIS OF REFERENCE

The center of gravity of a body is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference). The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating *y* and the left line of the figure for calculating *x*.

CENTRE OF GRAVITY OF PLANE FIGURES

The plane geometrical figures (such as *T*-section, *I*-section, *L*-section etc.) have only areas but no mass. The center of gravity of such figures is found out in the same way as that of solid bodies. The center of area of such figures is known as centroid, and coincides with the center of gravity of the Figure. It is a common practice to use center of gravity for centroid and vice versa.

Let *x* and *y* be the co-ordinates of the center of gravity with respect to some axis of reference, then

$$\bar{x} = \frac{a_1\,x_1 + a_2\,x_2 + a_3\,x_3}{a_1 + a_2 + a_3}$$

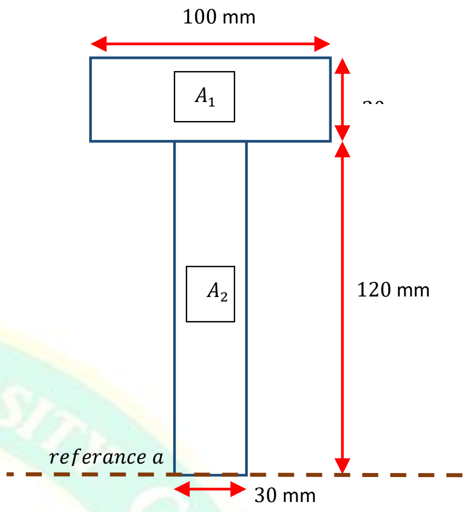
$$\bar{y} = \frac{a_1\,y_1 + a_2\,y_2 + a_3\,y_3}{a_1 + a_2 + a_3}$$

Where a_1, a_2, a_3....... etc., are the areas into which the whole figure is divided x_1, x_2, x_3 ..... etc., are the respective co-ordinates of the areas a_1, a_2, a_3........ on X-X axis with respect to same axis of reference. y_1, y_2, y_3....... etc., are the respective co-ordinates of the areas $a1$, $a2$, $a3$....... on Y-Y axis with respect to same axis of the reference.

Important Note While using the above formula, x_1, x_2, x_3..... or y_1, y_2, y_3 or x and y must be measured from the same axis of reference (or point of reference) and on the same side of it. However, if the figure is on both sides of the axis of reference, then the distances in one direction are taken as positive and those in the opposite directions must be taken as negative.

Example 1

Find the center of gravity of a 100 mm × 150 mm × 30 mm T-section.



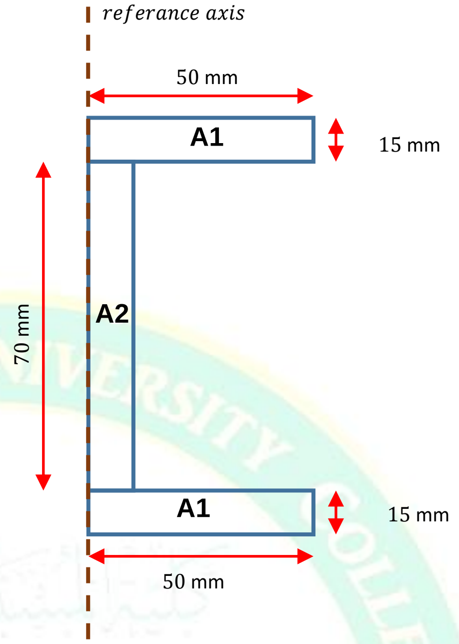
Solution

$$\bar{y} = \frac{621000}{6600} = 94.1$$

No.	Area (mm^2	$\bar{y}$	$a * \bar{y}$
1	$30 * 100 = 3000$	$120 + 1 = 135$	$3000 * 135 = 405000$
2	$30 * 120 = 3600$	$\frac{120}{2} = 60$	$3600 * 60 = 216000$
Total	$3000 + 3600 = 6600$		$405000 + 216000 = 621000$

Example 2

Find the center of gravity of a channel section 100 mm × 50 mm × 15 mm.



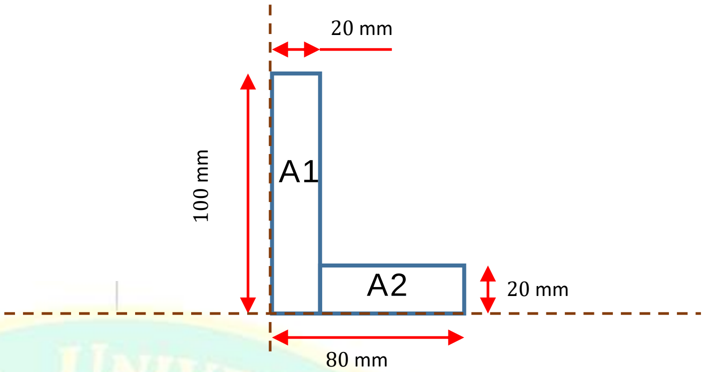
Solution /

$$\bar{x} = \frac{45375}{2550} = 17.8 \text{ mm}$$

No.	Area (mm^2)	$\bar{x}$	$a * \bar{x}$
1	$50 * 15 * 2 = 1500$	$\frac{50}{2} = 25$	$1500 * 25 = 37500$
2	$70 * 15 = 1050$	$\frac{15}{2} = 7.5$	$1050 * 7.5 = 7875$
Total	$1500 + 1050 = 2550$		$37500 + 7875 = 45375$

Example 3

Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.



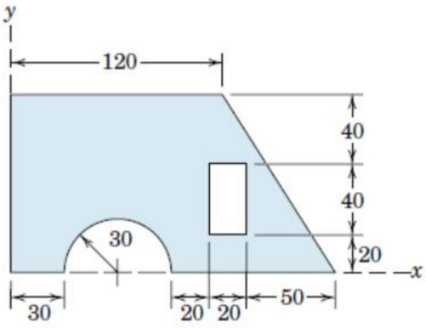
$$\bar{x} = \frac{68000}{3200} = 25 \text{ mm}$$

$$\bar{y} = \frac{112000}{3200} = 35 \text{ mm}$$

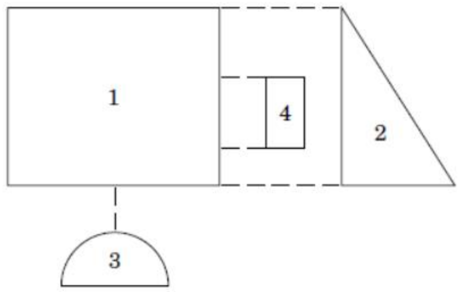
No.	Area (mm^2)	$\bar{x}$	$\bar{y}$	$a * \bar{x}$	$a * \bar{y}$
1	$100 * 20 = 2000$	$\frac{20}{2} = 10$	$\frac{100}{2} = 50$	$2000 * 10 = 20000$	$2000 * 50 = 100000$
2	$20 * 60 = 1200$	$\frac{60}{2} + 20 = 40$	$\frac{20}{2} = 10$	$1200 * 50 = 60000$	$1200 * 10 = 12000$
Total	$1200 + 2000 =$ 3200			$20000 + 60000 = 80000$	$100000 + 12000$ $= 112000$

Example 4

Locate the centroid of the shaded area.



Dimensions in millimeters



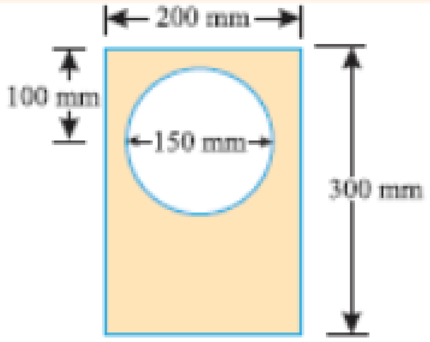
$$\bar{x} = \frac{959160}{12790} = 74.99$$

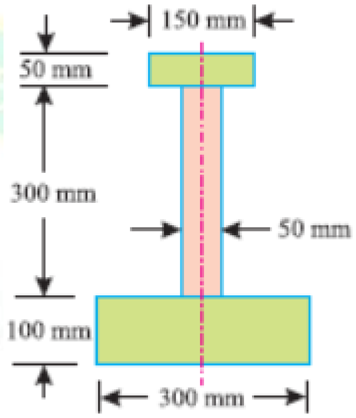
$$\bar{y} = \frac{649989.78}{12790} = 50.8$$

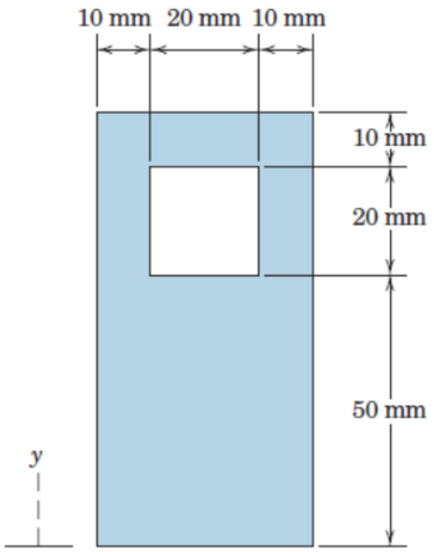
No.	Area (mm^2)	$\bar{x}$	$\bar{y}$	$a * \bar{x}$	$a * \bar{y}$
1	$120 * 100 = 12000$	$\frac{120}{2} = 60$	$\frac{100}{2} = 50$	$12000 * 60 = 720000$	$12000 * 50 = 600000$
2	$\frac{1}{2} * 60 * 100$ $= 3000$	$\frac{60}{3} + 120 = 140$	$\frac{1}{3} * 100 = 33.33$	$3000 * 140 = 420000$	$3000 * 33.33 = 99990$
3	$\frac{1}{2} * \pi * 30^2$=-1414	$30 + 30 = 60$	$\frac{4 * 30}{3 * \pi} = 12.73$	$-1414 * 60 = -84840$	$-1414 * 12.73 = -18000.22$
4	$40 * 20 = -800$	$30 + 60 + 20$ $+ 10 = 120$	$20 + 20 = 40$	$-800 * 120 = -96000$	$-800 * 40 = -32000$
Total	12000 +3000 −1414 −800 = 12790			720000 +420000 −84840 −96000 = 959160	600000 +99990 -18000.22 -32000 = 649989.78

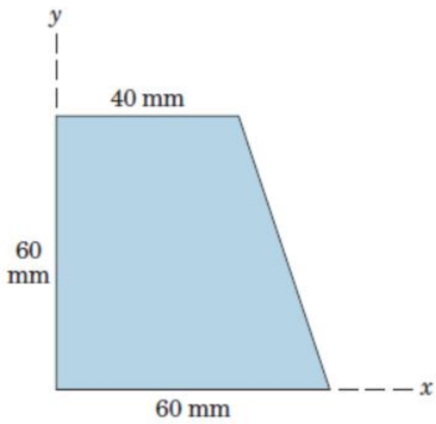
Homework

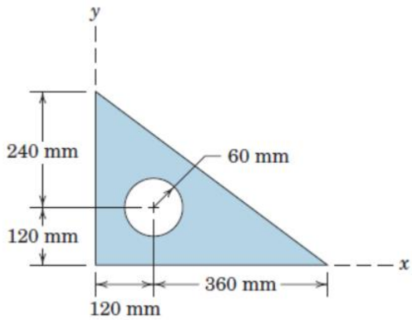
Locate the centroid of the following figures

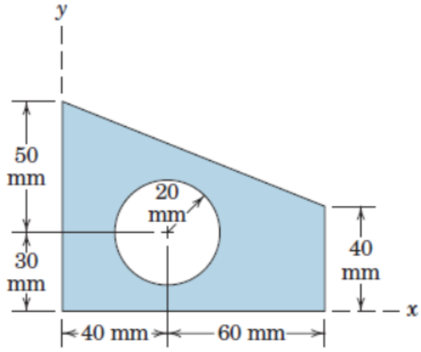


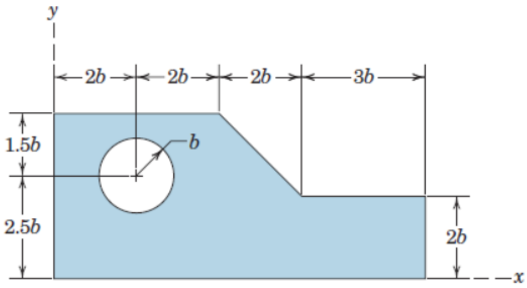











y
2b
2b
2b
3b
1.5b
b
2.5b
2b
x