Applications of derivative / Related rates of changes.

let $Q = f(t)$ where t is time

$\frac{\Delta Q}{\Delta t}$ = average rate change

$\frac{dQ}{dt}$ = instantaneous rate of change

if $\frac{dQ}{dt}$ is +ve $\Rightarrow$ Q increasing

if $\frac{dQ}{dt}$ is -ve $\Rightarrow$ Q decreasing

To Solve the problem of related rates

1- Write an equation that relates the variable quantities

2- Differentiate both sides with respect to the time t

3- Substitute ~~[illegible]~~ the given information and Solve for unkown rate.

Ex / How fast is The radius of a ball changing where the air is blown into it at the rate of 10 cm^3/Sec ? find rate where r = 6 cm?

Sol: r = radius (variable) dr

V = Volume of the air (ball), dv

$\frac{dv}{dt} = 10\ cm^3/Sec$, $\frac{dr}{dt} = ?$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = \frac{4}{\cancel{3}}\pi\left(\cancel{3}r^2 \frac{dr}{dt}\right)$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

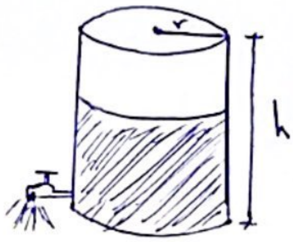
$$10 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{10}{4\pi r^2} = \frac{5}{2\pi r^2}$$

$\frac{dr}{dt}$ at $r = 6 \Rightarrow \frac{dr}{dt} = \frac{5}{2\pi(6)^2} = 0.022$ cm/Sec

EX2/ How fast does the water level drop when a cylindrical tank is drained at the rate of 3 L/sec?

Sol:- let V = Volume of water



$r = \pi r^2 h$ (r: constant).

[change in water vol.] $\frac{dv}{dt} = \pi r^2 \frac{dh}{dt}$ [$\frac{dh}{dt}$ = chang in water level]

$\frac{dv}{dt} = -3$ L/Sec (Volume is decreasing).

$$-3 = \pi r^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-3}{\pi r^2}$$