Parametric equations

If $x = f(t)$ and $y = g(t)$, then these equations are called parametric equations and the variable $t$ is called parameter.

from chain Rule $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$\therefore \frac{dy}{dx} = \frac{dy/du}{dx/du}$$

$x = f(t)$ , $y = g(t)$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

the 1st derivative for parametric equation

* for Second derivative

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} \quad , y' = \frac{dy}{dx}$$

Ex/ If $y = 2t^3 + 3$, $x = \frac{t}{t-1}$, find $\frac{dy}{dx}$

Sol:- $y = 2t^3 + 3$ , $x = \frac{t}{t-1}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dt} = 6t^2$$

$$\frac{dx}{dt} = \frac{(t-1) - t}{(t-1)^2} = \frac{-1}{(t-1)^2}$$

$$\frac{dy}{dx} = \frac{6t^2}{-1/(t-1)^2}$$

Ex/ If a point traces the circle $x^2+y^2=25$ and if $dx/dt = 4$ when the point reaches $(3,4)$ find $\frac{dy}{dt}$.

Sol:- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$x^2+y^2=25 \rightarrow 2x + 2y\frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

at point $(3,4) \rightarrow \frac{dy}{dx} = \frac{-3}{4}$

$\frac{-3}{4} = \frac{dy/dt}{4} \Rightarrow \frac{dy}{dt} = -3$

Ex/ If $x = \cos 3t$, $y = \sin^2 3t$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$

$\frac{dy}{dt} = 2\sin 3t(\cos 3t).3 = 6\sin 3t \cos 3t$

$\frac{dx}{dt} = -\sin 3t.3 = -3\sin 3t$

$\frac{dy}{dx} = \frac{+6\sin 3t \cos 3t}{-3\sin 3t} = -2\cos 3t = -2x$

$\frac{d^2y}{dx^2} = \frac{d/dt(dy/dx)}{dx/dt}$

$= -2(+\sin 3t.3).6\sin 3t$

$\frac{d^2y}{dx^2} = \frac{6\sin 3t}{-3\sin 3t} = -2$

or $\frac{dy}{dx} = -2x$, $\frac{d^2y}{dx^2} = -2$