اسم التدريسي	رامي قيس مالك
اسم المادة	تقنيات رقمية
عنوان المحاضرة	**Combinational Logic Circuits**
رقم المحاضرة	**9**
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AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :9th – Combinational Logic Circuits

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COMBINATIONAL LOGIC CIRCUITS

INTRODUCTION:

Complex digital systems like computers are made up of many simple combinatory, sequential and hybrid (combinatory and sequential) circuits connected together. A combinatory logic circuit is a logic circuit whose outputs depends only on the combination of its inputs logic states. One of the most important circuits of a computer, which is the Arithmetical and Logical Unit (ALU), is made up of simple circuits capable of performing operations such as addition, multiplication, subtraction and division.

1. PRINCIPLE OF ADDITION IN DIGITAL SYSTEMS:

Computers can add only two binary numbers at once. These numbers can have up to 64 bits, with respect to the width of the data bus of the mother board. Let us consider two binary numbers of 5 bits each to be added.

	1 0 1 1 1	Number A
+	0 0 1 0 1	Number B
	1 1 1 0 0	Sum (A + B)
	0 0 1 1 1	Carry out

We start the operation by adding the two LSB of the numbers A and B: 1 + 1 = 102 (Sum = 0; Carry out = 1). The carry out is added to the bits of the

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rank situated immediately at the left. The same operation will be repeated for the other ranks, until the addition is completed. There is a special circuit which is able to perform this operation: It is the full adder. The following figure gives us the principle of the addition involving many full adders:

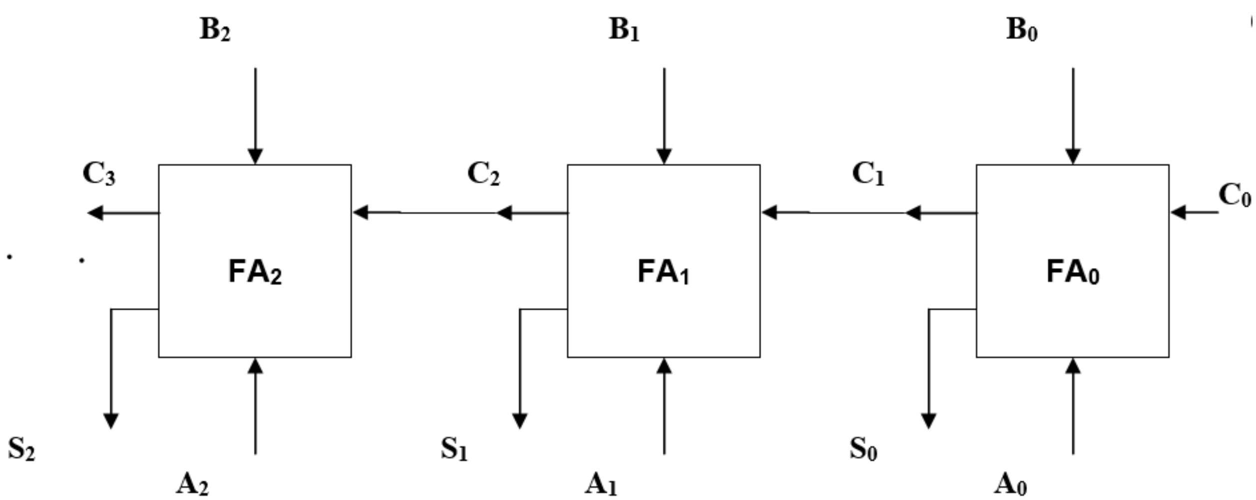

Figure 6.1: Principle of addition

Each element of the circuit above represents a full adder. Each full adder is intended to add one bit of the number A to the bit of the number B having the same weight. The two numbers to be added should have the same number of bits. So, if the two numbers to be added have five bits each, the addition will be performed using five full adders connected in parallel as shown above.

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2.1THE FULL ADDER:

As presented above, the full adder is combinatory logic circuit intended to add two bits. The principle diagram of a full adder is given by the figure bellow:

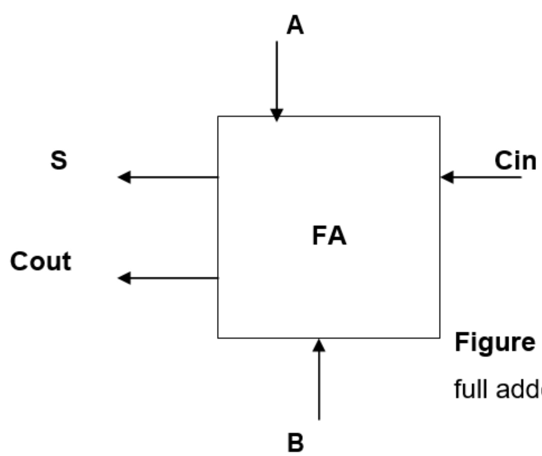

Figure 6.2: Principle diagram of a full adder

The full adder has three inputs: A: bit from number A; B: Bit from number B; Cin: Carry out coming from the previous rank. And two outputs: S: the bit of sum; Cout: The carry out (to be added to the bits of the next rank).

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- Truth table:

A	B	Ci	S	Co
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Equations of the outputs: We can either use Boolean algebra or k-map to determine the equations of the outputs.

- Using Boolean algebra:

$$S = \bar{A}.\bar{B}C_i + \bar{A}B\overline{C_i} + A\bar{B}.\overline{C_i} + ABC_i$$

$$= \bar{A}\left(\bar{B}C_i + B\overline{C_i}\right) + A\left(\bar{B}.\overline{C_i} + BC_i\right)$$

$$= \bar{A}(B \oplus C_{in}) + A\left(\overline{B \oplus C_i}\right)$$

Let $X = \left(\overline{B \oplus C_i}\right)$

$$S = \bar{A}X + A\bar{X}$$

$$= A \oplus X$$

$$= A \oplus B \oplus C_i$$

$$\boxed{S = A \oplus B \oplus C_i}$$

$$Co = \bar{A}.BC_i + A\bar{B}C_i + AB\overline{C_i} + ABC_i$$

The expression will not change if one of the elements of the sum of products is duplicated (After the Boolean additive identity according to which A + A = A, A being a Boolean variable). So we will duplicate the product simplify the expression easily.

$$Co = \overline{A}.BC_i + A\overline{B}C_i + AB\overline{C_i} + ABC_i + ABC_i + ABC_i$$
$$= BC_i(A + \overline{A}) + AC_i(B + \overline{B}) + AB(C_i + \overline{C_i})$$
$$= BC_i + AC_i + AB$$

$$\boxed{Co = BC_i + AC_i + AB}$$

- Using k-map:

S: A \ BCi	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$S = \overline{A}.\overline{B}C_i + \overline{A}B\overline{C_i} + A\overline{B}.\overline{C_i} + ABC_i$$

$$\boxed{S = A \oplus B \oplus C_i}$$

Co: A \ BCi	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$\boxed{Co = BC_i + AC_i + AB}$$

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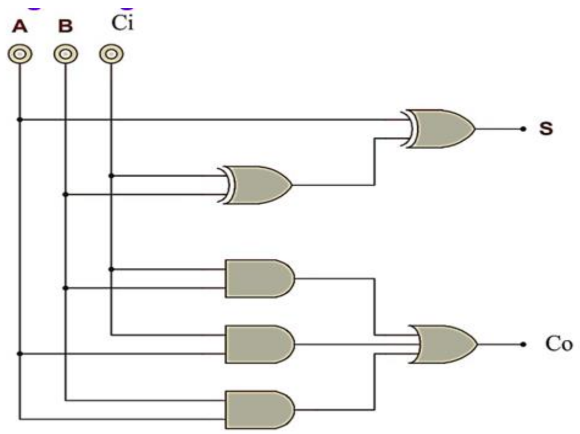

2.2 THE HALF ADDER:

The half adder has two inputs A and B which are the two bits to be added, and two outputs which are the sum output S and the carry output Co. The principle diagram of the half adder is given by the following diagram.

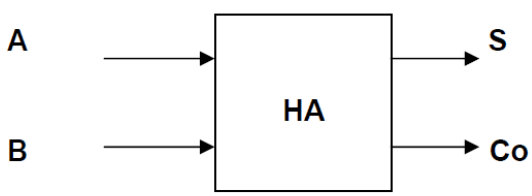

Figure 6.3: Principle diagram of a half adder

- Truth table:

A	B	S	Co
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

- Equations of the outputs

$$S = \overline{A}.B + A.\overline{B}$$

$$S = A \oplus B$$

$$C_o = A.B$$

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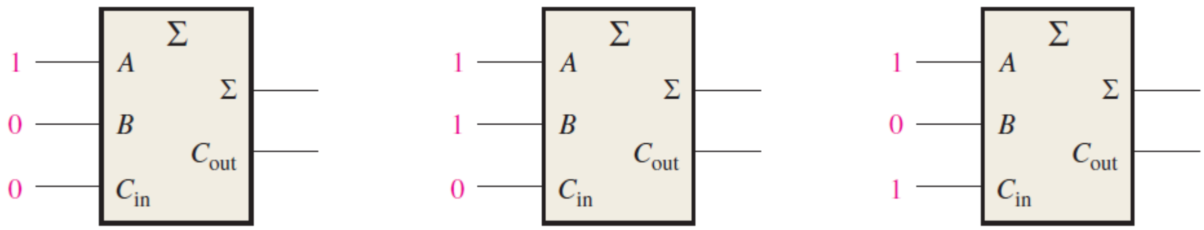
EXAMPLE 1: For each of the following figure determine the output for the input showing:



(a) (b) (c)

Solution

(a) The input bits are $A = 1$, $B = 0$, and $C_{in} = 0$.

$$1 + 0 + 0 = 1 \text{ with no carry}$$

Therefore, $\Sigma = \mathbf{1}$ and $C_{out} = \mathbf{0}$.

(b) The input bits are $A = 1$, $B = 1$, and $C_{in} = 0$.

$$1 + 1 + 0 = 0 \text{ with a carry of } 1$$

Therefore, $\Sigma = \mathbf{0}$ and $C_{out} = \mathbf{1}$.

(c) The input bits are $A = 1$, $B = 0$, and $C_{in} = 1$.

$$1 + 0 + 1 = 0 \text{ with a carry of } 1$$

Therefore, $\Sigma = \mathbf{0}$ and $C_{out} = \mathbf{1}$.