Electricity and Magnetism

Lecture Four

Flux of the Electric Field and Gauss's Law

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Outline

1. **Flux of the Electric Field**
2. **Gauss' Law**
3. **Gauss' Law and Coulomb's Law**
4. **A Charged Isolated Conductor**
5. **References**

1. Flux of the Electric Field

Electric flux **is the rate of flow of the electric field through a given area** (Fig. 1). Electric flux is proportional to the number of electric field lines going through a virtual surface.

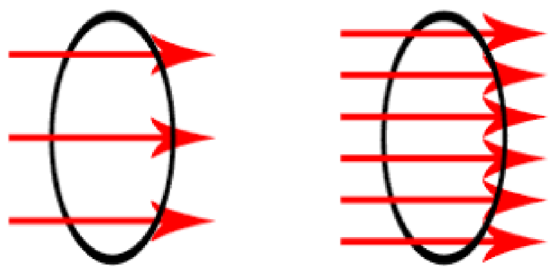


Figure 1: Electric Flux: Electric flux visualized. The ring shows the surface boundaries. The red arrows for the electric field lines.

Flat Surface, Uniform Field: We begin with a flat surface (Fig. 2) with area A in a uniform electric field $\vec{E}$. The total flux Φ is then:

$$\Phi = \int \vec{E}.d\vec{A} \quad (total\ flux)$$

$$\Phi = \int (E\ cos\theta)\ d\vec{A} \quad (total\ flux)$$

When the electric field is uniform and the surface is flat:

$$\Phi = (E\ cos\theta)A \quad (\text{uniform field, flat surface})$$

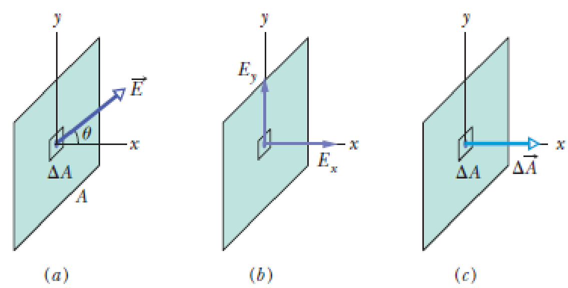


Figure 2: An electric field vector pierces a small square patch on a flat surface.

Example: A nonuniform electric field given by $\vec{E} = 3x\hat{\imath} + 4\hat{\jmath}$ pierces the Gaussian Square area $d\vec{A}$ with length 2 m. what is the flux throw the surface when x= 3m point in the positive direction of the x axis.?

Solution:

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}) \cdot (dA\hat{\mathrm{i}})$$

$$= \int [(3.0x)(dA)\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} + (4.0)(dA)\hat{\mathrm{j}} \cdot \hat{\mathrm{i}}]$$

$$= \int (3.0x\, dA + 0) = 3.0 \int x\, dA.$$

The integral $\int dA$ gives us the area A = 4 m^2 of the surface:

$$\Phi_r = 3.0 \int (3.0)\, dA = 9.0 \int dA.$$

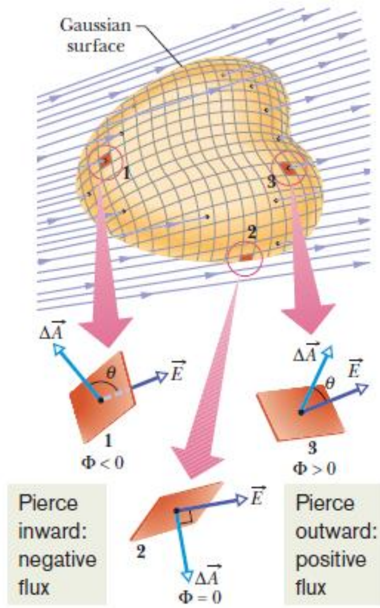
$$\Phi_r = (9.0\ \mathrm{N/C})(4.0\ \mathrm{m}^2) = 36\ \mathrm{N \cdot m^2/C}.$$

Closed Surface. Let’s use the closed surface in (Fig.3) that sits in a nonuniform electric field. To use Gauss’ law to relate flux and charge, we need a closed surface.

An inward field is negative flux. An outward field is positive flux

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux)}.$$

The loop on the integral sign indicates that we must integrate over the entire closed surface, to get the net flux through the surface (flux might enter on one side and leave on another side).



Figur 3: A Gaussian surface of arbitrary shape immersed in an electric field.

2. Gauss' Law

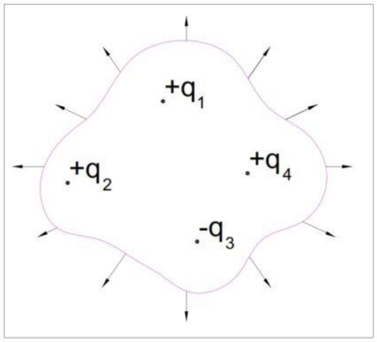
Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface. It tells us that:

$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss' law).}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{(Gauss' law).}$$

The net charge q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero.

If q_{enc} is positive, the net flux is outward; if q_{enc} is negative, the net flux is inward.



q_{enc} in figure above given below:

$$q_{enc} = q_1 + q_2 - q_3 + q_4$$

3. Gauss' Law and Coulomb's Law

we rewrite Gauss' law as:

$$\varepsilon_0 \oint \vec{E}.d\vec{A} = \varepsilon_0 \oint E \cos\theta \, .dA = q_{enc}$$

As shown in (Fig.4) for a particle with positive charge q. Then the electric field has the same magnitude E at any point on the sphere (all points are at the same distance r).

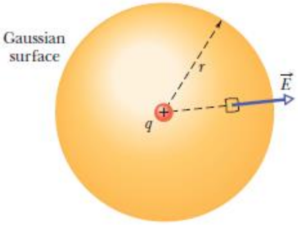


Figure 4: A spherical Gaussian surface centered on a particle with charge q.

we know that the electric field at $\vec{E}$ the patch is also radially outward and thus at angle $\theta = 0$ with $d\vec{A}$.So, we rewrite Gauss' law as

$$\varepsilon_0 \oint \vec{E}.d\vec{A} = \varepsilon_0 \oint E \, dA = q_{enc}$$

Here q_{enc} = q. Because the field magnitude E is the same at every patch element, E can be pulled outside the integral:

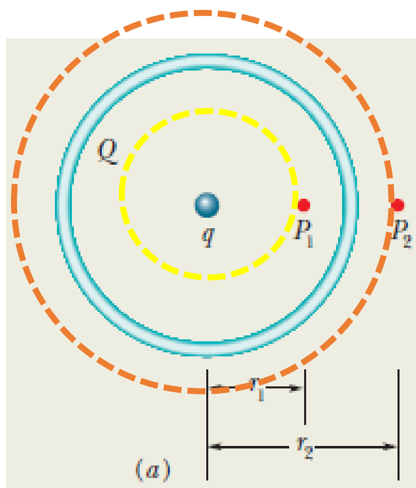
$$\varepsilon_0 E \oint dA = q$$

We already know that the total area of the sphere is $4\pi r^2$:

$$\varepsilon_0 E\,(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Example: Figure below shows, in cross section, a plastic, spherical shell with uniform charge $Q = -16e$ and radius R = 10 cm. A particle with charge $q = 5e$ is at the center. What is the electric field magnitude at (a) point P_1 at radial distance r_1 = 6 cm and (b) point P_2 at radial distance r_2 = 12 cm?



Solution:

The only charge enclosed by the Gaussian surface through P_1 is that of the particle. $q_{enc} = 5e$ and $r = r_1 = 0.06$ m.

$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2} = \frac{5(1.60\times10^{-19}\,\text{C})}{4\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.0600\,\text{m})^2} = 2.00\times10^{-6}\,\text{N/C}. \quad \text{(Answer)}$$

The only charge enclosed by the Gaussian surface through P_2 is that of the particle. $q_{enc} = q + Q = 5e - 16e = -11e$ and $r = r_2 = 0.12$ m.

$$E = \frac{-q_{enc}}{4\pi\varepsilon_0 r^2}$$
$$= \frac{-[-11(1.60 \times 10^{-19}\,\mathrm{C})]}{4\pi(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(0.120\,\mathrm{m})^2}$$
$$= 1.10 \times 10^{-6}\,\mathrm{N/C}. \qquad \text{(Answer)}$$

4. A Charged Isolated Conductor

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

Thus, the magnitude of the electric field just outside a conductor is proportional to the surface charge density on the conductor. The sign of the charge gives us the direction of the field. If the charge on the conductor is positive, the electric field is directed away from the conductor toward the conductor if the charge is negative.

If σ is the charge per unit area, then q_{enc} is equal to σA. When we substitute σA for q_{enc} and EA for Φ:

$$\varepsilon_0 EA = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface).}$$

5. Refrences

Walker, Jearl, Robert Resnick, and David Halliday. Halliday and resnick fundamentals of physics. Wiley, 2014.