Al-Mustaqbal University College

ميكانيك- المرحلة الاولى-فيزياء طبية

المحاضرة الرابعة

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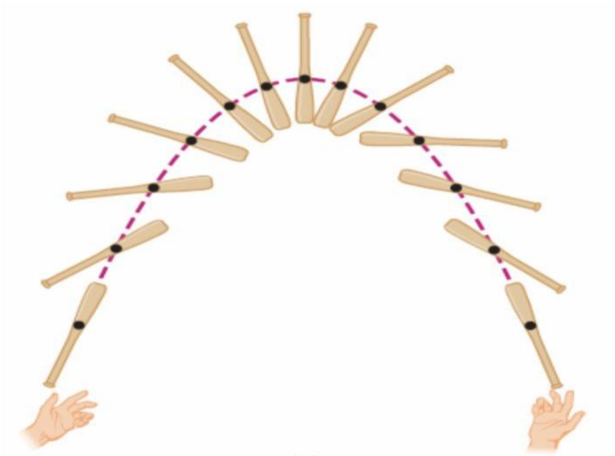
Center of mass

The center of mass of a system of particles is the point that moves as though

(a) *all of the mass were concentrated there;*

(b) *all external forces were applied there.*



The center of mass of system of N particles is a weighted average of their positions:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + \cdots + m_Nx_N}{m_1 + m_2 + \cdots + m_N}.$$

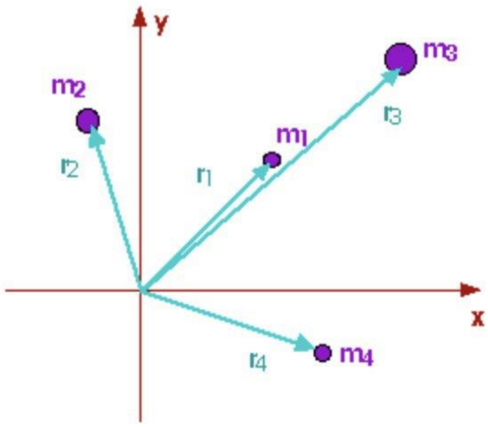
In fact, we can do this in any dimension:

$$y_{com} = \frac{m_1y_1 + m_2y_2 + \cdots + m_Ny_N}{m_1 + m_2 + \cdots + m_N},$$

$$z_{com} = \frac{m_1z_1 + m_2z_2 + \cdots + m_Nz_N}{m_1 + m_2 + \cdots + m_N}.$$

In three dimensions, the center of mass is:

$$\vec{r}_{com} = x_{\text{com}}\hat{i} + y_{\text{com}}\hat{j} + z_{\text{com}}\hat{k}$$



$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i}^{N} m_i\vec{r}_i$$

For solid bodies, the summation becomes an integral:

$$x_{\text{com}} = \frac{1}{M}\int x\,dm,$$
$$y_{\text{com}} = \frac{1}{M}\int y\,dm,$$
$$z_{\text{com}} = \frac{1}{M}\int z\,dm.$$

The body is sectioned into point masses dm.

The term **dm** *represents a small mass and depends on the problem at hand:*

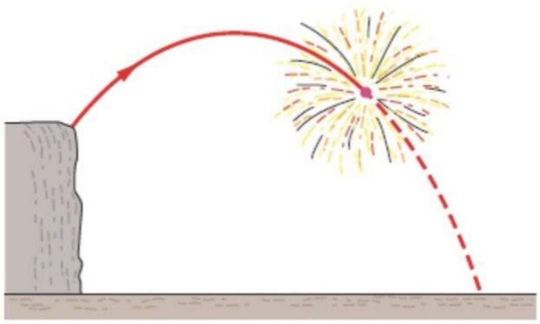
$$\begin{aligned} \text{1D:} \quad dm &= \lambda\, dx, \\ \text{2D:} \quad dm &= \sigma\, dA, \\ \text{3D:} \quad dm &= \rho\, dV. \end{aligned}$$

- λ is linear mass density kg/m
- σ is surface mass density kg/m^2
- ρ is volume mass density kg/m^3

For a system of particles (connected or not), Newton's 2nd Law applies to the center of mass:

$$\vec{F}_{net} = M\vec{a}_{\text{com}}$$

- $\vec{F}_{net}$ is the sum of all external forces on the particles
- M is the total mass of the particles
- $\vec{a}_{com}$ is the acceleration of the com



Linear momentum

The momentum of a particle is defined to be

$$\vec{p} = m\vec{v}.$$

If we take a derivative:

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

We can re-write Newton's 2nd Law using $\vec{p}$:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

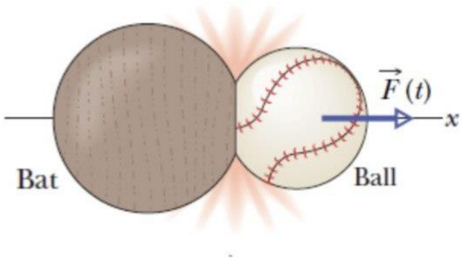
The momentum of a **system** *of particles is just*

$$\vec{P} = M\vec{v}_{\text{com}}.$$

We then get

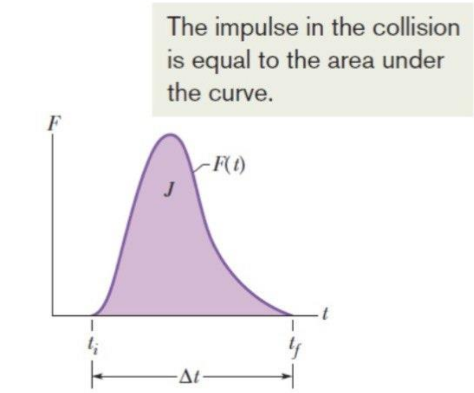
$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

When two objects collide there is a time varying force between them:

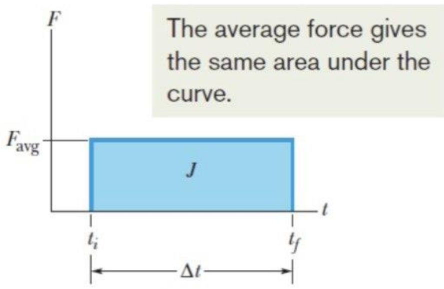


The impulse is a summation of the total change in momentum.

$$
\begin{aligned}
d\vec{p}/dt &= \vec{F} \\
d\vec{p} &= \vec{F}dt \\
\int_{t_i}^{t_f} d\vec{p} &= \int_{t_i}^{t_f} \vec{F}(t)dt \\
\vec{J} &= \int_{t_i}^{t_f} \vec{F}(t)dt \\
\vec{J} &= \vec{p}_f - \vec{p}_i
\end{aligned}
$$



(a)



(b)

If there are no external forces then momentum is conserved.

$$\frac{d\vec{P}}{dt} = \vec{F}_{net} = 0 \rightarrow \vec{P} = \text{constant}$$

Mathematically, we can write $\vec{P}_{before} = \vec{P}_{after}$ for

- Collisions
- Explosions

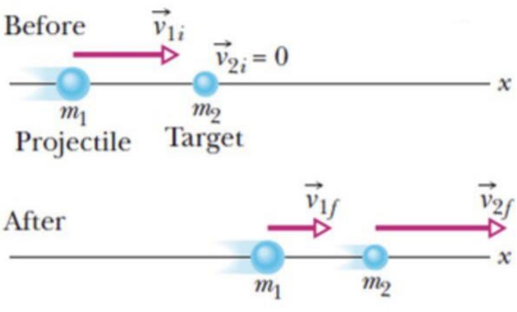
During an **elastic collision**, *kinetic energy is conserved.*

$$K_i = K_f \quad \text{or} \quad K = K'$$
$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1{v'_1}^2 + \frac{1}{2}m_2{v'_2}^2$$
$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

Examples:

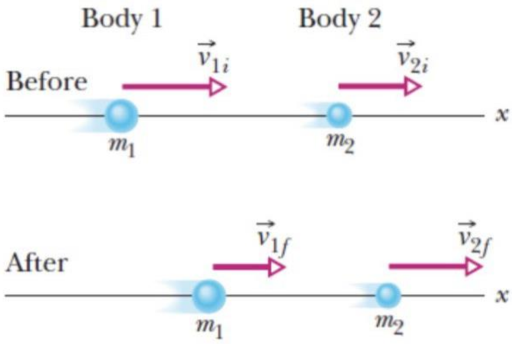
- Bouncy-balls colliding
- Carts with springs for bumpers

If an object of mass m_1 is shot at a stationary target of mass m_2 at a speed of v_{1i}, what are the speeds of the two objects after an elastic collision?



If an object of mass m_1 is shot with speed v_{1i} at a moving target of mass m_2 at a speed of v_{2i}, what are the speeds of the two objects after an elastic collision?

During an **inelastic collision**, *some kinetic energy is transferred to another form (e.g. heat or sound).*



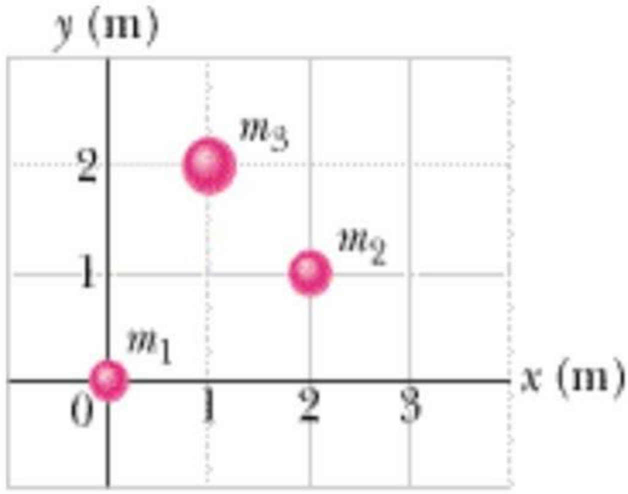
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Example: two pool balls striking

Examples:

1. Three particles of masses m_1=2 kg, m_2=3 kg, and m_3=5 kg are arranged in the xy plane, as shown in the figure below. Find the position vector of the center of mass.

Solution



We know that the position vector of the center of mass is defined as:

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

The components of the coordinate of the center of mass are

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M} = \frac{2\times0 + 3\times2 + 5\times1}{2+3+5} = 1.1\, m$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M} = \frac{2\times0 + 3\times1 + 5\times2}{2+3+5} = 1.3\, m$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3}{M} = \frac{2\times0 + 3\times0 + 5\times0}{2+3+5} = 0.0\, m$$

4. Four particles of masses m_1=2 kg, m_2=4 kg, and m_3= m_4= 3 kg have the following velocities: $\vec{v}_1 = 3\hat{i} + 4\hat{j}$, $\vec{v}_2 = 5\hat{i} - \hat{j}$, $\vec{v}_3 = -4\hat{i}$, and $\vec{v}_4 = 2\hat{j}$, where the velocities are measured in m/s. Find the linear momentum of the center of mass of the system.

Solution

The linear momentum of the center of mass of the system is

$$\vec{p}_{cm} = M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4$$

$$\vec{p}_{cm} = 2 \times \left(3\hat{i} + 4\hat{j}\right) + 4 \times \left(5\hat{i} - \hat{j}\right) + 3 \times \left(-4\hat{i}\right) + 3 \times \left(-2\hat{j}\right)$$

$$\vec{p}_{cm} = 14\hat{i} - 2\hat{j}$$

5. A motorcycle of mass 120 kg moves with a fixed speed of 15 m/s. Calculate the magnitude of its linear momentum.

Solution

The magnitude of the linear momentum is

$$p = mv = 120 \times 15 = 1800\ kg.m/s$$

6. A car is moving with a constant speed of 27 m/s. If its momentum is 21600 kg.m/s, what is its mass?

Solution

The magnitude of the linear momentum is defined as

$$p = mv$$

Therefore the mass is obtained by

$$m = \frac{p}{v} = \frac{21600}{27} = 800\ kg$$