The value of y_{t-1} is obtained from equation (i) as:

$$y_{t-1} = (0.75 \times 0.79) + 0.225 = (0.593 + 0.225) = 0.818$$

x_{t-1} is obtained from the equilibrium curve as 0.644

$$y_{t-2} = (0.75 \times 0.644) + 0.225 = (0.483 + 0.225) = 0.708$$

x_{t-2} from equilibrium curve = 0.492

$$y_{t-3} = (0.75 \times 0.492) + 0.225 = (0.369 + 0.225) = 0.594$$

x_{t-3} from the equilibrium curve = 0.382

This last value of composition is sufficiently near to that of the feed for the feed to be introduced on plate $(t-3)$. For the lower part of the column, the operating line equation (ii) will be used.

Thus:

$$y_{t-4} = (1.415 \times 0.382) - 0.042 = (0.540 - 0.042) = 0.498$$

x_{t-4} from the equilibrium curve = 0.298

$$y_{t-5} = (1.415 \times 0.298) - 0.042 = (0.421 - 0.042) = 0.379$$

x_{t-5} from the equilibrium curve = 0.208

$$y_{t-6} = (1.415 \times 0.208) - 0.042 = (0.294 - 0.042) = 0.252$$

x_{t-6} from the equilibrium curve = 0.120

$$y_{t-7} = (1.415 \times 0.120) - 0.042 = (0.169 - 0.042) = 0.127$$

x_{t-7} from the equilibrium curve = 0.048

This liquid x_{t-7} is slightly weaker than the minimum required and it may be withdrawn as the bottom product. Thus, x_{t-7} will correspond to the reboiler, and there will be seven plates in the column.

The method of McCabe and Thiele

The simplifying assumptions of constant molar heat of vaporisation, no heat losses, and no heat of mixing, lead to a constant molar vapour flow and a constant molar reflux flow in any section of the column, that is $V_n = V_{n+1}$, $L_n = L_{n+1}$, and so on. Using these simplifications, the two enrichment equations are obtained:

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D}{V_n} x_d \qquad \text{(equation 11.35)}$$

and:

$$y_m = \frac{L_m}{V_m} x_{m+1} - \frac{W}{V_m} x_w \qquad \text{(equation 11.37)}$$

These equations are used in the Lewis–Sorel method to calculate the relation between the composition of the liquid on a plate and the composition of the vapour rising to that plate. McCabe and Thiele[27] pointed out that, since these equations represent straight lines connecting y_n with x_{n+1} and y_m with x_{m+1}, they can be drawn on the same diagram as the equilibrium curve to give a simple graphical solution for the number of stages required. Thus, the line of equation 11.35 will pass through the points 2, 4 and 6 shown in Figure 11.14, and similarly the line of equation 11.37 will pass through points 8, 10, 12 and 14.

If $x_{n+1} = x_d$ in equation 11.35, then:

$$y_n = \frac{L_n}{V_n} x_d + \frac{D}{V_n} x_d = x_d \tag{11.38}$$

and this equation represents a line passing through the point $y_n = x_{n+1} = x_d$. If x_{n+1} is put equal to zero, then $y_n = Dx_d/V_n$, giving a second easily determined point. The top operating line is therefore drawn through two points of coordinates (x_d, x_d) and $(0, (Dx_d/V_n))$.

For the bottom operating line, equation 11.30, if $x_{m+1} = x_w$, then:

$$y_m = \frac{L_m}{V_m} x_w - \frac{W}{V_m} x_w \tag{11.39}$$

Since $V_m = L_m - W$, it follows that $y_m = x_w$. Thus the bottom operating line passes through the point C, that is (x_w, x_w), and has a slope L_m/V_m. When the two operating lines have been drawn in, the number of stages required may be found by drawing steps between the operating line and the equilibrium curve starting from point A.

This method is one of the most important concepts in chemical engineering and is an invaluable tool for the solution of distillation problems. The assumption of constant molar overflow is not limiting since in very few systems do the molar heats of vaporisation differ by more than 10 per cent. The method does have limitations, however, and should not be employed when the relative volatility is less than 1.3 or greater than 5, when the reflux ratio is less than 1.1 times the minimum, or when more than twenty-five theoretical trays are required[13]. In these circumstances, the Ponchon–Savarit method described in Section 11.5 should be used.

Example 11.8. The McCabe-Thiele Method

Example 11.7 is now worked using this method. Thus, with a feed composition, $x_f = 0.4$, the top composition, x_d is to have a value of 0.9 and the bottom composition, x_w is to be 0.10. The reflux ratio, $L_n/D = 3$.

Solution

a) From a material balance for a feed of 100 kmol:

$$V_n = V_m = 150;\ L_n = 112.5;\ L_m = 212.5;\ D = 37.5 \text{ and } W = 62.5 \text{ kmol}$$

b) The equilibrium curve and the diagonal line are drawn in as shown in Figure 11.15.
c) The equation of the top operating line is:

$$y_n = 0.75x_{n+1} + 0.225 \tag{i}$$

Thus, the line AB is drawn through the two points A (0.9, 0.9) and B (0, 0.225).

d) The equation of the bottom operating line is:

$$y_m = 1.415x_{m+1} - 0.042 \qquad \text{(ii)}$$

This equation is represented by the line CD drawn through C (0.1, 0.1) at a slope of 1.415.

e) Starting at point A, the horizontal line is drawn to cut the equilibrium line at point 1. The vertical line is dropped through 1 to the operating line at point 2 and this procedure is repeated to obtain points 3–6.

f) A horizontal line is drawn through point 6 to cut the equilibrium line at point 7 and a vertical line is drawn through point 7 to the lower enrichment line at point 8. This procedure is repeated in order to obtain points 9–16.

g) The number of stages are then counted, that is points 2, 4, 6, 8, 10, 12, and 14 which gives the number of plates required as 7.

Enrichment in still and condenser

Point 16 in Figure 11.15 represents the concentration of the liquor in the still. The concentration of the vapour is represented by point 15, so that the enrichment represented by the increment 16–15 is achieved in the boiler or still body. Again, the concentration on the top plate is given by point 2, but the vapour from this plate has a concentration given by point 1, and the condenser by completely condensing the vapour gives a product of equal concentration, represented by point A. The still and condenser together, therefore, provide enrichment (16 − 15) + (1 − A), which is equivalent to one ideal stage. Thus, the actual number of theoretical plates required is one less than the number of stages shown on the diagram. From a liquid in the still, point 16 to the product, point A, there are eight steps, although the column need only contain seven theoretical plates.

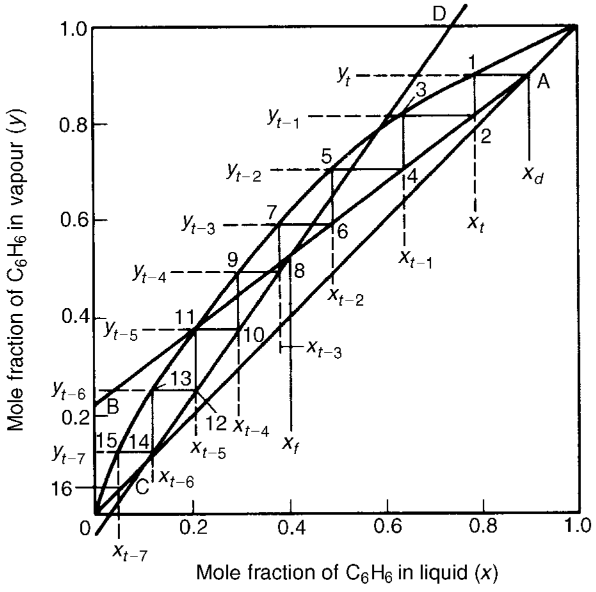

Figure 11.15. Determination of number of plates by the McCabe–Thiele method (Example 11.8)