Lecture 2

Optimisation formulation problem

Modelling optimization problem

- Mathematical modelling of optimisation problem

❖ A collection of mathematical equations that help to explain a system and to study the effects of different components and to make predictions about behaviour.

❖ Several approximations and assumption often made.

❖ It is often possible to construct more than one from of an mathematical model that represents the same problem equally accurately.

❖ Investigate the class of problem encountered.

$$\begin{array}{ccc}\textbf{Real problem} & \xrightarrow{\textbf{Formulating}} & \textbf{Mathematical model}\\ & & \Big\downarrow \textbf{Solving}\\ & & \textbf{Solution}\end{array}$$

Standard forms of the optimisation problem

1. An objective function $Min (max) f(x_1, x_2, \ldots, x_n)$

2. decision variables

3. Constraints

A set of inequality $g(x) \geq 0$

A set of equality $h(x) = 0$

$\therefore$ Every problems must be consisted of

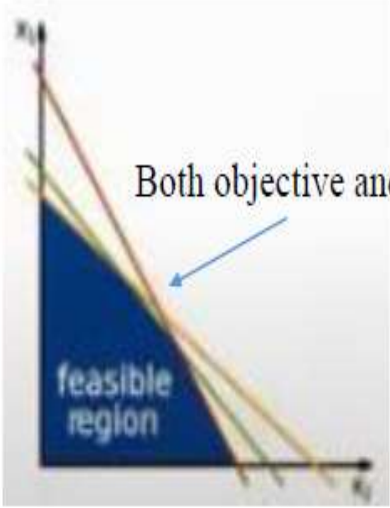
$$\left.\begin{array}{ll} \text{objective} & \min\limits_{x} f(x) \\ \text{Constrains} & g(x) \geq 0 \\ & h(x) = 0 \\ & LB \leq X \leq UB \end{array}\right\} \Rightarrow \text{general problem statement.}$$

Optimasation programing

Linear programing — Non-Linear programing

A- Linear programing

❖ If the objective functions and all the constraint functions in the optimization problem is linear, the problem is called a linear programming problem.



Both objective and constraint

B- Non-linear programing

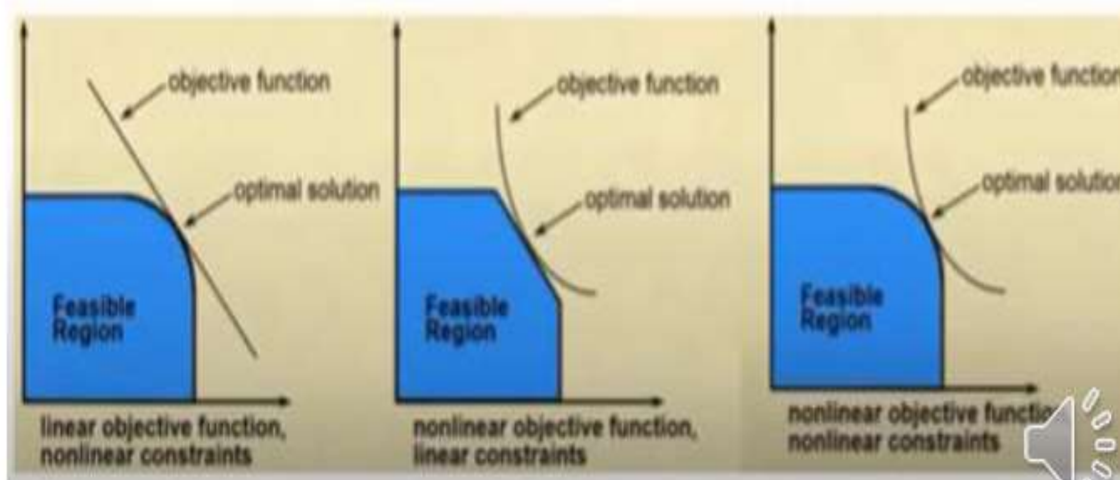
❖ If may of the function amoung the objuctive and constraints functions in the optimisation proplem is non-linear, the proplem is called a non-linear programmming proplem.

$$Min\ (Max)\ f(x_1, x_2, \dots, x_n)$$

Subject to the constraints

$$g(x_1, x_2, \dots, x_n)$$

$$h(x_1, x_2, \dots, x_n)$$



Example: Linear programing example. A company has requested a manufacture to produce formal and sports jackets, for materials, the manufacturer has 750 m^2 of cotton textile and 1000 m^2 of polyester. Every formal jackets needs 1 m2 of cotton and 2 m^2 of polyester. Every sport jacket needs 1.5 m^2 of cotton and 1 m^2 of polyester. The price of the formal jackets is 50$ and the sport jackets is 40$. The company want to know that the number of formal jackets and the sport jackets must request so that these items obtain a maximum scale. Formulate the problem as an optimisation problem.

Solution:

1- objective Function $\max F(x_1, X_2) = 50X_1 + 40X_2$

2- Decision variables X_1 = No. of formal jackets

X_2 = No. of sports jackets

3- Constraint $X_1 + 1.5X_2 \leq 75$ Cotton

$2X_1 + X_2 \leq 1000$ polyster

Example: A neighbourhood health clinic has a budget of 600,000$ per quarter. The director of the clinic wants to allocate the budget to maximize the number of patient visits, V, which is given as function of the number of the doctor, D and the number of the nurses, N, by $V=100\ D^6N^3$. A doctor gets 40.000 per quarter, nurses gets 10,000 per quarter. Formulate the formation mathematically?

Solution:

1- objective function Max $V(D,N) = 1000\, D^6 N^3$

2- Decision variables D = No. of doctor

N = No. of Nurses

3- Constraint $40{,}000\, D + 10{,}000\, N \leq 600{,}000$

OR

$40{,}000\, D + 10{,}000\, N = 600{,}000$

Example:

		Price	Demand	Seats to Sell
JFK	Regular	617	100	
LAX	Discount	238	150	

Capacity 166

Find the optimal number of discounted seats and regular seats to sell to maximums revenue? Formulate the problem mathematically

Maximize Total airline revenue
Subject to Seats sold can not exceed the capacity
Seats sold can not exceed the demand
Seats sold can not be negative

Solution:

First step: decisions

- Number of regular seats to sell R
- Number of discount seats to sell D

Second step: the objective

Maximizing the total airline revenue

- Revenue from each type of seat equal to the number of that type of seat sold times the seat price?

$$\max \; 617 * R + 238 * D$$

Step three: Constrains

The total number of seats sold can not exceed the capacity

$$R+D \leq 166$$

- Airline can not sell more seats than the demand:
  - Regular seats sold can not exceed 100

$$R \leq 100$$

Discount seats sold can not exceed 150

$$D \leq 150$$

Step four: Non-negativity

Airline can not sell a negative number of seats

$$R \geq 0 \quad D \geq 0$$

The problem mathematically:

$$\begin{array}{ll} \text{Maximize} & 617R + 238D \\ \text{Subject to} & R + D \leq 166 \\ & R \leq 100, D \leq 150 \\ & R \geq 0, D \geq 0 \end{array}$$