Lecture 1

Introduction

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1-Introduction to Optimization

2-Organization of Optimization Problems

General procedure for optimization, Formulation of optimization problems, Unconstrained and constrained problems.

3- Optimization Methods for Single Variable Problems

Analytical, Graphical methods, Numerical methods (fixed step, DSC and Newton methods for unconstrained functions) and (Sequential, Dichotomous, Fibonacci; Golden ratio search methods for constrained functions).

4- Optimization Techniques for Multivariable Problems

a- Unconstrained minimization and maximization procedure.

b- Solution of constrained multivariable problems by: Analytical method, Lagrangeion duality, Linearization, simplex method, Pivot table formulation.

c- Linear programming (LP) Formulation.

Text/ references books

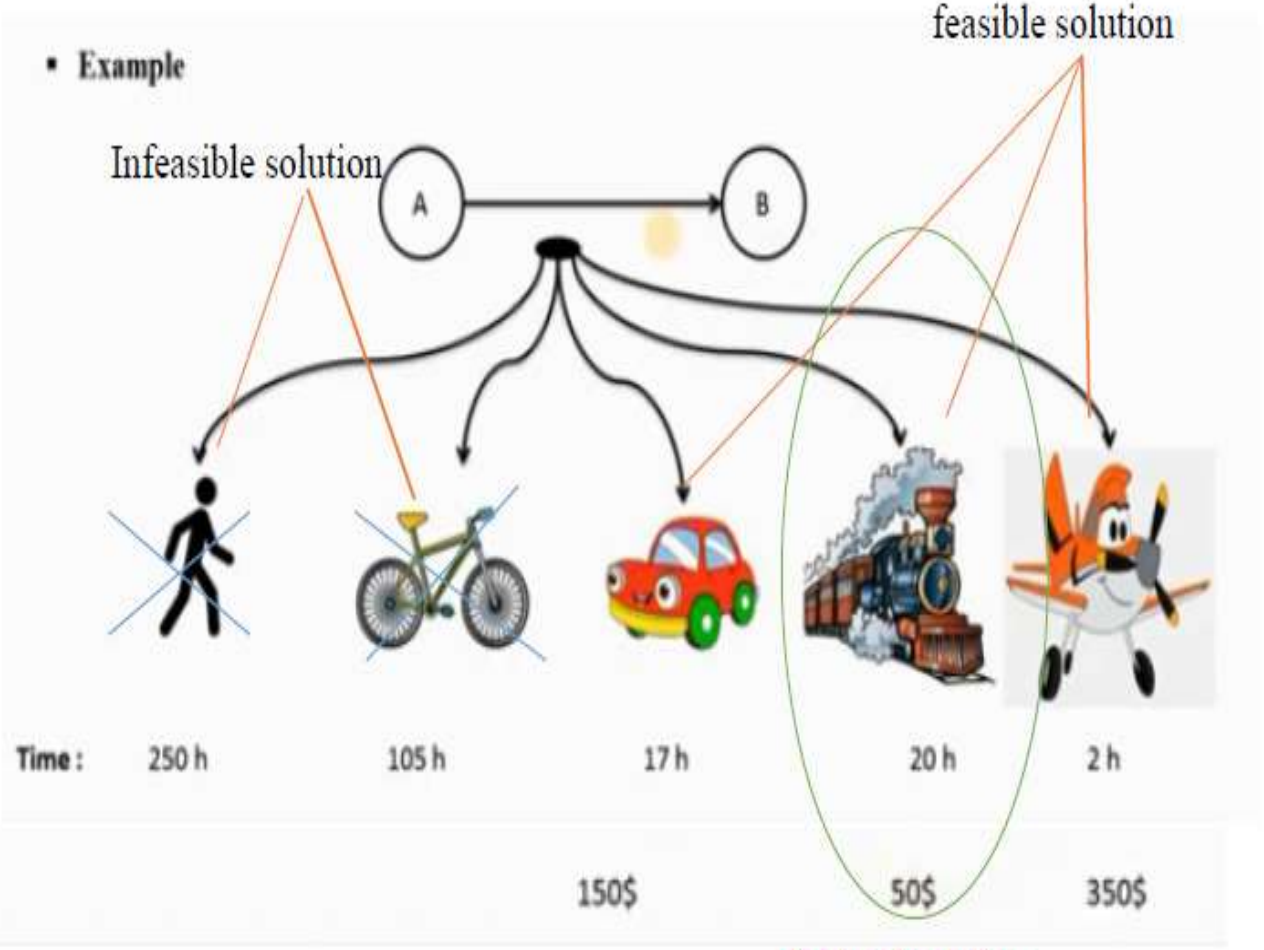
1- Optimasation of chemical processes-T.F.Edgar, D.M.Himmelblau and L.S.Lasdon, 2nd edition,McGraw Hill,2001.

2- Engineering optimization: methods and applications-A.Ravindran, K.M.Ragsdell,G.V.Reklaitis, 2nd edition,,Wiiley india,2006.

3- Engineering optimization: theory and practice-s.S.Rao, 4th edition, John Wiley & Sons,Inc,2009.

Introduction

- Finding the best solution among set of all feasible solutions.
- Deals with the problem of minimizing or maximizing a function with several variables subject to some constrains.
- Every, almost daily, solves optimisation problems in formal ways by using mental models.
- Optimisation plays a central rule in operation research, managing science and engineering.



1- Time 25 h

2- objective : min cost

Optimal solution

Why optimize

- Best design
- solving for the number of pipes in a piping network that result in minimal pressure drop
- solving for the size of an engine block that results in the best combination of speed and efficiency (using some weighting on each of these metrics its importance)

❖ Lowest cost

- Find the combination of truck routs to transport a set amount of raw material between points that results in the lowest total cost (labour, truck maintained fuel)

❖ Highest efficiency

- Maximising the efficiency of a power production system by finding the right combination of inputs (flow rate, temperatures, pressure,..)
- In Chemical Engineering

❖ Utilize resource/energy/utility in the most efficiency way.

❖ Reduce waste generation; minimize the environmental impact.

❖ Determine the most desirable operating conditions; safe operation

❖ Meet product specification; maximize profit.

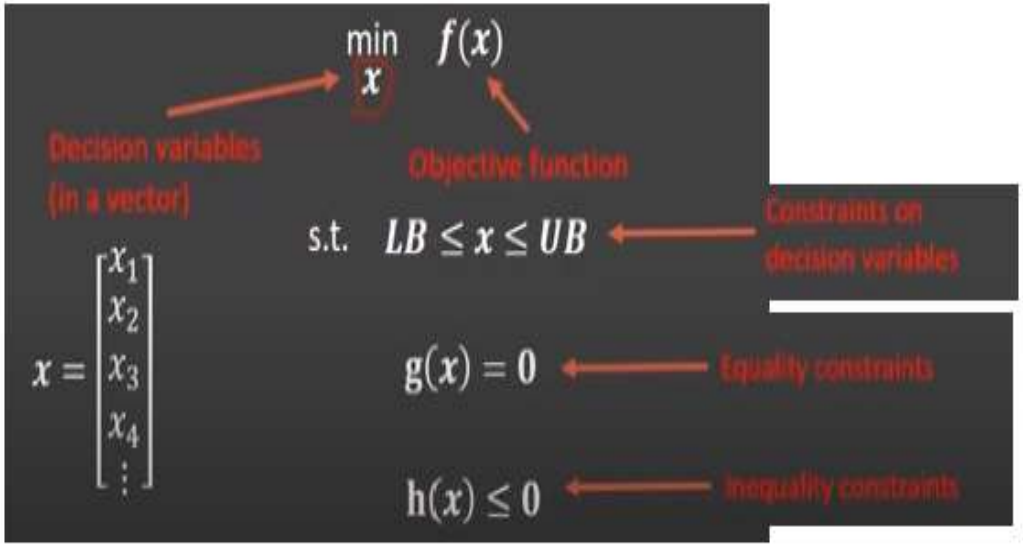
❖ deciding on the most effective allocation of limited resources

❖ choosing control variables that will cause a system to behave as desired.

- Some objectives in optimisation problems may be:
  - minimise cost
  - Maximise the profit
  - planning production
  - increase process efficiency

Optimisation Terminologies

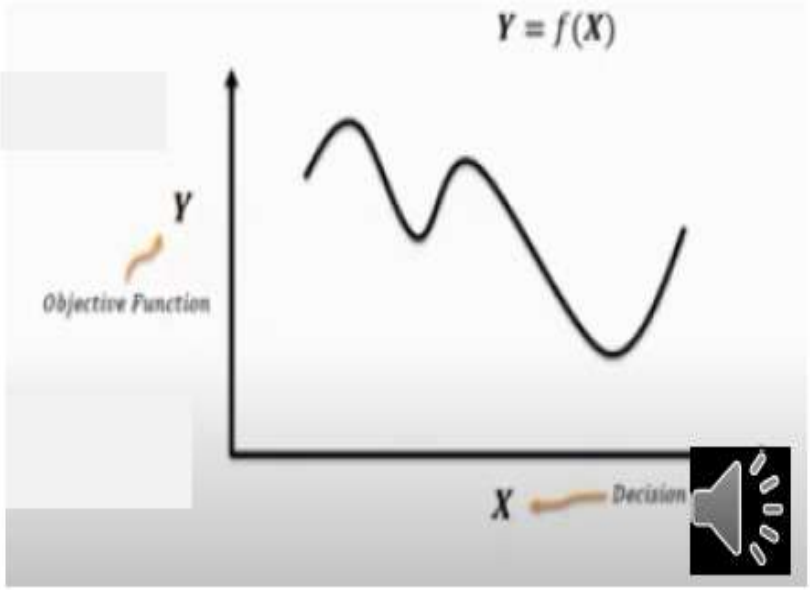
Objective function : the function that it is desired to maximize or minimize

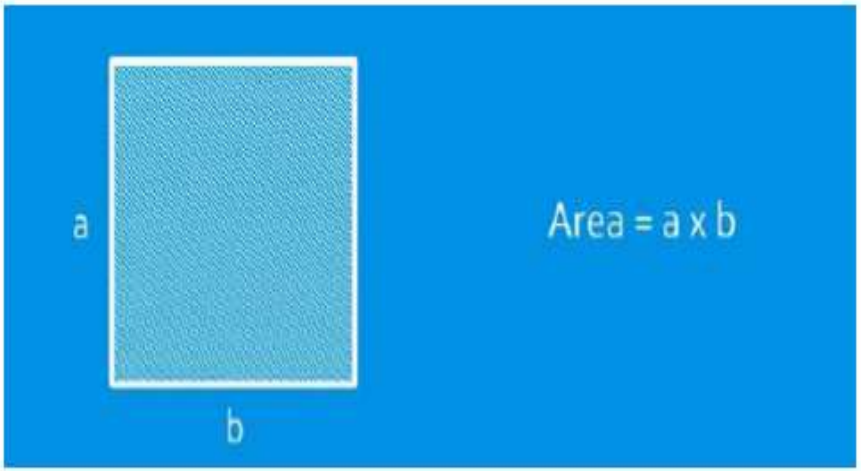


Decision variables :

a- are the inputs to your problem that your optimizer can change to try to improve the objective function

b- all decision variables should be independent of each other.





$$\underset{x}{minimize}\ f(x)$$

Objective = a + b

Objective = a x b

Objective = 3a + 4ab

$$\underset{x}{minimize}\ f(x_1, x_2, x_3, x_4, x_5 \ldots)$$

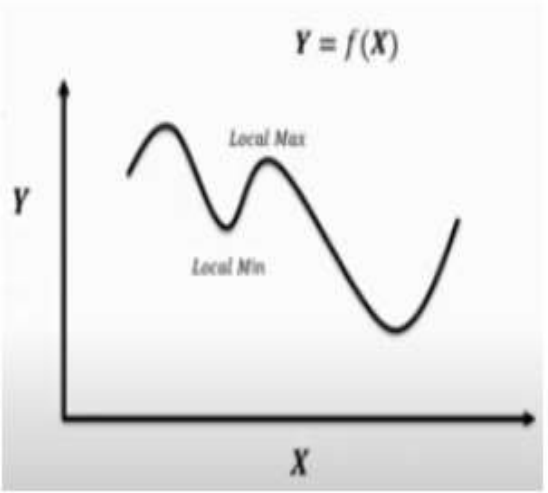
- Objective Function
  - The value you are trying to optimize
  - Minimized or maximized
- Decision Variables
  - The values the optimizer can change
  - Also called design or manipulated variables

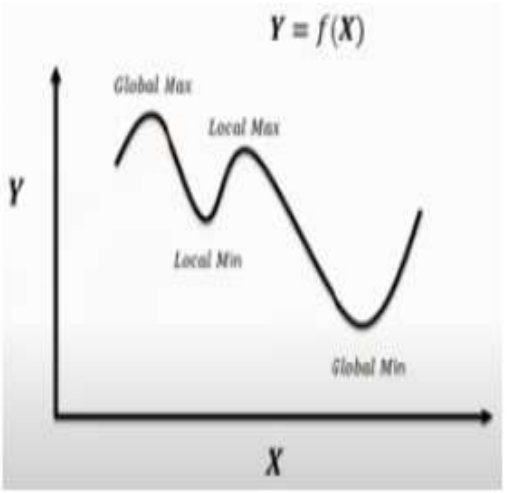
value in the example the decision variables would be the length of the two sides

(variable may be called design variable s or manipulated variables).

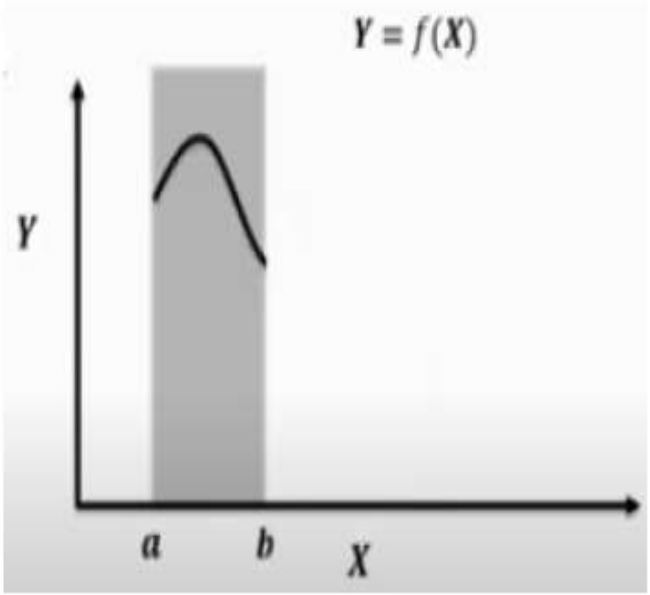
Some other important definitions:

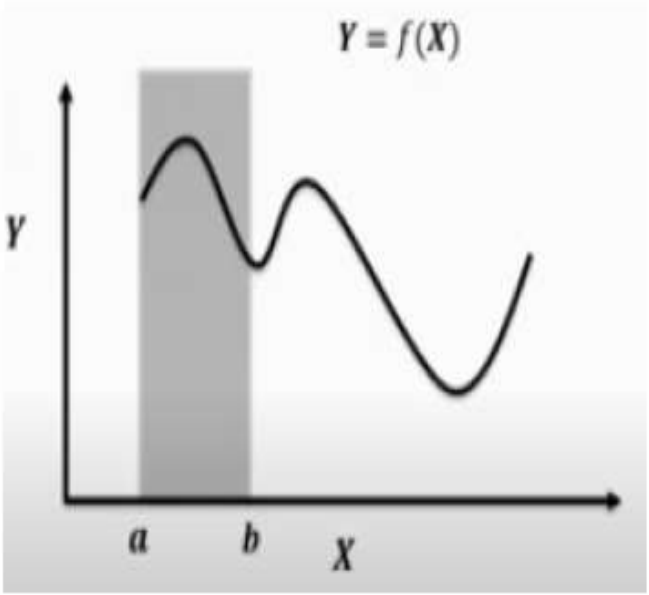
- Optimum value: it is a technical term including quantities measurements and mathematical analysis to determine the best setting (maximum or minimum) of a dependent variables.
- Optimization procedure: it is the process of determining the optimum value (maximum or minimum) of some criterion function.
- Optimization problem: is the specification of the variables that need to be optimized.
- Local minimum (maximum) a point where the function value is smaller (greater) than or equal to the value at nearby points.
- Global minimum (maximum) a point where the function value is smaller (greater) than or equal to the value at all other feasible points.



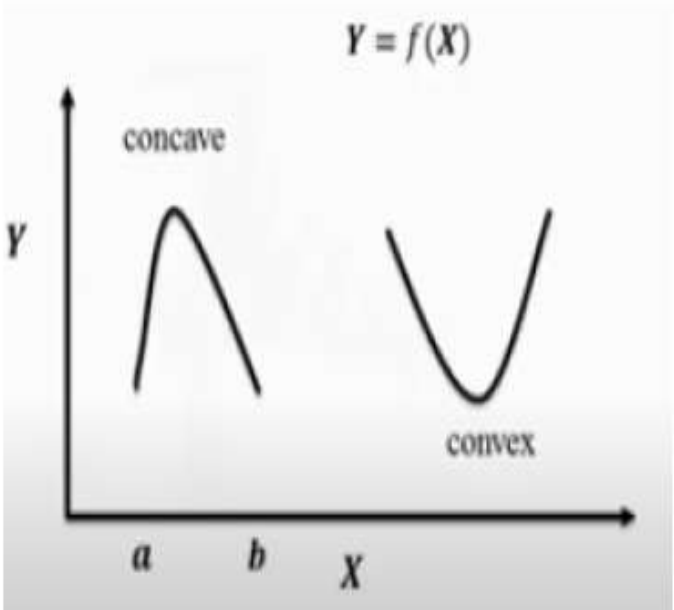


❖ Search space : a function $f(X)$ defined at a closed interval $[a, b]$.



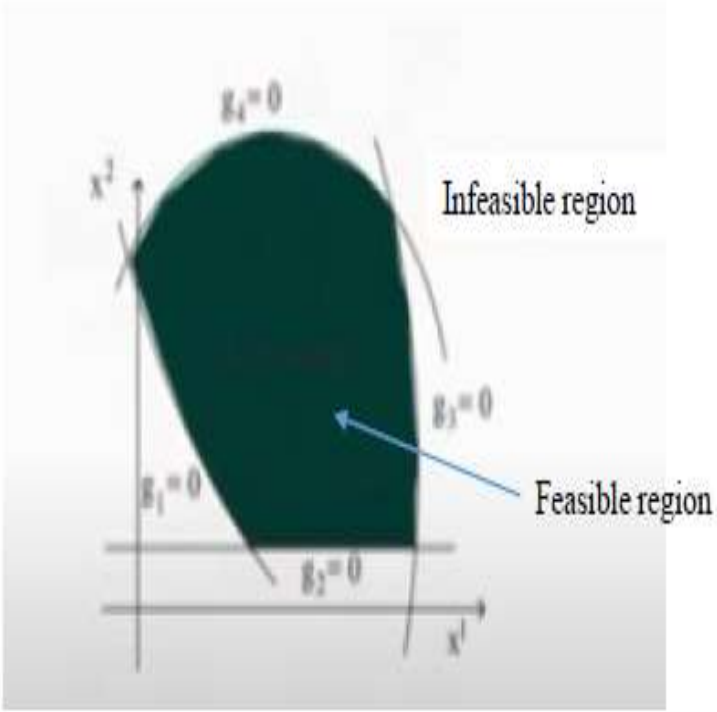


❖ Unimodal function : a function has only one peak (maximum, concave) or valley (minimum, concave) in a given interval



Constraint: The constrains represent some functional relationships among the design variables and other design parameters satisfying certain physical phenomenon and certain resource limitations. Figure below shows the feasible regain in a two dimensional space.

The set of values of X that satisfy the equation gi (x) = 0 forms a hyper surface in the design space and is called a constraint surface.



Classification of the optimization problems

- In an optimization problem, the types of mathematical relationships between the objective and constrains and decision variables determine how hard it is to solve, the solution methods or algorithms that can be used for optimisation and the confidence you can have that the solution is truly optimal.

Classification based on the number of decision variables

- ❖ single variable
- ❖ Multi variables

- Classification based on the existence of constrains

- ❖ Unconstraint optimisation
- ❖ constraint optimisation

- Classification based on the existence of constrains

- ❖ Linear optimisation
- ❖ Nonlinear optimisation

- Classification based on the number objective function

- ❖ single variable
- ❖ Multi variables

- Classification based on the deterministic nature of the variables

- Static
- ❖ Dynamic

- Classical optimization techniques
  - ❖ single variable optimisation
  - ❖ Multi variables optimisation
- Multi variables optimisation with no constrained
- Multi variables optimisation with equality constraints
- Multi variables optimisation with inequality constraints
- Numerical optimization techniques
  - ❖ elimination method
  - ❖ Golden section method
- Interpolation methods
  - ❖ Newton method
- Advanced optimisation techniques
  - ❖ Heuristic

How to solve an optimisation problem

- Examine the structure of the system and the inter-relationship of the system elements.
- Develop a model of your system
  - consider the model complexity
  - consider the model form (linear, quadratic, nonlinear, mixed-integer,…), these are listed in order of increasing complexity, general speaking.
- Determine your objective function (what are you trying to maximise or minimise)
  - This is usually a combination of additives terms, all put on the same basis (profit= sum of revenue –sum of costs)
  - By conversion, most solvers will require your problem to be a minimisation problem. For example: if you want to
  - maximise profit, you actually need to formulate you problem so that the profit is minimised.
- Examine the restrictions imposed upon the problem.
- Determine your constrains (a flow rate ≥ 0, temperature $\leq$ some explosive limit, ..)
- Put the math representing your model into the form required by the solver
  - Many solvers require putting the problem into matrix form.
  - Determine the optimum solution and discuss the nature of system conditions.

Setting up the problem in the right format is most of the battle.

Thank you