Lecture 3

Optimisation formulation problem

- # Extension of linear programming

❖ Goal programming

➢ Special class of linear programming where decision maker set multiple objectives (often conflicting) instead of one.

➢ Procedure:

✓ Each objective is considered as a goal or target to be achieved.

✓ Deviations from these goals in case of above (d+) and below (d-) the target are measured.

✓ Unwanted deviations from this set of target values are then minimised.

- **Linear Programming vs. Goal Programming**

Linear Programming	Goal Programming
One objective	Multi objectives
Objective (Max or Min)	Objectives (only Min)
Relative simple problem	Relative complex problem
Write objectives Then constraints	Write constraints Then objectives
Optimal solution	Near optimal solution
Constraints sings ($\leq, \geq$ *and* $=$)	Constraints sings (*only* $=$)

❖ Goal programming example. Company produce two products (A and B) . The two products require a one hour labour work. The available labour work is 300 hours per month. The demand product A and B in a month are 140 and 200 respectively. The profit from the sale product A is 600 $ and product B is 200 $. The manger has set the following goals:

P1: avoid any underutilisation of normal production capacity.

P2: sell maximum possible units of product A and B.

P3: minimums the overtime operation of the plant as much as possible.

Formulate the given problem as an optimisation problem?

Solution :

Decision variables X_1 = No. of product A
X_2 = No. of product B

Goal 1 :- $(P_1)\ X_1 + X_2 + d_1^+ + d_1^- = 300$

unwanted deviations Goal 1 $\rightarrow d_1^-$ ---- (1)

Goal 2 $(P_2)\ X_1 + d_2^- = 140$

Goal 2 $(P_2)\ X_2 + d_3^+ = 200$

unwanted deviations Goal 2 $\rightarrow 3d_2^- + d_3^-$

Goal 3 $X_1 + X_2 + d_1^+ + d_1^- = 300$

unwanted deviations Goal 3 $\rightarrow d_1^+$

$\therefore$ Min $P_1(d_1^-)$

$P_2(3d_2^- + d_3^+)$

$P_3(d_1^+)$

subjected to

$X_1 + X_2 + d_1^+ + d_1^- = 300$

$X_2 + d_2^- = 140$

$X_2 + d_3 = 200$

All variables ≥ 0

Examples for non-linear programming:

A company is planning to introduce two new products: A and B. It is estimated that the price of product A is 339 $, It also estimated that the cost of product A is 195$ per unit, and the cost of product b IS 225$ per unit, plus additional cost of 400000 $. In competitive market the number of sales will effect the seles price. It is estimated that for each product, the sales price drops by one cent for each additional unit sasold. Furthermore, sales of the product will affect sales of product B and vise versa. It is estimated that the price for the product A will reduced by an additional 0.3 cents for each product B sold and the price for product B will decrease for by 0.4 cent for each of product A sold. Formulate the given problem to maximize the profit of the company?

(1) Decision variables

X_1 = No. of product A

X_2 = No. of product B

price of product A → $P_1 = 339 - 0.01 X_1 - 0.003 X_2$

price of product B → $P_2 = 399 - 0.01 X_2 - 0.004 X_1$

∴ Revenue → $R = P_1 X_1 + P_2 X_2$

$R = (339 - 0.01 X_1 - 0.003 X_2) X_1 + (399 - 0.01 X_2 - 0.004 X_1) X_2$

∴ $R = (339 X_1 - 0.01 X_1^2 - 0.003 X_1 X_2) + (399 X_2 - 0.01 X_2^2 - 0.004 X_1 X_2)$

non-linear

$R = 339 X_1 + 399 X_2 - 0.01 X_1^2 - 0.01 X_2^2 - 0.007 X_1 X_2$

profit = R − C (R → Revenue, C → Cost)

Costs $C = 195 X_1 + 225 X_2 + 400000$

∴ $\text{profit} = 339 X_1 + 399 X_2 - 0.01 X_1^2 - 0.01 X_2^2 - 0.007 X_1 X_2 - 195 X_1 - 225 X_2 - 400000$

∴ $\text{Max profit}(X_1, X_2) = 144 X_1 + 174 X_2 - 0.01 X_1^2 - 0.01 X_2^2 - 0.007 X_1 X_2 - 225 X_2 - 400000$

Thank you