المحاضرة الثانية

الدوال المثلثيةTrigonometric functions

$$1.\ \sin(x \mp y) = \sin x \cos y \mp \sin y \cos x \qquad 2.\ \sin 2x = 2\sin x \cos x$$

$$3.\ \cos(x \mp y) = \cos x \cos y \pm \sin x \sin y \qquad 4.\ \cos 2x = \cos^2 x - \sin^2 x$$

$$5.\ \sin^2 x + \cos^2 x = 1 \quad , \quad \tan^2 x + 1 = \sec^2 x \quad , \quad 1 + \cot^2 x = \csc^2 x$$

$$6.\ \sin^2 x = \frac{1 - \cos 2x}{2} \quad , \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

مشتقات الدوال المثلثية :

$$1.\ \frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx} \qquad 2.\ \frac{d}{dx}(\cos u) = -\sin u \cdot \frac{du}{dx}$$

$$3.\ \frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx} \qquad 4.\ \frac{d}{dx}(\cot u) = -\csc^2 u \cdot \frac{du}{dx}$$

$$5.\ \frac{d}{dx}(\sec u) = \sec u \tan u \cdot \frac{du}{dx} \qquad 6.\ \frac{d}{dx}(\csc u) = -\csc u \cot u \cdot \frac{du}{dx}$$

Examples

$$1)\ \frac{d}{dx}(\sin y) = \cos y \frac{dy}{dx}$$

مثال / جد $\frac{d}{dx}\sin 5x$

الحل / $\frac{d}{dx}\sin 5x = \cos 5x \cdot 5 = 5\cos 5x$

$$2)\ \frac{d}{dx}(\cos y) = -\sin y \frac{dy}{dx}$$

مثال / جد $\frac{d}{dx}\cos \frac{x}{2}$

الحل / $\frac{d}{dx}\cos \frac{x}{2} = -\sin \frac{x}{2} \cdot \frac{1}{2} = \frac{1}{2}\sin \frac{x}{2}$

$$3)\ \frac{d}{dx}(\tan y) = \sec^2 y \frac{dy}{dx}$$

مثال / جد $\frac{d}{dx}\tan x^2$

$\frac{d}{dx}\tan x^2 = \sec^2 x^2 (2x) = 2x \sec^2 x^2$

$$4)\ \frac{d}{dx}(\cot y) = -\csc^2 y \frac{dy}{dx}$$

مثال / جد $\frac{d}{dx}\cot 8x$

$\frac{d}{dx}\cot 8x = -\csc^2 8x \cdot 8 = -8\csc^2 8x$

$$5)\ \frac{d}{dx}(\sec y) = \sec y \tan y \frac{dy}{dx}$$

مثال / جد $\frac{d}{dx}\sec 4x$

الحل $\frac{d}{dx}\sec 4x = sec\ 4x\ tan\ 4x \cdot 4 = 4sec\ 4x\ tan\ 4x$

$$6)\ \frac{d}{dx}(\csc y) = -\csc y \cot y \frac{dy}{dx}$$

مثال / جد $\frac{d}{dx}\csc 5x$

الحل /

$$\frac{d}{dx}\csc 5x = -csc\ 5x\ cot\ 5x\ \cdot 5 = -5csc\ 5x\ cot\ 5x$$

مثال / اذا كانت $f(x) = \text{Sin}(7x^2 + 4x + 1)$ جد $f'(x)$

الحل /

$$f'(x) = \cos(7x^2 + 4x + 1)(14x + 4)$$
$$=(14x + 4)\cos(7x^2 + 4x + 1)$$

مثال / اذا كانت $f(x) = \text{Sin}\sqrt[3]{x}$ جد $f'(x)$

الحل /

$$f(x) = \text{Sin}\ x^{\frac{1}{3}} \Longrightarrow \quad f'(x) = \cos x^{\frac{1}{3}}\left(\frac{1}{3}x^{-\frac{2}{3}}\right) = \frac{1}{3}x^{-\frac{2}{3}}\cos\sqrt[3]{x}$$

مثال / اذا كانت $f(x) = \cos^3 7x$ جد $f'(x)$

$$f'(x) = 3\cos 2\ 7x\ (-\sin 7x\ \cdot 7)$$
$$f'(x) = -21\cos 2\ 7x \sin 7x$$

مثال / اذا كانت $f(x) = \cos 3x - \tan 5x + \sec 4x$ جد $f'(x)$

$$f'(x) = -3\sin 3x - 5\sec^2 5x + 4\sec 4x \tan 4x$$

مثال / اذا كانت $f(x) = (\text{Sec } 5x)^3$ جد $f'(x)$

الحل /

$$f'(x) = 3(\text{Sec } 5x)^2\ (\sec 5x \tan 5x)\ 5 = 15\sec^3 5x \tan 5x$$

اذا كانت $y = \text{Sin}(5 - x^3)$ جد y'

$$y' = \cos(5 - x^3)(-3x^2) = -3x^2 \cos(5 - x^3)$$

اذا كانت $y = \sqrt{cos(4x + 2)}$ جد y'

الحل /

$$y' = \frac{-sin\,(4x+2)(4)}{2\sqrt{cos(4x+2)}} = \frac{-2\,sin(4x+2)}{\sqrt{cos(4x+2)}}$$

اذا كانت $y = x \text{ Sec } x^2$ جد y'

الحل /

$$y' = x \text{ Sec } x^2 \tan x^2 (2x) + \sec x^2 (1) = 2x^2 \text{ Sec } x^2 \tan x^2 + \sec x^2$$

اذا كانت $y = \text{Sin } 3x \cos 3x$ جد y'

$$y' = \text{Sin } 3x (-\sin 3x) (3) + \cos 3x \cos 3x (3) = -3\text{Sin}^2 3x + 3\cos^2 3x$$

اذا كانت $y = \sqrt[3]{cot^2\,4x}$ جد y'

الحل /

$$y = (cot^2\,4x)^{\frac{1}{3}} = (cot\,4x)^{\frac{2}{3}}$$

$$y' = \frac{2}{3}(cot\,4x)^{-\frac{1}{3}}(-csc\,4x)(4) = \frac{-8csc\,4x}{3\sqrt[3]{cot\,4x}}$$

اذا كانت $y = \csc^5 (x^2 + 1)$ جد y'

الحل /

$$y' = 5\csc^4 (x^2 + 1)[- \csc (x^2 + 1) \cot (x^2 + 1)] (2x)$$

$$= -10\csc^5 (x^2 + 1) \cot (x^2 + 1)$$

/ أذا كانت $y = (\sin 3x - \cos 3x)^2$ جد y'

الحل /

$$y = \sin^2 3x - 2 \sin 3x \cos 3x + \cos^2 3x$$

$$y = \sin^2 3x + \cos^2 3x - 2 \sin 3x \cos 3x$$

$$y = 1 - \sin 6x$$

$$y' = -\cos 6x\,(6) = -6 \cos 6x$$

/ اثبت صحة $\frac{d}{dx}[\sin ax - \frac{1}{3}\sin^3 ax] = a\cos^3 ax$

الحل /

$$\frac{d}{dx}[\sin ax - \frac{1}{3}\sin^3 ax] = \cos ax \cdot a - \sin^2 ax \cdot \cos ax \cdot a$$
$$= a\cos ax - a\sin^2 ax \cos ax$$
$$= a\cos ax\,(1 - \sin^2 ax)$$
$$= a\cos ax \cos^2 ax = a\cos^3 ax$$

اثبت صحة $\frac{d}{dx}(\frac{2-\cos x}{2+\cos x}) = \frac{4\sin x}{(2+\cos x)^2}$

الحل

$$L \cdot H = \frac{d}{dx}(\frac{2-\cos x}{2+\cos x}) = \frac{(2+\cos x)(\sin x) - (2-\cos x)(-\sin x)}{(2+\cos x)^2}$$
$$= \frac{2\sin x + c\cos x \sin x + 2\sin x - \cos x \sin x}{(2+\cos x)^2}$$
$$= \frac{4\sin x}{(2+\cos x)^2}$$

اذا كانت $y = \cos 2x$ فجد $\frac{d^4y}{dx^4}$

الحل /

$$\frac{dy}{dx} = -2\sin 2x$$

$$\frac{d^2y}{dx^2} = -4\cos 2x$$

$$\frac{d^3y}{dx^3} = 8\sin 2x$$

$$\frac{d^4y}{dx^4} = 16\cos 2x$$

/ اذا كانت $y = \tan x$ فبرهن $\frac{d^2y}{dx^2} = 2y(1 + y^2)$

الحل /

$$\frac{dy}{dx} = sec^2x$$

$$\frac{d^2y}{dx^2} = 2\sec x \cdot \sec x \tan x$$

$$\frac{d^2y}{dx^2} = 2\ \sec^2 x \tan x$$

$$\frac{d^2y}{dx^2} = 2\ (1 + \tan^2 x) \tan x$$

نعوض $y = \tan x$

$$\frac{d^2y}{dx^2} = 2\ (1 + y^2)\, y = 2y(1 + y^2)$$

اذا كانت $y = x\ sin\ x$ فبرهن ان $y^{(4)} - y + 4\cos x = 0$

الحل /

$$\frac{dy}{dx} = x\cos x + \sin x\,(1) = x\cos x + \sin x$$

$$\frac{dy^2}{d^2x} = x(-\sin x) + \cos x\,(1) + \cos x$$

$$\frac{dy^2}{d^2x} = -x\sin x + 2\cos x$$

$$\frac{dy^2}{d^2x} = -y + 2\cos x$$

$$\frac{dy^2}{d^2x} + y = 2\cos x$$

$$\frac{dy^3}{d^3x} + y^{/} = -2\sin x$$

$$\frac{dy^4}{d^4x} + y^{//} = -2\cos x$$

$$\frac{dy^4}{d^4x} + 2\cos x - y = -\cos x$$

$$\frac{dy^4}{d^4x} + 2\cos x - y + 2\cos x = 0$$

$$\frac{dy^4}{d^4x} + 4\cos x - y = 0$$

Home Work

Differentiate the following:

1. $\cos 3x$
2. $\sin(4x+5)$
3. $\sin^3 x$
4. $\sin x \cos x$
5. . $x^2 \sin x$
6. $\cos(x^2+1)$
7. $\frac{\sin x}{x}$
8. $\sin \frac{1}{x}$
9. $\tan(\sqrt{x})$
10. $\frac{1}{x} \sin \frac{1}{x}$

Prove that

$$y = \tan x + \frac{1}{3} tan^3 x = sec^4 x$$