Heat Exchanger Effectiveness (NTU method)

If more than one of the inlet and outlet temperature of the heat exchanger is unknown, LMTD may be obtained by trial and errors solution. Another approach introduce the definition of heat exchanger effectiveness (Є), which is a dimensionless with ranging between 0 to1.

$$\epsilon = \frac{q_{act}}{q_{\max}}$$

Where, q_{max} is the maximum possible heat transfer for the exchanger. The maximum value could be attained if one of the fluids were to undergo a temperature change equal to the maximum temperature difference present in the exchanger, which is the difference in the entering temperatures for the hot and cold fluids.

Let C = mCp

$$q_{act} = C_h(Th_i - Th_o) = Cc(Tc_o - Tc_i)$$

The maximum possible heat transfer when the fluid of small C undergoes the maximum temperature difference available

$$q_{\max} = C_{\min}(Th_i - Tc_i)$$

$$q_{act} = \varepsilon\, C_{\min}(Th_i - Tc_i)$$

For parallel flow H.E with combining the last three equations, we get two expressions for effectiveness

$$\epsilon = \frac{C_h(Th_i - Th_o)}{C_{\min}(Th_i - Tc_i)} = \frac{Cc(Tc_o - Tc_i)}{C_{\min}(Th_i - Tc_i)}$$

For $C_h < Cc$: $$\epsilon_h = \frac{Th_i - Th_o}{Th_i - Tc_i}$$

For $C_h > Cc$: $$\epsilon_c = \frac{Tc_o - Tc_i}{Th_i - Tc_i}$$

Using the following equation,

$$\ln\frac{Th_o - Tc_o}{Th_i - Tc_i} = -UA\left(\frac{1}{C_h} + \frac{1}{Cc}\right)$$

We get

$$\frac{Th_o - Tc_o}{Th_i - Tc_i} = \exp\left[-\frac{UA}{C_h}(1+\frac{C_h}{Cc})\right]$$

From energy balance,

$$Tc_o = Tc_i + \frac{C_h}{Cc}(Th_i - Th_o)$$

By using the last two equations, we obtained

$$\epsilon_h = \frac{1-\exp[(-UA/C_h)(1+(C_h/Cc)]}{1+(C_h/Cc)}$$

and

$$\epsilon_c = \frac{1-\exp[(-UA/Cc)(1+(Cc/C_h)]}{1+(Cc/C_h)}$$

The last two equations may be written as

$$\epsilon = \frac{1-\exp[(-UA/C_{\min})(1+C_{\min}/C_{\max})]}{1+C_{\min}/C_{\max}}$$

The terms *UA/Cmin* is called the number of transfer units (NTU) since it is indicative of the size of the heat exchanger, i.e

$$NTU = \frac{UA}{C_{\min}}$$

Figures 10.12 to 10.17 are available to obtain the value ϵ instead of using equations.

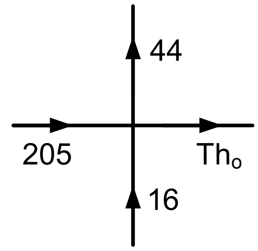
Note, in a boiler or condenser, *Cmin/Cmax* → 0 and all the heat-exchanger effectiveness relations approach a single simple equation,

$$\epsilon = 1 - e^{-\mathrm{NTU}}$$

Examples 10.10, 10.11, 10.13 and 10.14 are requested.

Example: A double pipe parallel flow H.E. use oil (cp = 1.88 kJ/kg.K) at an initial temperature of 205°C to heat water, flowing at 225kg/hr from 16°C to 44°C. The oil flow rate is 270 kg/hr. a) what is the heat transfer area required for an overall heat transfer coefficient of 340 W/m^2.K. b) Determine the number of transfer unit (NTU). c) Calculate the effectiveness of the H.E.

Solution:



$(mcp\Delta T)_{oil} = (mcp\Delta T)_{water}$

$cp_{water} = 4.18kJ/kg.K$

$\therefore\ 270\times 1.88\times(205 - Th_o) = 225\times 4.18\times(44-16)$

$\Rightarrow Th_o = 153°C$

$\therefore \Delta T_1 = 205 - 16 = 189°C, \Delta T_2 = 153 - 44 = 109°C,$

$$\therefore \Delta TLM = \frac{189 - 109}{\ln\frac{189}{109}} = 145.4°C$$

a) $A = q / U \cdot \Delta TLM = m_w cp_w \Delta T_w = 0.148m^2$

b) $(mcp)_{water} = 225\times 4.18 = 9.405\times 10^5\ J/hr.K,\ (mcp)_{oil} = 270\times 1.88 = 5.076\times 10^5\ J/hr.K$

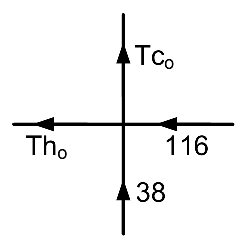
$\therefore Cmin = 5.076\times 10^5\ J/hr.K = 141 W/K$

$NTU = UA/C\min = 340\times 0.148/141 = 0.36$

c) $$\epsilon = \frac{1-\exp[(-UA/C_{\min})(1+C_{\min}/C_{\max})]}{1+C_{\min}/C_{\max}} = 28\%$$

Example: water enters a counter flow double pipe H.E at 38°C flowing at the rate of 0.75 kg/s. it is heated by oil (cp = 1.884 kJ/kg.K) flowing at the rate 1.5 kg/s from an inlet temperature of 116°C. For an area of 13m^2 and overall heat transfer coefficient of 340 W/m^2.K, determine the total heat transfer rate?

Solution:



$mcp_{oil} = 1.5 \times 1.88 = 2.82$ kJ/s.K

$mcp_{water} = 0.75 \times 4.18 = 3.14$ kJ/s.K

$\therefore$ Cmin = 2.82 kJ/s.K

C = Cmin/Cmax = 0.9

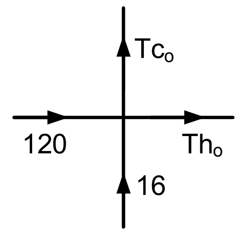
NTU = UA/Cmin = 1.57

From figure 10.13, Є = 0.62

$\therefore q_{act} = Є \times Cmin \times (Th_i - Tc_i) = 0.62 \times 2820 \times (116 - 38) = 136400$ W

Example: water enters a cross flow H.E (both fluid unmixed) at 16°C and flow at the rate 7.5 kg/s to cool 10 kg/s of air from 120°C. For an overall heat transfer coefficient of 225 W/m^2.K and an exchanger surface area of 240m^2. What is the exit air temperature?

Solution:



$mcp_{air} = 10 \times 1.014 = 10.14$ kJ/s.K = 10140 W/K

$mcp_{water} = 7.5 \times 4182 = 31370$ W/K

$\therefore$ Cmin = 10.14 kJ/s.K

C = Cmin/Cmax = 10.14/31370 = 0.32

NTU = UA/Cmin = 240 × 225 / 10140 = 5.32

From figure 10.15, $\epsilon = 0.94$

$\therefore q_{act} = 0.94 \times 10140 \times (120-16) = 9.91 \times 10^5$ W

$9.91 \times 10^5 = 10140\,(120 - Tho) \Rightarrow Tho = 32.6$ °C

Example: Hot water at 80°C enters the tube of two shell pass, eight tubes pass H.E at the rate 0.375 kg/s heating helium from 20°C. The overall heat transfer coefficient is 155 W/m^2.K and the exchanger area is 10m^2. If the water exits at 44°C, determine the exit temperature of the helium and its mass flow rate.

Solution:

Since the flow rate of helium is not given, we first assume that the min fluid is water with cp = 4.185 kJ/kg.K.

$mcp_{water} = 0.375 \times 4185 = 1570$ W/K

NTU = UA/Cmin = 155 × 10 / 1570 = 0.99

The effectiveness can be obtained by the equation

$$\epsilon_h = \frac{Th_i - Th_o}{Th_i - Tc_i} = \frac{80-44}{80-20} = 0.6$$

From figure 10.17, Cmin/Cmax = 0.25, which validates the initial assumption of the water as minimum fluid

Cmin/Cmax = 0.25 $\Rightarrow$ Cmax = C_{helium} = 6280 W/K

$cp_{helium} = 5.2$ kJ/kg.K $\Rightarrow m_{helium} = 1.21$ kg/s

$$(mcp\Delta T)_{water} = (mcp\Delta T)_{He} \Rightarrow (T_{helium})_{out} = 29°C$$

Figure 10-12 | Effectiveness for parallel-flow exchanger performance.

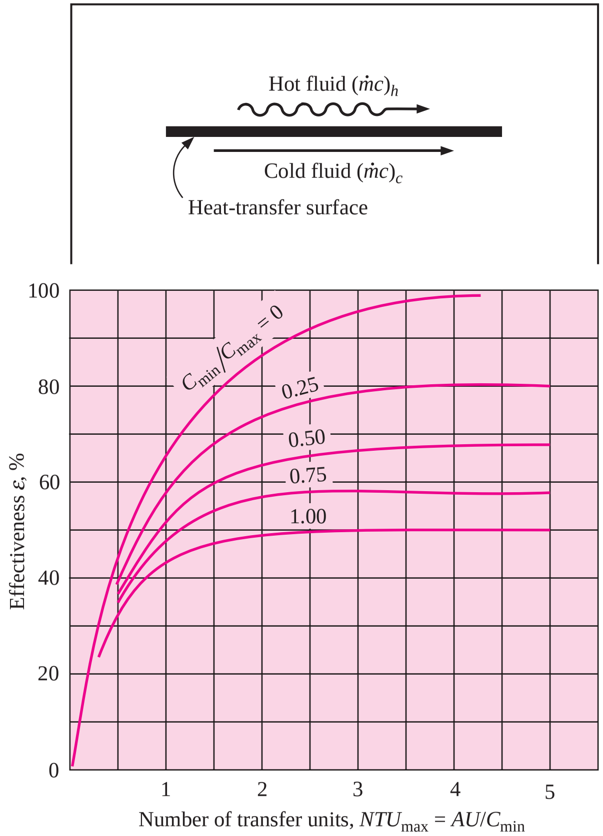


Figure 10-13 | Effectiveness for counterflow exchanger performance.

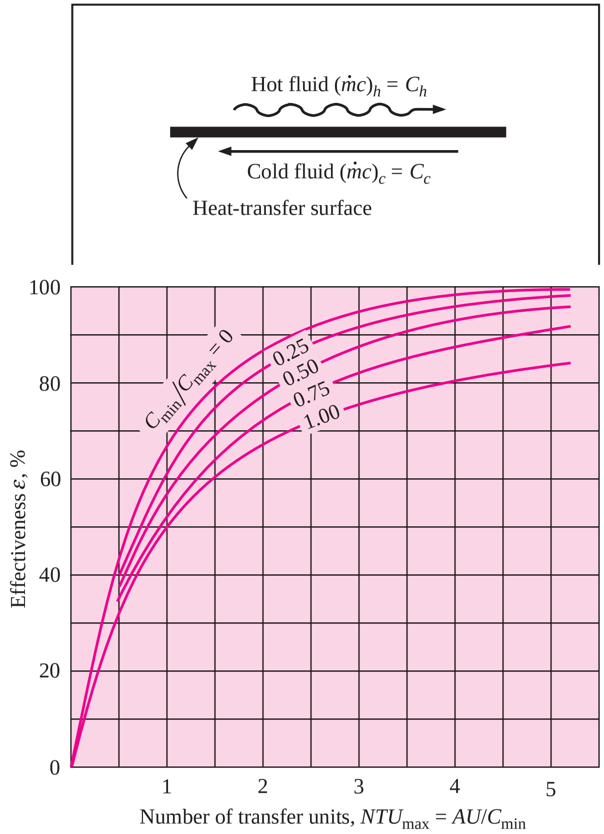


Figure 10-14 | Effectiveness for cross-flow exchanger with one fluid mixed.

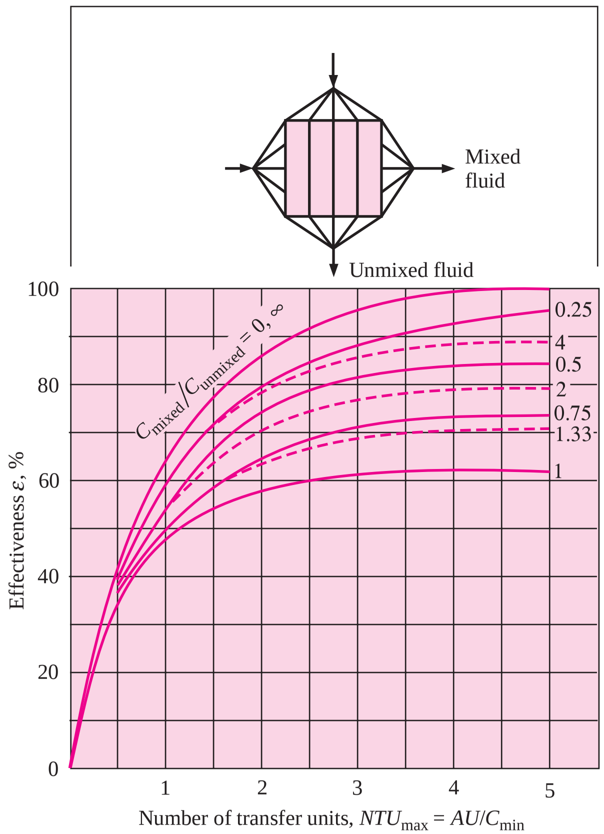


Figure 10-15 | Effectiveness for cross-flow exchanger with fluids unmixed.

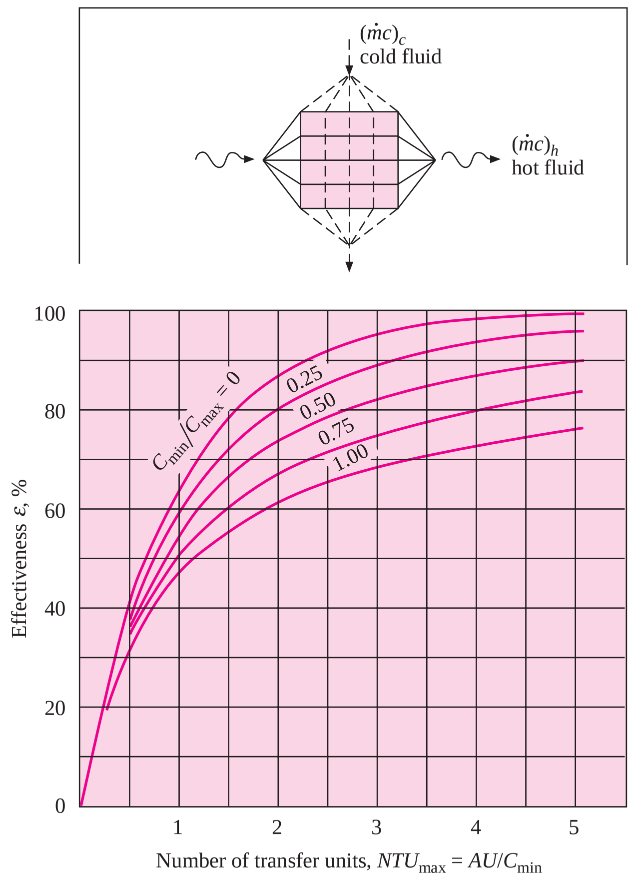


Figure 10-16 | Effectiveness for 1-2 parallel counterflow exchanger performance.

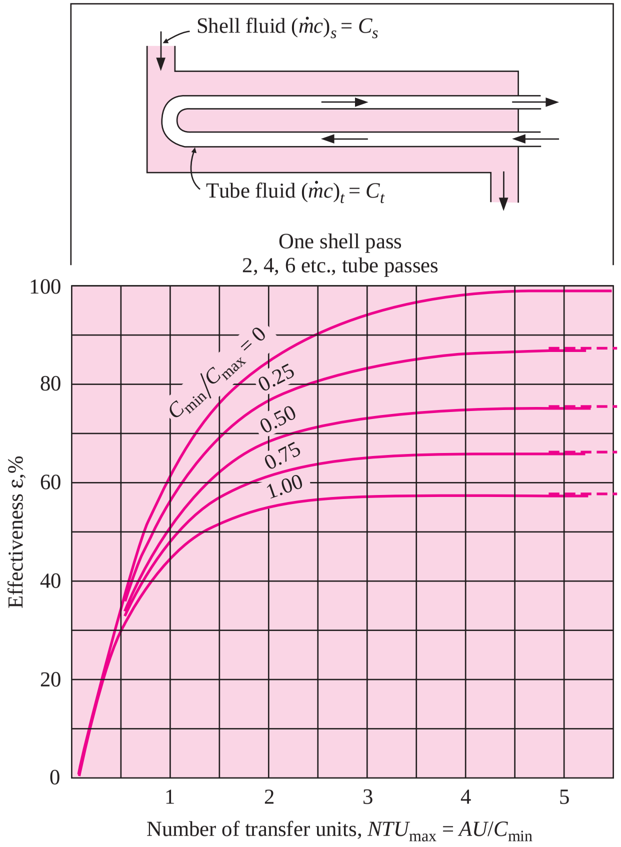


Figure 10-17 | Effectiveness for 2-4 multipass counterflow exchanger performance.

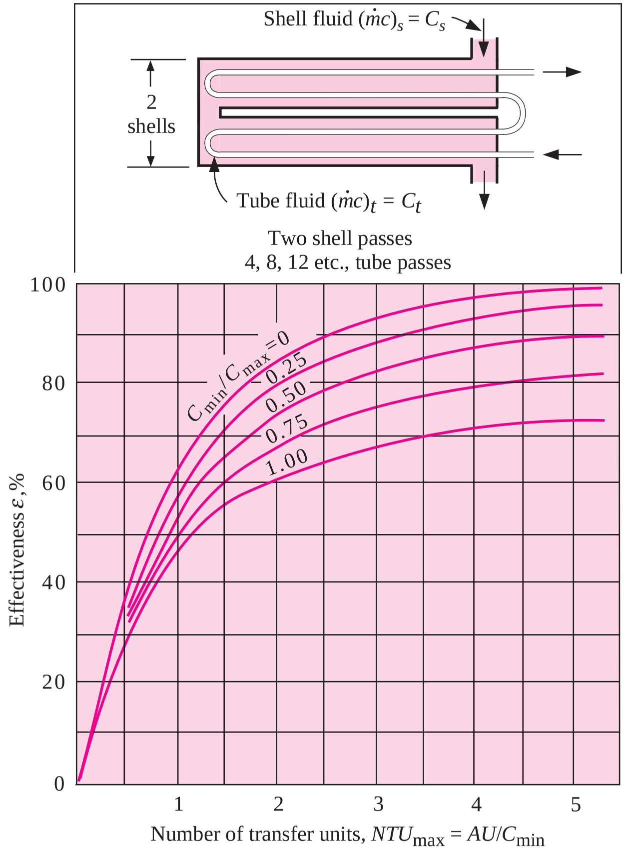


Table 10-3 | Heat-exchanger effectiveness relations.

$N = \text{NTU} = \dfrac{UA}{C_{min}}$ $C = \dfrac{C_{min}}{C_{max}}$

Flow geometry	Relation
Double pipe:	
Parallel flow	$\epsilon = \dfrac{1-\exp[-N(1+C)]}{1+C}$
Counterflow	$\epsilon = \dfrac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}$
Counterflow, $C=1$	$\epsilon = \dfrac{N}{N+1}$
Cross flow:	
Both fluids unmixed	$\epsilon = 1-\exp\left[\dfrac{\exp(-NCn)-1}{Cn}\right]$ where $n = N^{-0.22}$
Both fluids mixed	$\epsilon = \left[\dfrac{1}{1-\exp(-N)} + \dfrac{C}{1-\exp(-NC)} - \dfrac{1}{N}\right]^{-1}$
C_{max} mixed, C_{min} unmixed	$\epsilon = (1/C)\{1-\exp[-C(1-e^{-N})]\}$
C_{max} unmixed, C_{min} mixed	$\epsilon = 1-\exp\{-(1/C)[1-\exp(-NC)]\}$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$\epsilon = 2\left\{1+C+(1+C^2)^{1/2} \times \dfrac{1+\exp[-N(1+C^2)^{1/2}]}{1-\exp[-N(1+C^2)^{1/2}]}\right\}^{-1}$
Multiple shell passes, $2n$, $4n$, $6n$ tube passes ($\epsilon_p =$ effectiveness of each shell pass, $n =$ number of shell passes)	$\epsilon = \dfrac{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - 1}{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - C}$
Special case for $C=1$	$\epsilon = \dfrac{n\epsilon_p}{1+(n-1)\epsilon_p}$
All exchangers with $C=0$	$\epsilon = 1-e^{-N}$

Table 10-4 | NTU relations for heat exchangers.

$C = C_{min}/C_{max}$ $\epsilon =$ effectiveness $N = \text{NTU} = UA/C_{min}$

Flow geometry	Relation
Double pipe:	
Parallel flow	$N = \dfrac{-\ln[1-(1+C)\epsilon]}{1+C}$
Counterflow	$N = \dfrac{1}{C-1}\ln\left(\dfrac{\epsilon-1}{C\epsilon-1}\right)$
Counterflow, $C=1$	$N = \dfrac{\epsilon}{1-\epsilon}$
Cross flow:	
C_{max} mixed, C_{min} unmixed	$N = -\ln\left[1+\dfrac{1}{C}\ln(1-C\epsilon)\right]$
C_{max} unmixed, C_{min} mixed	$N = \dfrac{-1}{C}\ln[1+C\ln(1-\epsilon)]$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$N = -(1+C^2)^{-1/2} \times \ln\left[\dfrac{2/\epsilon-1-C-(1+C^2)^{1/2}}{2/\epsilon-1-C+(1+C^2)^{1/2}}\right]$
All exchangers, $C=0$	$N = -\ln(1-\epsilon)$