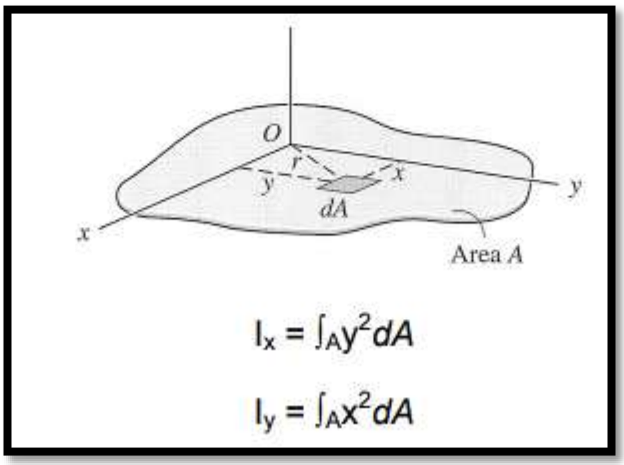
Moment of Inertia:

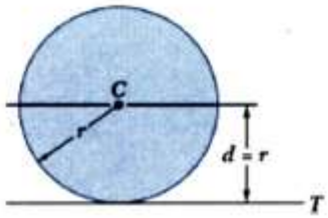
The Moment of Inertia (I) is a term used to describe the capacity of a cross-section to resist bending. It is always considered with respect to a reference axis such as X-X or Y-Y. It is a mathematical property of a section concerned with a surface area and how that area is distributed about the reference axis (axis of interest). The reference axis is usually a centroidal axis. The moment of inertia is also known as the Second Moment of the Area and is expressed mathematically as:



Where y = distance from the x axis to area dA x = distance from the y axis to area dA

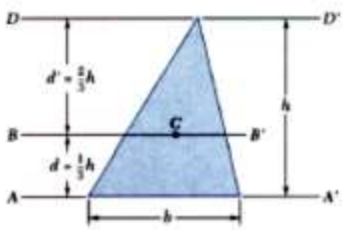
Area Moments of Inertia

Parallel Axis Theorem



- Moment of inertia I_T of a circular area with respect to a tangent to the circle,

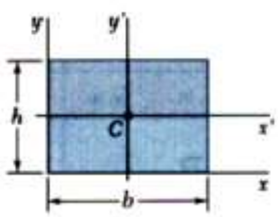
$$I_T = \bar{I} + Ad^2 = \tfrac{1}{4}\pi r^4 + \left(\pi r^2\right)r^2$$
$$= \tfrac{5}{4}\pi r^4$$



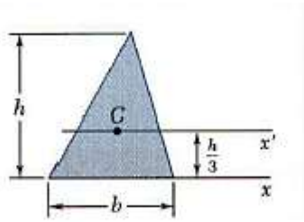
- Moment of inertia of a triangle with respect to a centroidal axis,

$$I_{AA'} = \bar{I}_{BB'} + Ad^2$$
$$I_{BB'} = I_{AA'} - Ad^2 = \tfrac{1}{12}bh^3 - \tfrac{1}{2}bh\left(\tfrac{1}{3}h\right)^2$$
$$= \tfrac{1}{36}bh^3$$

Area Moments of Inertia: Standard MIs

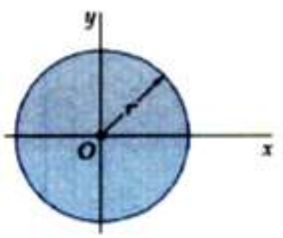


	Answer
Moment of inertia about x-axis	$I_x = \dfrac{1}{3}bh^3$
Moment of inertia about y-axis	$I_y = \dfrac{1}{3}b^3h$
Moment of inertia about x'-axis	$I_{x'} = \dfrac{1}{12}bh^3$
Moment of inertia about y'-axis	$I_{y'} = \dfrac{1}{12}b^3h$
Moment of inertia about z-axis passing through C	$I_C = \dfrac{1}{12}bh(b^2 + h^2)$



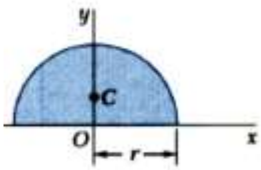
Moment of inertia about x-axis $I_x = \frac{1}{12} bh^3$

Moment of inertia about x'-axis $I_{x'} = \frac{1}{36} bh^3$



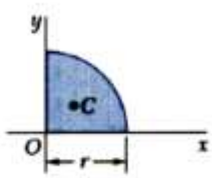
Moment of inertia about x'-axis $I_x = \frac{1}{4}\pi r^4$

Moment of inertia about z-axis passing through O $I_0 = \frac{1}{2}\pi r^4$



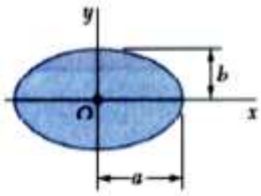
Moment of inertia about x'-axis $I_x = I_y = \frac{1}{8}\pi r^4$

Moment of inertia about z-axis passing through O $I_0 = \frac{1}{4}\pi r^4$



Moment of inertia about x'-axis $I_x = I_y = \frac{1}{16}\pi r^4$

Moment of inertia about z-axis passing through O $I_0 = \frac{1}{8}\pi r^4$



Moment of inertia about x-axis $I_x = \frac{1}{4}\pi ab^3$

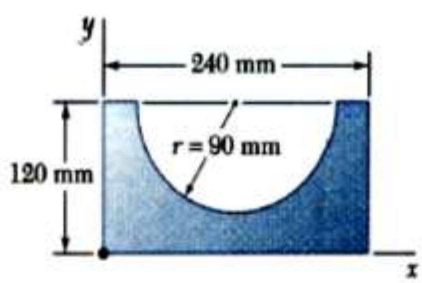
Moment of inertia about y-axis $I_y = \frac{1}{4}\pi a^3 b$

Moment of inertia about z-axis passing through O $I_0 = \frac{1}{4}\pi ab(a^2 + b^2)$

Area Moments of Inertia

Example:

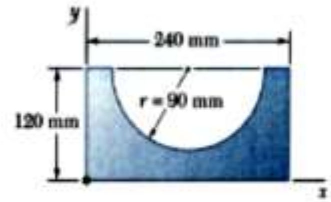
Determine the moment of inertia of the shaded area with respect to the *x* axis.



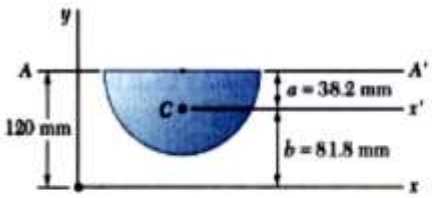
SOLUTION:

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the *x* axis.
- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.

Area Moments of Inertia



Example: Solution



$$a = \frac{4r}{3\pi} = \frac{(4)(90)}{3\pi} = 38.2\, mm$$

$$b = 120 - a = 81.8\, mm$$

$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi (90)^2$$
$$= 12.72\times 10^3\, mm^2$$

SOLUTION:

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the x axis.

Rectangle:

$$I_x = \tfrac{1}{3}bh^3 = \tfrac{1}{3}(240)(120)^3 = 138.2\times 10^6\, mm^4$$

Half-circle:

moment of inertia with respect to AA',

$$I_{AA'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi (90)^4 = 25.76\times 10^6\, \mathrm{mm}^4$$

Moment of inertia with respect to x',

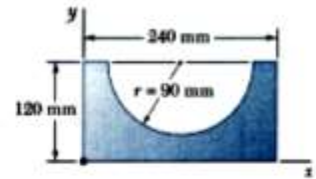
$$\bar{I}_{x'} = I_{AA'} - Aa^2 = (25.76\times 10^6) - (12.72\times 10^3)(38.2)^2$$
$$= 7.20\times 10^6\, mm^4$$

moment of inertia with respect to x,

$$I_x = \bar{I}_{x'} + Ab^2 = 7.20\times 10^6 + (12.72\times 10^3)(81.8)^2$$
$$= 92.3\times 10^6\, mm^4$$

Area Moments of Inertia

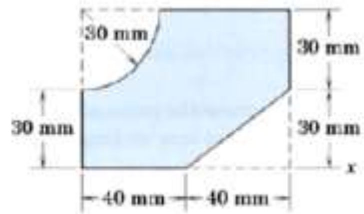
Example: Solution



- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.

$$I_x = 138.2\times10^6\,\text{mm}^4 - 92.3\times10^6\,\text{mm}^4$$

$$\boxed{I_x = 45.9\times10^6\,\text{mm}^4}$$



Consider area (1)

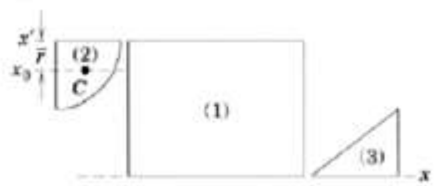
$$I_x = \frac{1}{3}bh^3 = \frac{1}{3} \times 80 \times 60^3 = 5.76 \times 10^6\ mm^4$$

Consider area (2)

$$I_{x'} = \frac{1}{4}\left(\frac{\pi r^4}{4}\right) = \frac{\pi}{16}(30)^4 = 0.1590 \times 10^6\ mm^4$$

$$\bar{I_x} = 0.1590 \times 10^6 - \frac{\pi}{4}(30)^2 \times (12.73)^2 = 0.0445 \times 10^6\ mm^4$$

$$I_x = 0.0445 \times 10^6 + \frac{\pi}{4}(30)^2(60 - 12.73) = 1.624 \times 10^6\ mm^4$$



Consider area (3)

$$I_x = \frac{1}{12}bh^3 = \frac{1}{12} \times 40 \times 30^3 = 0.09 \times 10^6\ mm^4$$

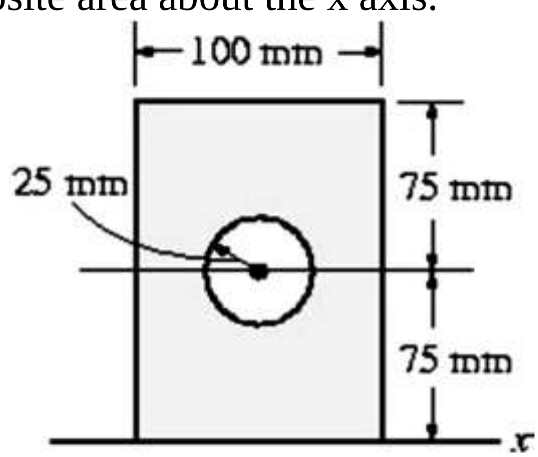
$$\boldsymbol{I_x = 5.76 \times 10^6 - 1.624 \times 10^6 - 0.09 \times 10^6 = 4.05 \times 10^6\ mm^4}$$

$$A = 60 \times 80 - \frac{1}{4}\pi(30)^2 - \frac{1}{2}40 \times 30 = 3490mm^2$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.05 \times 10^6}{3490}} = \mathbf{34.00mm}$$

Example

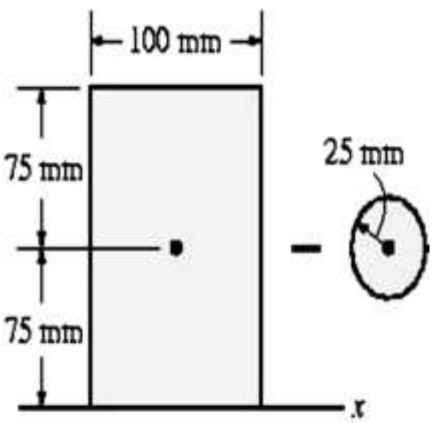
Compute the moment of inertia of the composite area about the x axis.



(a)

Solution: Composite Parts

- Composite area obtained by subtracting the circle form the rectangle
- Centroid of each area is located in the figure



(b)

Solution:

Parallel Axis Theorem.

***Circle:**

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$= \frac{1}{4}\pi(25)^4 + \pi(25)^2(75)^2 = 11.4(10^6)mm^4$$

*** Rectangle:**

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$= \frac{1}{12}(100)(150)^3 + (100)(150)(75)^2 = 112.5(10^6)mm^4$$

Summation For moment of inertia for the composite area.

$$I_x = -11.4(10^6) + 112.5(10^6)$$
$$= 101(10^6)mm^4$$