Work and gravitational potential energy

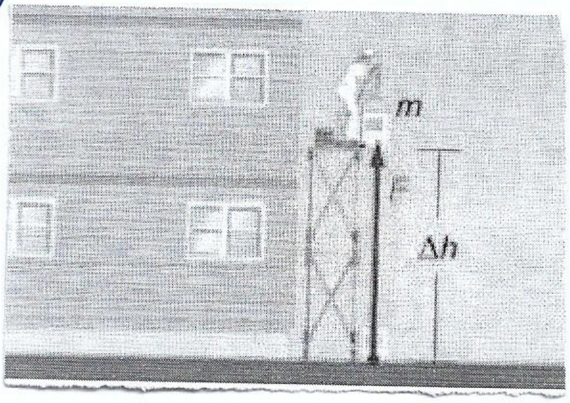
- work equals chang in potential energy

$W = \Delta PE$

where

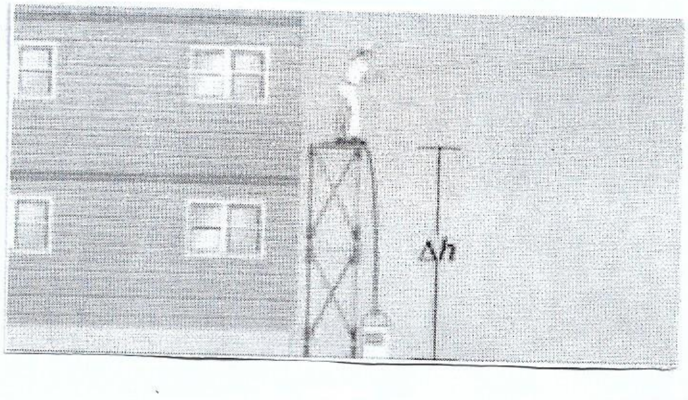
W: work done against gravity.

PE: potential energy of system.



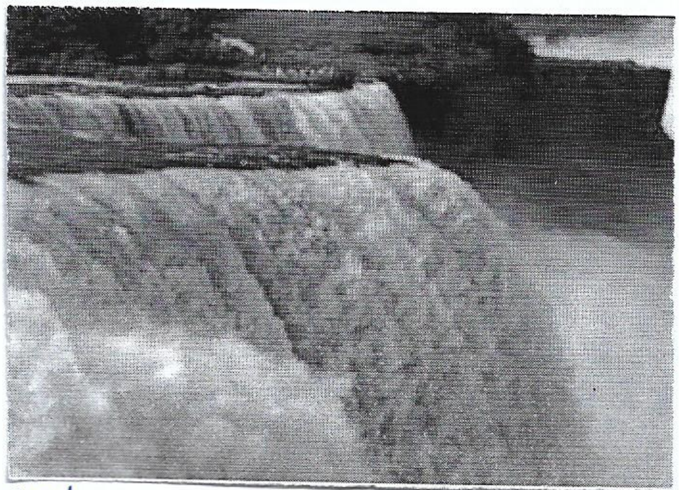
- The system we consider consists of two objects, the bucket and Earth, illustrated in the above figure. The painter applies an external force to this system (via a rope) when she raises or lowers the bucket. The bucket starts at rest on the ground, and she raises it up and places it on the scaffolding. That means the work she does as she moves the bucket from its initial to its final position changes only it gravitational potential energy (PE). The system's kinetic energy is zero at the begining and the end of this process.
- As she raises the bucket, the painter does work on it. She pulls the bucket up against the force of gravity, which is equal in magnitude to the bucket's weight, mg. She pulls in the direction of the bucket's displacement, Δh. The work equals the force multiplied by the displacement: $mg\Delta h$. The paint bucket's change in gravitational potential energy also equals $mg\Delta h$.
- The above analysis lets us reach an important conclusion: The work done on the system, against gravity, equals the system's increase in gravitational potential energy.
- Imagine that the painter drops the bucket from the scaffolding. Only the force of gravity does work on the bucket as it falls. The system has more potential energy when the bucket is at the top of scaffolding than when it is at the bottom, so the work done by gravity has lowered the system's PE: the change in PE due to the work done by gravity is negative:

$W = -\Delta PE$ (work done by gravity)



Sample problem: Potential energy and Niagara Falls

An average of 5.71×10^{6} kg of water flowed per second over Niagara Falls, falling 51.0 m. If all the work done by gravity could be converted into electric power as the water fell to the bottom, how much power would the falls generate?



Solution:

mass flow rate, $\dot{m} = \frac{Mass}{time} = \frac{m}{t} = 5.71 \times 10^{6}$ kg/s

$h = 51$ m

Since

$$P = \frac{W}{\Delta t} \quad \text{—— (1)}$$

change in gravitational potential energy, $PE = mg\Delta h$

work done by gravity

$$W = -\Delta PE \quad \text{—— (2)} \text{ sub. into Eq. (1)}$$

$$P = \frac{-\Delta PE}{\Delta t} = \frac{-mg(h_f - h_i)}{\Delta t} = \frac{mg(h_i - h_f)}{\Delta t} = \frac{m}{\Delta t} g(h_i - h_f)$$

$$P = \dot{m} g (h_i - h_f)$$

$$P = 5.71 \times 10^{6} \times 9.8 (51 - 0) \Rightarrow P = 2.85 \times 10^{9} \text{ W}$$

This is the theoretical maximum power that could be generated. A real power plant cannot be 100% efficient.

Work and energy

We have discussed work on a particle increasing its KE, and work on a system increasing its PE. Now we discuss what happens when work increases both forms of mechanical energy.

- Because we are considering only KE and PE in this chapter, we can say the net work done on an object equals the change in the sum of its KE and PE

$W = \Delta PE + \Delta KE$ [work on system equals its change in total energy]

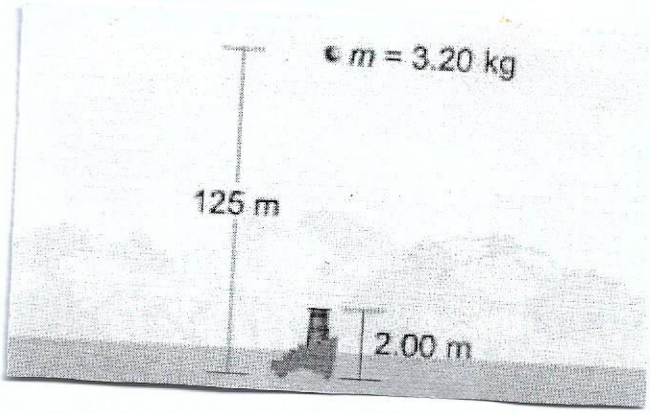
- Positive work done on an object increases it energy; negative work decreases its energy.

Example: A cannon shoots a 3.20 kg cannonball straight up. The barrel of the cannon is 2.00 m long, and it exerts an average force of 6250 N while the cannonball is in the cannon. Negelect air resistance. Determine the cannonball's velocity when it has traveled 125 m upward?

Solution:

Assumption:

- we assume the cannon does no work on the cannonball after it leaves the cannon.



$W = F \cdot \Delta X = 6250 * 2 = 12500 \ J$

Since

$W = \Delta PE + \Delta KE$

$W = mg\Delta h + \Delta KE$

$12500 = 3.2 * 9.8 * 125 + \Delta KE$

$\Delta KE = 8580 \ J$

Since

$\Delta KE = \frac{1}{2} m v^2$

$8580 = \frac{1}{2} * 3.2 * v^2$

$v^2 = \frac{2 * 8580}{3.2}$

$v = 73.2 \ m/s$

Conservation of energy

The total energy in an isolated system remains constant.

-Energy never disappears. It only changes form and transfers between objects.

- Example on conservation of energy:
When a moving car hits a parked car and causes the parked car to move, energy is transferred from the moving car to the parked one.

- water falls from the sky, converting potential energy into kinetic energy. This kinetic energy from the water is used to rotate the turbine of a generator to produce electricity. This process turns kinetic energy into electrical energy.

- The law of conservation of energy applies to an isolated system. An isolated system is one that has no interactions with its environment. The particles within the system may interact with one another, but no net external force or field acts on an isolated system.

Conservation of energy

$E_f = E_i$

$PE_f + KE_f = PE_i + KE_i$

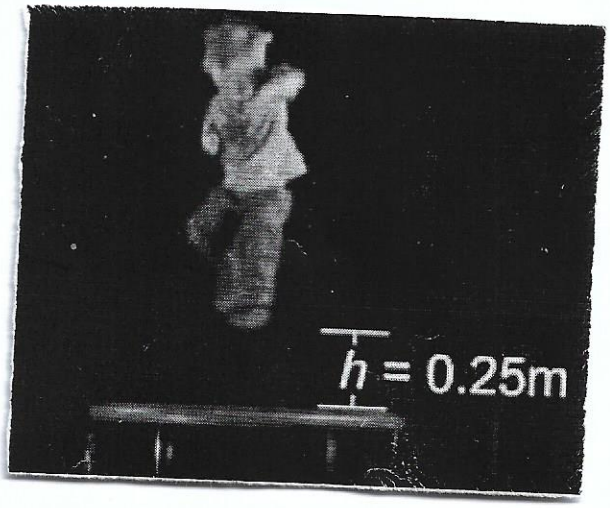
where

KE: kinetic energy

PE: potential energy

Sample problem: conservation of energy

Sam is at the peak of his jump. Calculate Sam's speed when he reaches the trampoline's surface.



Solution:

$h = 0.25\ m$

$v_{peak} = 0\ m/s$

$v_{bottom} = ?$

- By using the law of conservation energy, we can state that Sam's total energy at the peak of his jump is the same as his total energy at the surface of the trampoline.
- Sam's kinetic energy at peak = Zero
- Sam's potential energy at bottom = Zero

$E_f = E_i$ for isolated system

$KE_f + PE_f = KE_i + PE_i$

$KE_f + 0 = 0 + PE_i$

$KE_f = PE_i$

$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g h$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$v = \sqrt{2 * 9.8 * 0.25}$

$v = 2.2\ m/s$