When Z_1 and Z_2 are of different signs :

$$Pr[Z_1 < Z < Z_2] = A_{Z_2} + A_{Z_1}$$

* For bound of measurements, the bound in actual value is ∓0.5 units in L.S.D.

Example :

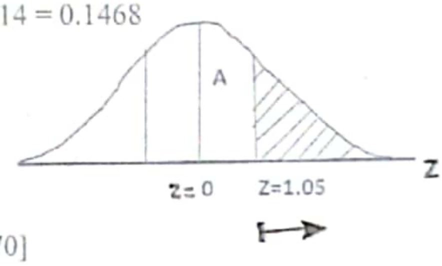
For measurements of $\mu = 160$, $\sigma = 10$, obtain the following :

1. Pr[X greater than 170] = Pr[X >170]

$$Z = \frac{X-\mu}{\sigma} = \frac{170.5-160}{10} = 1.05$$

$\therefore$ Pr[Z > 1.05] = 0.5 − 0.35314 = 0.1468

Where $A_{Z=1.05} = 0.35314$
من الجدول

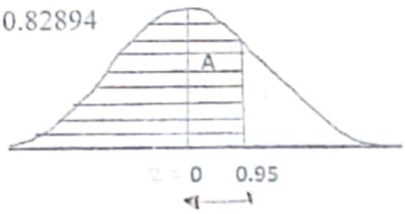


2. Pr[X less than 170] = Pr[X<170]

$x = 169.5 \rightarrow Z = 0.95$

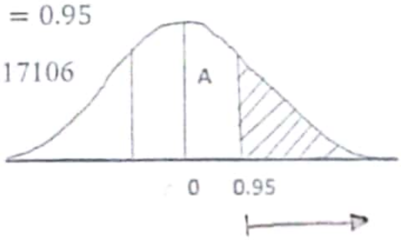
Pr [Z < 0.95] = 0.5 + 0.32894 = 0.82894

Where $A_{Z=0.95} = 0.32894$
من الجدول



3. Pr[X ≥ 170] → $x = 169.5 \rightarrow Z = 0.95$

Pr [Z > 0.95] = 0.5 − 0.32894 = 0.17106



$X > 170 \rightarrow 170.5$

$X < 170 \rightarrow 169.5$

$X \geq 170 \rightarrow 169.5$

$X \leq 170 \rightarrow 170.5$

When Z_1 and Z_2 are of different signs :

$$Pr[Z_1 < Z < Z_2] = A_{Z_2} + A_{Z_1}$$

* For bound of measurements, the bound in actual value is ∓0.5 units in L.S.D.

Example :

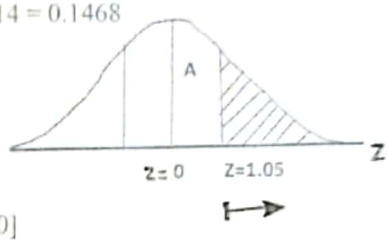
For measurements of $\mu = 160$, $\sigma = 10$, obtain the following :

1. Pr[X greater than 170] = Pr[X >170]

$$Z = \frac{X-\mu}{\sigma} = \frac{170.5-160}{10} = 1.05$$

$\therefore$ Pr[Z > 1.05] = 0.5 − 0.35314 = 0.1468

Where $A_{Z=1.05} = 0.35314$
من الجدول

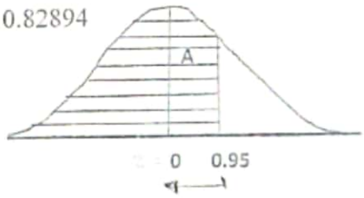


2. Pr[X less than 170] = Pr[X<170]

$x = 169.5 \rightarrow Z = 0.95$

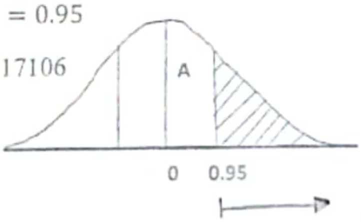
Pr [Z < 0.95] = 0.5 + 0.32894 = 0.82894

Where $A_{Z=0.95} = 0.32894$
من الجدول



3. Pr[X ≥ 170] → $x = 169.5$ → $Z = 0.95$

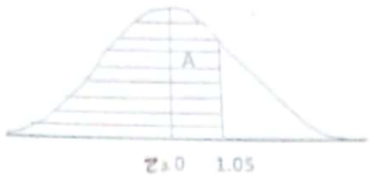
Pr [Z > 0.95] = 0.5 − 0.32894 = 0.17106



X > 170 → 170.5
X < 170 → 169.5
X ≥ 170 → 169.5
X ≤ 170 → 170.5

4. $\Pr[X \le 170] \to x = 170.5 \ \to Z = 1.05$

$\Pr[Z > 1.05] = 0.5 + 0.35314 = 0.85314$



Linear inter polation :

$\boxed{\text{interpolation طريقة}}$

When Z lies between successive Z_1 and Z_2 with respective A_1 and A_2 , A is obtained by linear interpolation "

$$\frac{Z - Z_1}{Z_2 - Z_1} = \frac{A - A_1}{A_2 - A_1} \qquad , Z_1 < Z < Z_2$$

e.g. for $\quad Z_1 = 2.32 \qquad A_1 = 0.48983$

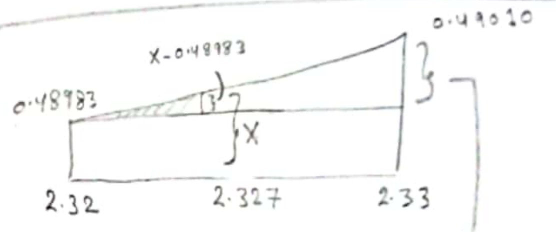
$\qquad\qquad Z_2 = 2.33 \qquad A_2 = 0.49010$

Then when $Z = 2.327 \to A = ?$

$$A = \frac{Z - Z_1}{Z_2 - Z_1}(A_2 - A_1) + A_1 = 0.49002$$

$$\frac{0.49010 - 0.48983}{2.33 - 2.32} = \frac{X - 0.48983}{2.327 - 2.32}$$

$X = 0.49002$



0.49010 - 0.48983 =

Example 1)

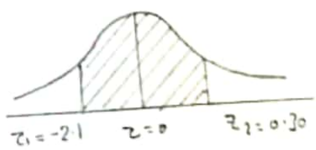
For a measurement of size (N)=500

$\mu = 151$, $\sigma = 15$, assuming normal dist. Find how many measurements :

a) between 120 and 155 = $\Pr[120 \le X \le 155]$

$x_1 = 119.5 \to z_1 = -2.1 \to A_1 = 0.4821$

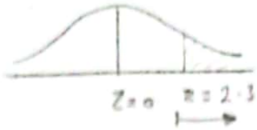
$x_2 = 155.5 \to z_2 = 0.30 \to A_2 = 0.1179$



Z_1 Z_2

Pr $[-2.1 < Z < 0.3] = 0.4821 + 0.1179 =$ No. of meas. $=$ $500[0.4821+0.1179]=300$

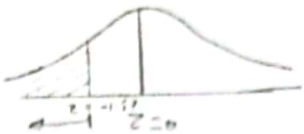
b) more than 185 = Pr[Z>185]



$x = 185.5 \rightarrow Z = 2.3 \rightarrow A = 0.4893$

Pr $[Z > 2.3] = 0.5 - 0.4893 =$ No. of meas. $=$ $500[0.51-0.4893]=5.$

c) Less than 128 = Pr [X < 128]



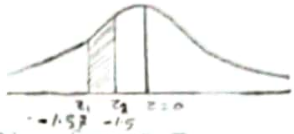
$x = 127.5 \rightarrow Z = -1.57 \rightarrow A = 0.4418$

No. of meas. = 500[0.5−0.4418]=29

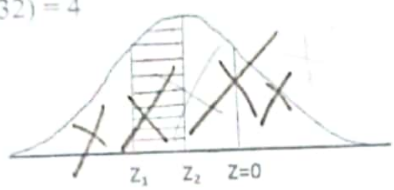
d) equal to 128 = Pr [X=128]

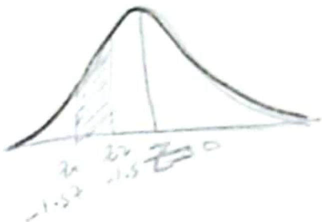
$x_1 = 127.5 \rightarrow z_1 = -1.57 \rightarrow A_1 = 0.4418$

$x_2 = 128.5 \rightarrow z_2 = -1.5 \rightarrow A_2 = 0.4332$



Pr $[-1.57 < Z < -1.5] = 0.4418-0.4332 =$ No. of meas. $=$ $500(0.4418-0.4332)=4$





e) Less than or equal to 128 = Pr [X≤128]

$x = 128.5 \rightarrow Z = -1.5 \rightarrow A = .0.4332$

No. = 500[0.5−0.4332]=33

f) Less than or equal to 185 = Pr [X≤185]

$x = 185.5 \rightarrow Z = 2.3 \rightarrow A = 0.4893$

No. = 500[0.5 + 0.4893] = 495

Example 2)

For a sample of washers produced by a machine the mean inside dia. (μ) is 5.02 mm and the standard deviation is 0.05 mm. The max. useful tolerance in the dia. Is 4.96 to 5.08 mm, otherwise the washers are considered defective. Determine % of defective washers.

Solu.)

Pr of max. to lerance = Pr $(4.96 \leq X \leq 5.08)$

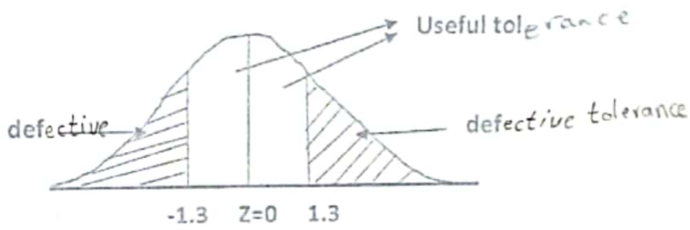
One unit in L.S.D. = 0.01

$x_1 = 4.955 \rightarrow z_1 = -1.3 \rightarrow A_1 = 0.4032$

$x_2 = 5.085 \rightarrow z_2 = +1.3 \rightarrow A_2 = 0.4032$

Pr $[-1.3 < Z < 1.3] = 2 * 0.4032 = 0.8064$

$\therefore$ % of defective washers = $(1-0.8064) * \overset{100\ \%}{} = 19.4$ %



Example 3)

Out of a large No. of examination applicant a samples of size 50 gave a mean mark of 64 and a standard dev. of 14 . What is the expected % of applicants achieving a min. pass mark of 50 ?

Solu. :

$\mu = 64$

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$$\sigma = S\sqrt{\frac{N}{N-1}} = 14\sqrt{\frac{50}{50-1}} = 14.1$$

applicant

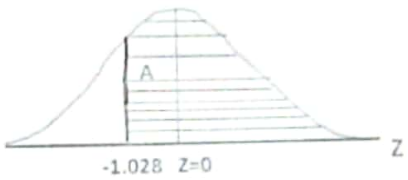
Pr [app. Have a min. pass mark of 50]= Pr [50≤X]

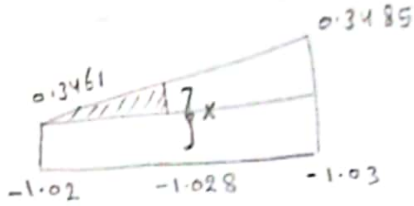
$x = 49.5 \rightarrow Z = -1.028 \rightarrow A = 0.3480$ interpolation

$\Pr[-1.028 < Z] = 0.348 + 0.5 = 0.848 = 84.8\ \%$

$$\frac{0.3485 - 0.3461}{-1.03 + 1.02} = \frac{X - 0.3461}{-1.028 + 1.02}$$

$X = 0.3480 = A$





Example 4)

The strength of individual bars made by a certain manufacturing process are approximate normally distributed with mean 28.4 μ and standard dev. σ 2.95 . To ensure safety, a customer requires at least 95% of the bars to be stronger than 24.0 . (one unit = 0.1)

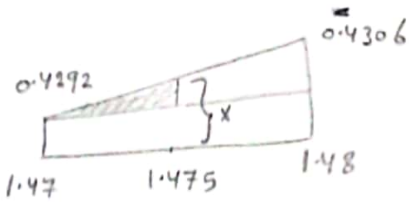
a) Do the bars meet the specification ?

b) By improved manufacturing techniques, the manufacturer make the bars more uniform (that is, decrease the standard dev.) what value of standard dev. will just meet the specification if the mean stays the same ?

Solu. :

$\Pr[X > 24.0] \rightarrow Z_1 = \frac{X-\mu}{\sigma} = \frac{24.05-28.4}{2.95}$

$Z_1 = -1.475 \rightarrow A_1 = 0.4299$



$$\frac{0.4306 - 0.4292}{1.48 - 1.47} = \frac{X - 0.4292}{1.475 - 1.47}$$

$X = 0.4299 = A_1$

0.4306

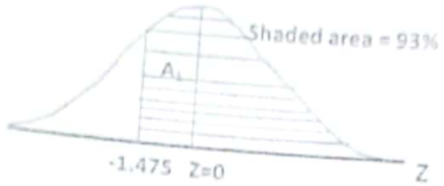
0.4292

1.47 1.475 1.48

$x = 0.4299$

$\Pr[Z > -1.475] = 0.5 + 0.4299 = 0.9299 \simeq 93\%$

Since (93%) less than 95%, the bars do not meet the specification.



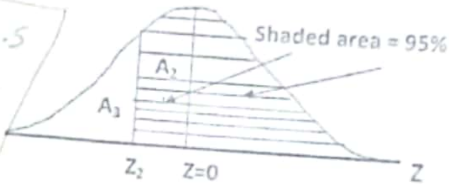
b) The specification is at least 95% of bars > 24.0

$A_3 = 1 - 0.95 = 0.05$

$\therefore\ A_2 = 0.5 - 0.05 = 0.45$

OR: $A_2 = 0.95 - 0.5$

$A_2 = 0.45$



At $A_2 = 0.45 \rightarrow Z_2 = -1.645$ (from table) لدول اكبر من ٠.٥ اذا فيه سالب

$$Z_2 = \frac{X - \mu}{\sigma} \rightarrow -1.645 = \frac{24.05 - 28.4}{\sigma}$$

$\sigma = 2.644$ (if the σ can be reduced to 2.644 while keeping the mean constant, the specification will just be met)