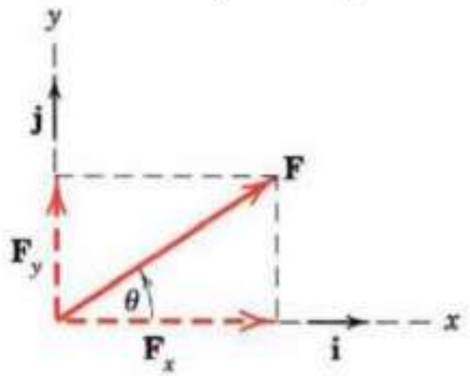
TWO-DIMENSIONAL FORCE SYSTEMS

RECTANGULAR COMPONENTS

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector F of Fig. may be written as



The scalar components can be positive or negative, depending on the quadrant into which F points.

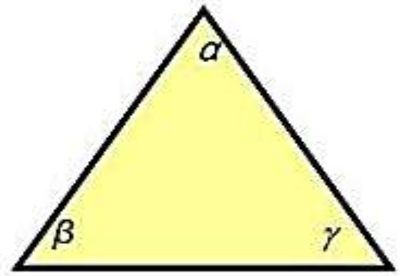
$$F_x = F\cos\theta \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

Determining the Components of a Force Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the x-axis, and the origin of coordinates need not be on the line of action of a force.

Symbols:

α	*ALPPHA*
β	*BETA*
γ	*GAMMA*
ϕ	*PHI*
π	*PI*
μ	*MU*



Trigonometric Relations for Right Angle's Triangles

$$\sin\beta = \frac{a}{c}$$

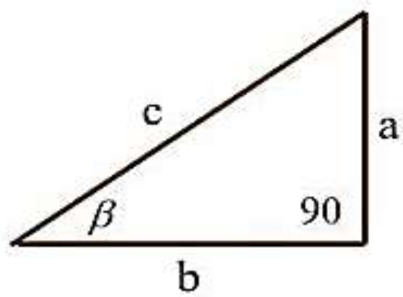
$$\cos\beta = \frac{b}{c}$$

$$\tan\beta = \frac{a}{b}$$

$$\sec\beta = \frac{c}{b} = 1/\cos\beta$$

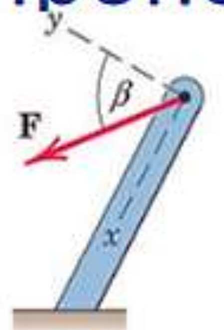
$$\cot\beta = \frac{b}{a} = 1/\tan\beta$$

$$c\sec\beta = \frac{c}{a} = 1/\sin\beta$$

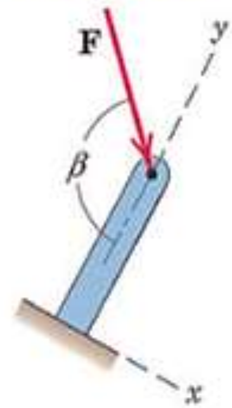


Components of Force

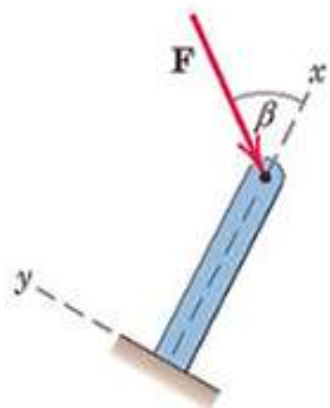
Examples



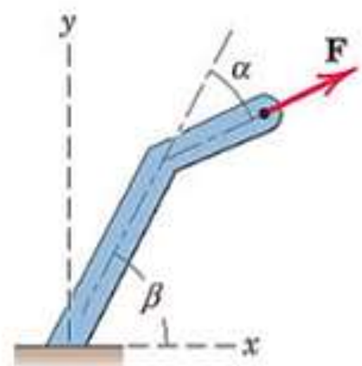
$F_x = F \sin \beta$
$F_y = F \cos \beta$



$F_x = F \sin(\pi - \beta)$
$F_y = -F \cos(\pi - \beta)$

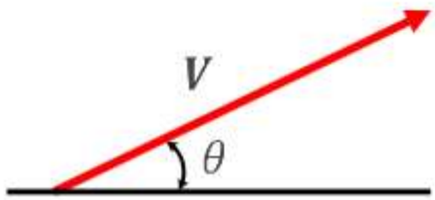


$F_x = -F \cos \beta$
$F_y = -F \sin \beta$

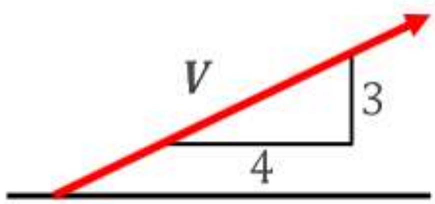


$F_x = F \cos(\beta - \alpha)$
$F_y = F \sin(\beta - \alpha)$

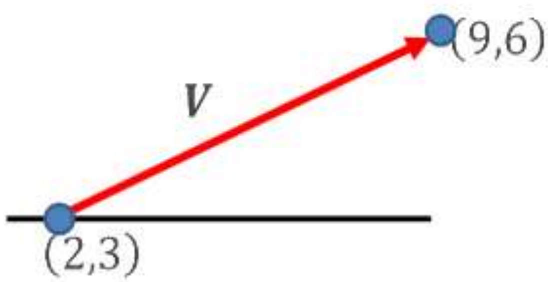
Vector



$$V = V(cos\theta\boldsymbol{i} + sin\theta\boldsymbol{j})$$



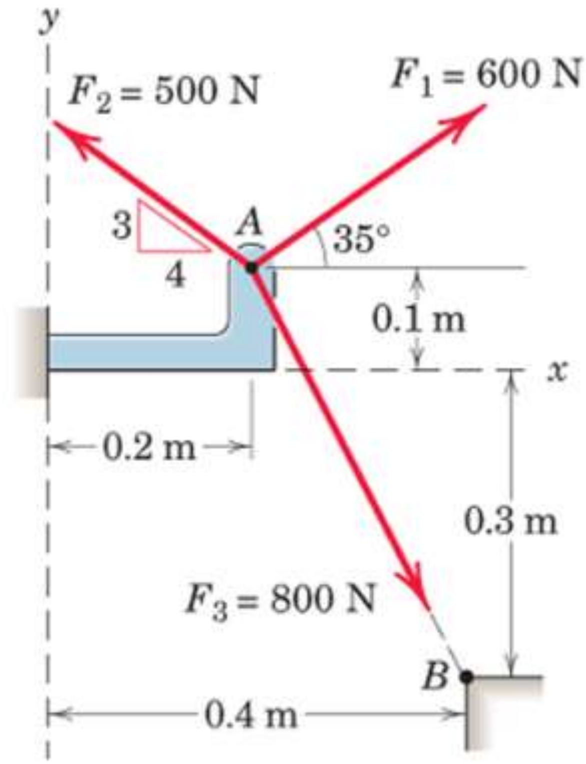
$$V = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$



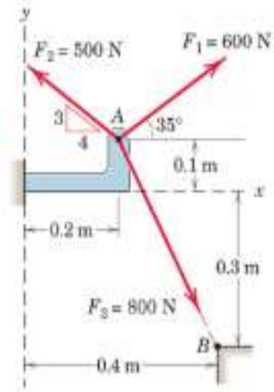
$$V = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

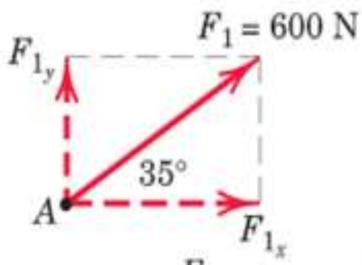
Components of Force

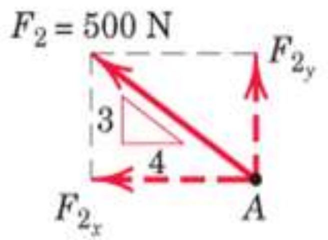
Example 1:Determine the x and y scalar components of F_1, F_2, and F_3actingat point A of the bracket

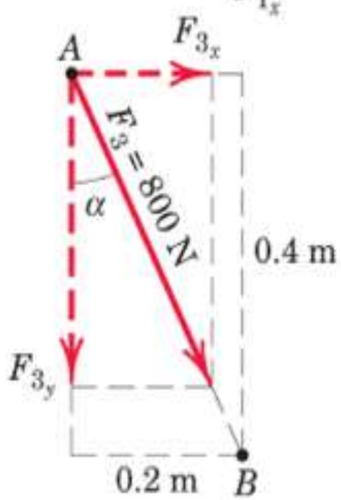


Components of Force









$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

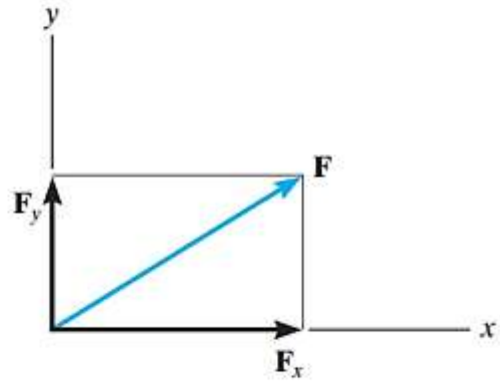
$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

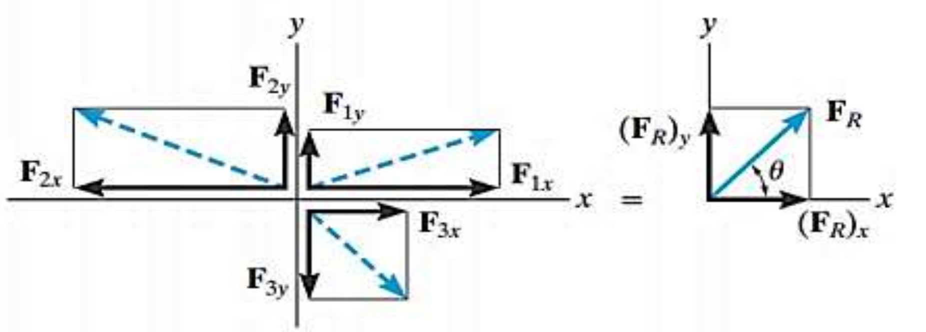
Resultants of two dimensional concurrent & non- concurrent force systems:

A. Rectangular Components : المركبات متعامدة

Vectors **Fx** and **Fy** are rectangular components of **F**.

The resultant force is determined from the algebraic sum of its components.





Rx = $(F_R)_x = \Sigma F_x$

RY= $(F_R)_y = \Sigma F_y$

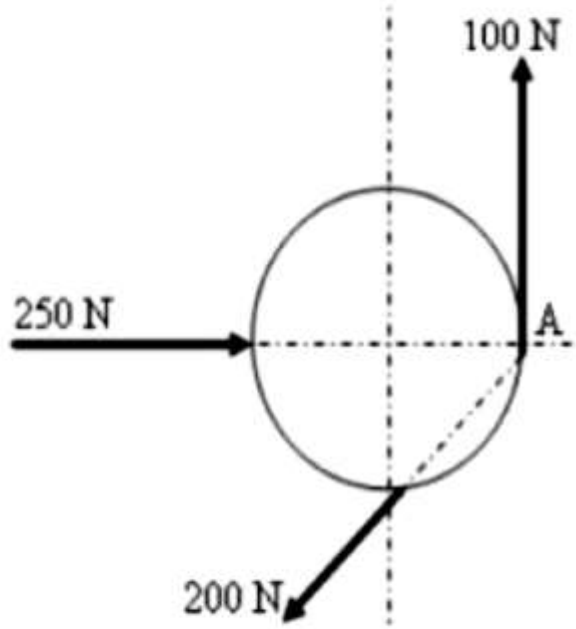
R= $F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$

The direction of R $\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$

Ex.(2):

Determine the resultant force for the forces system shown in fig.

Solution:



$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$

$$= 100\cos 90 + 250\cos(0) - 200\cos 45$$

$$= 192.5 \quad N$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$

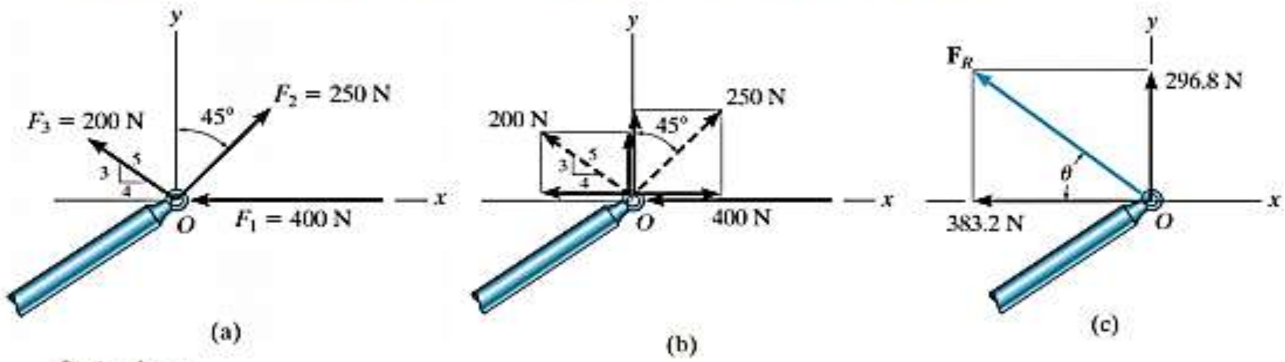
$$= 100\sin 90 + 250\sin(0) - 200\sin 45$$

$$= -\ 60.78 \quad N$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(192.5)^2 + (-60.78)^2} = 201.8 \ N$$

Example: The end of the boom O in Figure (a) below is subjected to three concurrent and coplanar forces. Determine the ***magnitude*** and ***direction*** of the resultant force.



Solution:

Each force is resolved into its x and y components, Figure (b), Summing the x-components and y-components:

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400\text{ N} + 250\sin 45^\circ\text{ N} - 200\left(\tfrac{4}{5}\right)\text{ N}$$
$$= -383.2\text{ N} = 383.2\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250\cos 45^\circ\text{ N} + 200\left(\tfrac{3}{5}\right)\text{ N}$$
$$= 296.8\text{ N}\uparrow$$

The resultant force, shown in Figure c, has a magnitude of:

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$
$$= 485\text{ N}$$

The direction angle θ is:

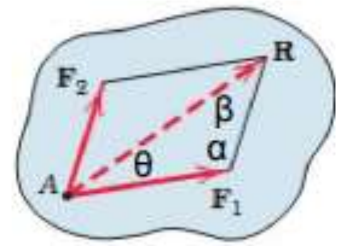
$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$

B. Non-rectangular Components: المركبات غير متعامدة

1- F1, F2: known Required: R, resultant force of F1, F2 Parallelogram low:

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\alpha$$

$$\frac{R}{\sin\alpha} = \frac{F_2}{\sin\theta} \Rightarrow \theta = \sin^{-1}\left(\frac{F_2}{R}\sin\alpha\right)$$



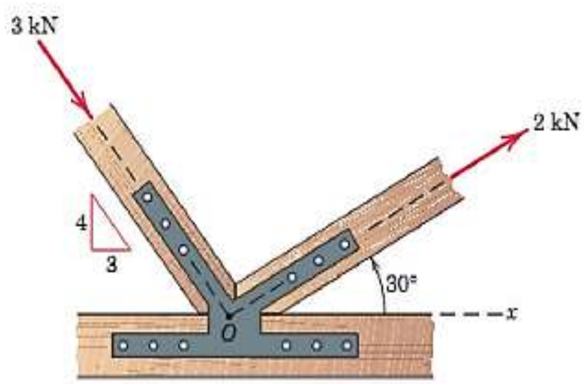
2- R : known

Required: force componentsF1, F2

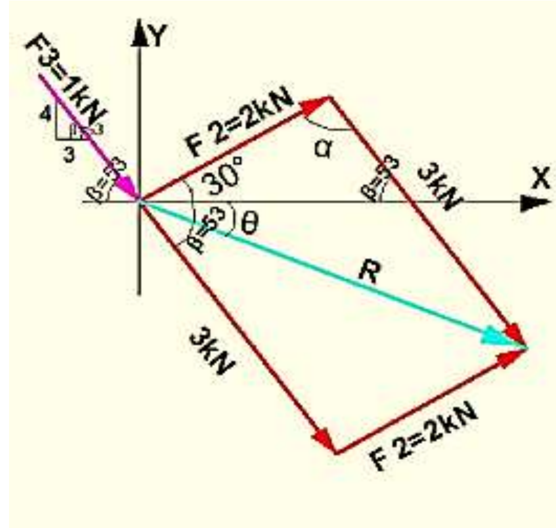
$$\frac{R}{\sin\alpha} = \frac{F_2}{\sin\theta} \Rightarrow F_2 = \frac{\sin\theta}{\sin\alpha}R$$

$$\frac{R}{\sin\alpha} = \frac{F_1}{\sin\beta} \Rightarrow F_1 = \frac{\sin\beta}{\sin\alpha}R$$

EXAMPLE: The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint **O**. Determine the magnitude of the resultant **R** of the two forces and the angle **θ** which R makes with the positive x-axis.



Procedure 1:



$\beta = \tan^{-1}\frac{4}{3} = 53.13°$

$\alpha = 180 - 53.13 - 30 = 96.87°$

$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\alpha$

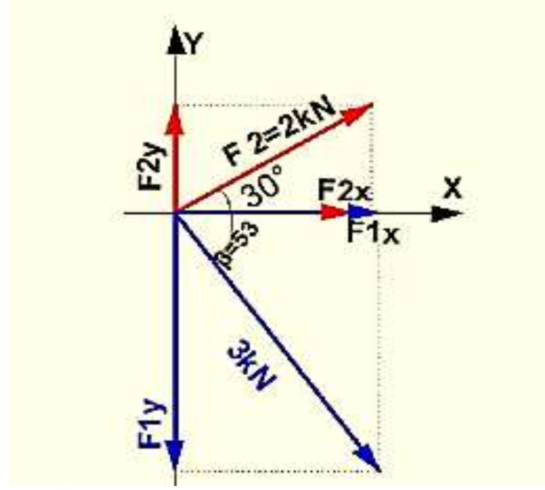
$R^2 = 2^2 + 3^3 - 2*2*3\cos 96.87$

$R = \sqrt{14.35} = 3.8N$

$$\frac{R}{\sin\alpha} = \frac{F_1}{\sin(\theta+30)} \Rightarrow \frac{3.8}{\sin 96.87} = \frac{3}{\sin(\theta+30)} \Rightarrow \theta + 30 = \sin^{-1}(0.78)$$

$\theta + 30 = 51.6 \Rightarrow \theta = 21.6°$

Procedure 2:



$F_{1x} = F_1\cos 53.13 = 3\cos 53.13 = 1.8N \rightarrow$

$F_{1y} = F_1\sin 53.13 = 3\sin 53.13 = -2.4N \downarrow$

$F_{2x} = F_2\cos 30 = 2\cos 30 = 1.73N \rightarrow$

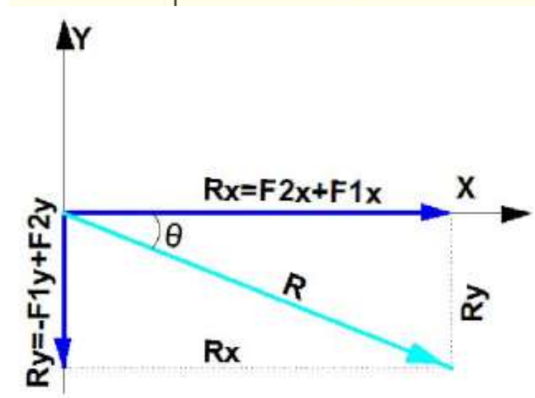
$F_{2y} = F_2\sin 30 = 2\sin 30 = 1N \uparrow$

$R_x = F_{1x} + F_{2x} = 1.8 + 1.73 = 3.53N \rightarrow$

$R_y = -F_{1y} + F_{2y} = -2.4 + 1 = -1.4N \downarrow$

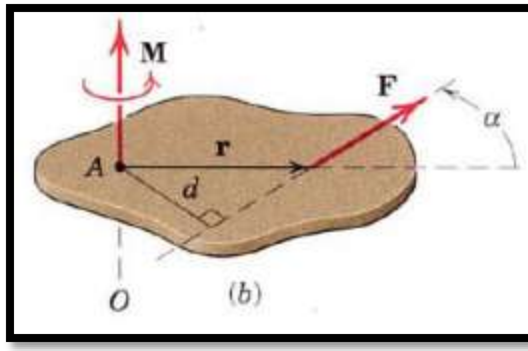
$R = \sqrt{R_1^2 + R_2^2} = \sqrt{3.53^2 + 1.4^2} = 3.8N$

$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{1.4}{3.53} = 21.6°$



Moment of a force:

It is ability of a force to turning or twisting a body about any axis.

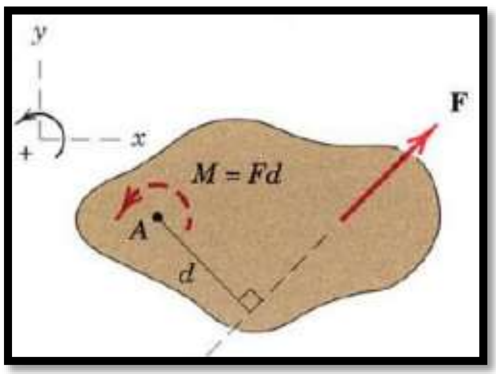


- **Magnitude of moment :**

M=F*d

The Moment of a force= The applied force × the perpendicular distance .

العزم =القوة المسلطة × المسافة العمودية بين نقطة تأثير القوة و مركز العزم



Where:

M:The Moment of a force (N.m).

F: The applied force (N).

d : Is the perpendicular distance from the axis of moment to the line of acting of the force.

- **Units :**

The basic units of moment according to SI units are newton-meters N.m, kN.m, N.mm…etc

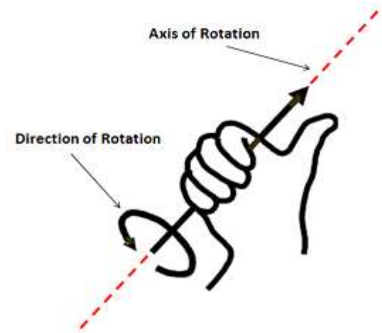
• Sense of Moment :

• minus sign(-) for counterclockwise moments . (سالب):عكس اتجاه عقرب الساعة

• a plus sign (+) for clockwise moments. (موجب):اتجاه عقرب الساعة

A clockwise rotation about the center of moments will be considered a positive moment; while a counter-clockwise rotation about the center of moments will be considered negative.

نستخدم قاعدة الكف اليمنى، حيث الابهام المبسوط يعبر عن اتجاه المتجه و اتجاه دوران بقية الاصابع يمثل اتجاه العزم.



• The Principle of Moments:

The moment of several forces about a point is simply the algebraic sum of their component moments about the same point.

$M = \Sigma F.d$

$= F_{1x} .y_{1} + F_{1y} .x_{1} + F_{2x} .y_{2} + F_{3y} .x_{3}$