Adding these quantities together gives the total kilogram moles.

## Example 14

In normal living cells, the nitrogen requirement for the cells is provided from protein metabolism (i.e., consumption of the protein in the cells). When individual cells are commercially grown, $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}$ is usually used as the source of nitrogen. Determine the amount of $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}$ consumed in a fermentation medium in which the final cell concentration is $35 \mathrm{~g} / \mathrm{L}$ in a 500 L volume of the fermentation medium. Assume that the cells contain 9 wt . \% N, and that $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}$ is the only nitrogen source.

## Solution

Basis: 500 L solution containing $35 \mathrm{~g} / \mathrm{L}$

$$
\underline{500 \mathrm{~L}}\left|\frac{35 \mathrm{~g} \text { cell }}{\mathrm{L}}\right| \frac{0.09 \mathrm{~g} \mathrm{~N}}{1 \mathrm{gcell}}\left|\frac{1 \mathrm{~g} \mathrm{~mol} \mathrm{~N}}{14 \mathrm{~g} \mathrm{~N}} \times \frac{1 \mathrm{~g} \mathrm{~mol}\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}}{1 \mathrm{~g} \mathrm{~mol} \mathrm{~N}}\right| \frac{132 \mathrm{~g}\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}}{1 \mathrm{~g} \mathrm{~mol}\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}}=14,850 \mathrm{~g}\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4}
$$

## Analyses of Multi Components Solutions and Mixtures

The composition of gases will always be presumed to be given in mole percent or fraction unless specifically stated otherwise.

The composition of liquids and solids will be given by mass (weight) percent or fraction unless otherwise specifically stated.
For Example Table below lists the detailed composition of dry air (composition of air $21 \% \mathrm{O}_{2}$ and $79 \% \mathrm{~N}_{2}$ ).
Basis 100 mol of air

| Component | Moles $=$ percent | Mol. wt. | Lb or kg | Weight $\%$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{2}$ | 21.0 | 32 | 672 | 23.17 |
| $\mathrm{~N}_{2}$ | $\underline{79.0}$ | 28.2 | $\underline{2228}$ | $\underline{76.83}$ |
| Total | 100 |  | 2900 | 100.00 |

The average molecular weight is $2900 \mathrm{lb} / 100 \mathrm{lb} \mathrm{mol}=29.0$, or $2900 \mathrm{~kg} / 100 \mathrm{~kg} \mathrm{~mol}=29$

### 1.2.3 Concentration

Concentration generally refers to the quantity of some substance per unit volume.
a. Mass per unit volume ( lb of solute/ $\mathrm{ft}^{3}$ of solution, g of solute $/ \mathrm{L}, \mathrm{lb}$ of solute/barrel, kg of solute $/ \mathrm{m}^{3}$ ).
b. Moles per unit volume ( lb mol of solute $/ \mathrm{ft}^{3}$ of solution, g mol of solute/L, g mol of solute $/ \mathrm{cm}^{3}$ ).
c. Parts per million (ppm); parts per billion (ppb), a method of expressing the concentration of extremely dilute solutions; ppm is equivalent to a mass (weight) fraction for solids and liquids because the total amount of material is of a much higher order of magnitude than the amount of solute; it is a mole fraction for gases.
d. Parts per million by volume ( ppmv ) and parts per billion by volume ( ppbv )
e. Other methods of expressing concentration with which you may be familiar are molarity ( $\mathrm{g} \mathrm{mol} / \mathrm{L}$ ), molality (mole solute/kg solvent), and normality (equivalents/L).

## Example 14

The current OSHA 8-hour limit for HCN (MW = 27.03) in air is 10.0 ppm . A lethal dose of HCN in air is (from the Merck Index) $300 \mathrm{mg} / \mathrm{kg}$ of air at room temperature. How many $\mathrm{mg} \mathrm{HCN} / \mathrm{kg}$ air is 10.0 ppm ? What fraction of the lethal dose is 10.0 ppm ?

## Solution

Basis: 1 kg mol of the air/ HCN mixture
The 10.0 ppm is $\frac{10.0 \mathrm{~g} \mathrm{~mol} \mathrm{HCN}}{10^{6}(\text { air }+\mathrm{HCN}) \mathrm{g} \mathrm{mol}}=\frac{10.0 \mathrm{~g} \mathrm{~mol} \mathrm{HCN}}{10^{6} \mathrm{~g} \mathrm{~mol} \mathrm{air}}$
a. $\left.\frac{10.0 \mathrm{~g} \mathrm{~mol} \mathrm{HCN}}{10^{6} \mathrm{~g} \mathrm{~mol} \text { air }}\left|\frac{27.03 \mathrm{~g} \mathrm{HCN}}{1 \mathrm{~g} \mathrm{~mol} \mathrm{HCN}}\right| \frac{1 \mathrm{~g} \mathrm{~mol} \text { air }}{29 \mathrm{~g} \text { air }} \right\rvert\, \frac{1000 \mathrm{mg} \mathrm{HCN}}{1 \mathrm{~g} \mathrm{HCN}} \times \frac{1000 \mathrm{~g} \text { air }}{1 \mathrm{~kg} \text { air }}=9.32 \mathrm{mg} \mathrm{HCN} / \mathrm{kg}$ air
b. $\frac{9.32}{300}=0.031$

## Example 15

A solution of $\mathrm{HNO}_{3}$ in water has a specific gravity of 1.10 at $25^{\circ} \mathrm{C}$. The concentration of the $\mathrm{HNO}_{3}$ is $15 \mathrm{~g} / \mathrm{L}$ of solution. What is the
a. Mole fraction of $\mathrm{HNO}_{3}$ in the solution?
b. ppm of $\mathrm{HNO}_{3}$ in the solution?

## Solution

Basis: 1 L of solution

$$
\begin{array}{r}
\frac{15 \mathrm{~g} \mathrm{HNO}_{3}}{1 \mathrm{~L} \text { soln }}\left|\frac{1 \mathrm{~L}}{1000 \mathrm{~cm}^{3}}\right| \frac{1 \mathrm{~cm}^{3}}{1.10 \mathrm{~g} \text { soln }}=0.01364 \frac{\mathrm{~g} \mathrm{HNO}_{3}}{\mathrm{~g} \text { soln }} \\
\text { Basis: } 100 \mathrm{~g} \text { solution }
\end{array}
$$

The mass of water in the solution is: $100-0.0134=99.986 \mathrm{~g} \mathrm{H}_{2} \mathrm{O}$.

|  |  | g |  | MW | g mol |
| :--- | :--- | :--- | :--- | :--- | :--- |
| mol fraction |  |  |  |  |  |
| a. | $\mathrm{HNO}_{3}$ | 0.01364 | 63.02 | $2.164 \times 10^{-4}$ | $3.90 \times 10^{-5}$ |
|  | $\mathrm{H}_{2} \mathrm{O}$ | 99.986 | 18.016 | $\frac{5.550}{5.550}$ | $\underline{1.00}$ |
|  | Total |  |  | 1.00 |  |
| b. |  | $\frac{0.01364}{1}$ | $=\frac{13,640}{10^{6}}$ or $13,640 \mathrm{ppm}$ |  |  |

## Example 16

To avoid the possibility of explosion in a vessel containing gas having the composition of $40 \% \mathrm{~N}_{2}, 45 \% \mathrm{O}_{2}$, and $15 \%$ $\mathrm{CH}_{4}$, the recommendation is to dilute the gas mixture by adding an equal amount of pure $\mathrm{N}_{2}$. What is the final mole fraction of each gas?

## Solution

The basis is 100 moles of initial gas

| Composition | Original Mixture mol\% | After Addition | Final Mixture |
| :---: | :---: | :---: | :---: |
| $\mathrm{N}_{2}$ | $40 \longrightarrow 100$ | $N_{2}$ | Mole Fraction |
| $\mathrm{O}_{2}$ | 45 | 140 | $140 / 200=0.70$ |
| $\mathrm{CH}_{4}$ | 15 | 45 | $45 / 200=0.23$ |
| Total | 100 | 15 | $15 / 200=0.07$ |

## Questions

1. Answer the following questions true or false:
a. The pound mole is comprised of $2.73 \times 10^{26}$ molecules
b. The kilogram mole is comprised of $6.022 \times 10^{26}$ molecules.
c. Molecular weight is the mass of a compound or element per mole.
2. What is the molecular weight of acetic acid $(\mathrm{CH} 3 \mathrm{COOH})$ ?
3. For numbers such as 2 mL of water +2 mL of ethanol, does the sum equal to 4 mL of the solution?
4. Answer the following questions true or false:
a. The inverse of the density is the specific volume.
b. Density of a substance is the mass per unit volume.
c. The density of water is less than the density of mercury.
5. A cubic centimeter of mercury has a mass of 13.6 g at Earth's surface. What is the density of mercury?
6. What is the approximate density of water at room temperature in $\mathrm{kg} / \mathrm{m}^{3}$ ?
7. For liquid HCN , a handbook gives: sp. gr. $10^{\circ} \mathrm{C} / 4^{\circ} \mathrm{C}=1.2675$. What does this statement mean?
8. Answer the following questions true or false:
a. The density and specific gravity of mercury are the same.
b. Specific gravity is the ratio of two densities.
c. If you are given the value of a reference density, you can determine the density of a substance of interest by multiplying by the specific gravity.
d. The specific gravity is a dimensionless quantity.
9. A mixture is reported as $15 \%$ water and $85 \%$ ethanol. Should the percentages be deemed to be by mass, mole, or volume?
10. Answer the following questions true or false:
a) In engineering practice the compositions of liquids and solids are usually denoted in weight (mass) fraction or percent.
b) In engineering practice the composition of gases is usually denoted in mole fraction or percent.
c) e. A pseudo-average molecular weight can be calculated for a mixture of pure components whether solid, liquid, or gases.
11. Do parts per million denote a concentration that is a mole ratio?
12. Does the concentration of a component in a mixture depend on the amount of the mixture?
13. Pick the correct answer. How many ppm are there in 1 ppb ? (a) 1000 , (b) 100 , (c) 1 , (d) 0.1 , (e) 0.01 , (f) 0.001 ?
14. How many ppb are there in 1 ppm ?
15. Does 50 ppm represent an increase of five times a value of 10 ppm ?

## Answers:

1. (a) T ; (b) T ; (c) T
2. 60.05
3. No
4. (a) T ; (b) T ; (c) T
5. $13.6 \mathrm{~g} / \mathrm{cm}^{3}$
6. $1000 \mathrm{~kg} / \mathrm{m}^{3}$
7. The statement means that the density at $10^{\circ} \mathrm{C}$ of liquid HCN is 1.2675 times the density of water at $4^{\circ} \mathrm{C}$.
8. (a) F - the units differ; (b) T; (c) T; (d) F.
9. Mass
10. (a) T ; (b) T ; (c) T
11. For gases but not for liquids or solids.
12. No
13. 0.001
14. 1000
15. No (4 times)

## Problems

1. Convert the following:
a) 120 g mol of NaCl to g .
b) 120 g of NaCl to g mol .
c) 120 lb mol of NaCl to lb .
d) 120 lb of NaCl to lb mol.
2. Convert 39.8 kg of NaCl per 100 kg of water to kg mol of NaCl per kg mol of water.
3. How many lb mol of $\mathrm{NaNO}_{3}$ are there in 100 lb ?
4. The density of a material is $2 \mathrm{~kg} / \mathrm{m}^{3}$. What is its specific volume?
5. An empty 10 gal tank weighs 4.5 lb . What is the total weight of the tank plus the water when it is filled with 5 gal of water?
6. If you add 50 g of sugar to 500 mL of water, how do you calculate the density of the sugar solution?
7. For ethanol, a handbook gives: sp. gr. $60^{\circ} \mathrm{F}=0.79389$. What is the density of ethanol at $60^{\circ} \mathrm{F}$ ?
8. The specific gravity of steel is 7.9. What is the volume in cubic feet of a steel ingot weighing 4000 lb ?
9. The specific gravity of a solution is 0.80 at $70^{\circ} \mathrm{F}$. How many cubic feet will be occupied by 100 lb of the solution at $70^{\circ} \mathrm{F}$ ?
10. A solution in water contains 1.704 kg of $\mathrm{HNO}_{3} / \mathrm{kg} \mathrm{H}_{2} \mathrm{O}$, and the solution has a specific gravity of 1.382 at $20^{\circ} \mathrm{C}$. What is the mass of $\mathrm{HNO}_{3}$ in kg per cubic meter of solution at $20^{\circ} \mathrm{C}$ ?
11. Forty gal/min of a hydrocarbon fuel having a specific gravity of 0.91 flows into a tank truck with a load limit of $40,000 \mathrm{lb}$ of fuel. How long will it take to fill the tank in the truck?
12. Pure chlorine enters a process. By measurement it is found that 2.4 kg of chlorine pass into the process every 3.1 minutes. Calculate the molar flow rate of the chlorine in $\mathrm{kg} \mathrm{mol} / \mathrm{hr}$.
13. Commercial sulfuric acid is $98 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ and $2 \% \mathrm{H}_{2} \mathrm{O}$. What is the mole ratio of $\mathrm{H}_{2} \mathrm{SO}_{4}$ to $\mathrm{H}_{2} \mathrm{O}$ ?
14. A compound contains $50 \%$ sulfur and $50 \%$ oxygen by mass. Is the empirical formula of the compound (1) SO , (2) $\mathrm{SO}_{2}$, (3) $\mathrm{SO}_{3}$, or (4) $\mathrm{SO}_{4}$ ?
15. How many kg of activated carbon (a substance used in removing trace impurities) must be mixed with 38 kg of sand so that the final mixture is $28 \%$ activated carbon?
16. A gas mixture contains 40 lb of $\mathrm{O}_{2}, 25 \mathrm{lb}$ of $\mathrm{SO}_{2}$, and 30 lb of $\mathrm{SO}_{3}$. What is the composition of the mixture in mole fractions?
17. Saccharin, an artificial sweetener that is 3000 times sweeter than sucrose, is composed of $45.90 \%$ carbon, $2.73 \%$ hydrogen, $26.23 \%$ oxygen, $7.65 \%$ nitrogen, and $17.49 \%$ sulfur. Is the molecular formula of saccharin (a) $\mathrm{C}_{14} \mathrm{H}_{10} \mathrm{O}_{6} \mathrm{~N}_{2} \mathrm{~S}_{2}$, (b) $\mathrm{C}_{5} \mathrm{H}_{7} \mathrm{O}_{3} \mathrm{NS}$, (c) $\mathrm{C}_{8} \mathrm{H}_{9} \mathrm{O}_{2} \mathrm{NS}$, and (d) $\mathrm{C}_{7} \mathrm{H}_{5} \mathrm{O}_{3} \mathrm{NS}$ ?
18. A mixture of gases is analyzed and found to have the following composition: $\mathrm{CO}_{2} 12.0 \%, \mathrm{CO} 6.0 \%, \mathrm{CH}_{4}$ $27.3 \%, \mathrm{H}_{2} 9.9 \%$ and $\mathrm{N}_{2} 44.8 \%$. How much will 3 lb mol of this gas weigh?
19. A liquefied mixture of $n$-butane, $n$-pentane, and $n$-hexane has the following composition: $n-\mathrm{C}_{4} \mathrm{H}_{10} 50 \%$, $n-\mathrm{C}_{5} \mathrm{H}_{12} 30 \%$, and $n-\mathrm{C}_{6} \mathrm{H}_{14} 20 \%$. For this mixture, calculate:
a) The weight fraction of each component.
b) The mole fraction of each component.
c) The mole percent of each component.
d) The average molecular weight of the mixture.
20. How many $\mathrm{mg} / \mathrm{L}$ is equivalent to a $1.2 \%$ solution of a substance in water?

## Answers:

1. (a) 7010 g ; (b) 2.05 g mol ; (c) 7010 lb ; (d) 2.05 lb mol
2. $0.123 \mathrm{~kg} \mathrm{~mol} \mathrm{NaCl} / \mathrm{kg} \mathrm{mol} \mathrm{H}_{2} \mathrm{O}$
3. 1.177 lb mol
4. $0.5 \mathrm{~m}^{3} / \mathrm{kg}$
5. 46.2 lb
6. Measure the mass of water (should be about 500 g ) and add it to 50 g . Measure the volume of the solution (will not be 450 mL ). Divide the mass by the volume.
7. $0.79389 \mathrm{~g} / \mathrm{cm}^{3}$ (assuming the density of water is also at $60^{\circ} \mathrm{F}$ )
8. $8.11 \mathrm{ft}^{3}$
9. $2 \mathrm{ft}^{3}$
10. $870 \mathrm{~kg} \mathrm{HNO} 3 / \mathrm{m}^{3}$ solution.
11. 132 min
12. $0.654 \mathrm{~kg} \mathrm{~mol} / \mathrm{hr}$
13. 9
14. $\mathrm{SO}_{2}$
15. 14.8 kg
16. $\mathrm{O}_{2} 0.62 ; \mathrm{SO}_{2} 0.19 ; \mathrm{SO}_{3} 0.19$
17. (d)
18. 72.17 lb
19. (a) $\mathrm{C}_{4}: 0.50, \mathrm{C}_{5}: 0.30, \mathrm{C}_{6}: 0.20$; (b) $\mathrm{C}_{4}: 0.57, \mathrm{C}_{5}: 0.28, \mathrm{C}_{6}: 0.15$; (c) $\mathrm{C}_{4}: 57, \mathrm{C}_{5}: 28, \mathrm{C}_{6}: 15$; (d) $66.4 \mathrm{~kg} / \mathrm{kg}$ mol
20. $12000 \mathrm{mg} / \mathrm{L}$.

### 1.3 Choosing a Basis

* A basis is a reference chosen by you for the calculations you plan to make in any particular problem, and a proper choice of basis frequently makes the problem much easier to solve.
* The basis may be a period of time such as hours, or a given mass of material, such as $\mathbf{5 \mathbf { k g }}$ of $\mathrm{CO}_{2}$, or some other convenient quantity.
* For liquids and solids in which a mass (weight) analysis applies, a convenient basis is often $\underline{\mathbf{1} \mathbf{~ o r ~} \mathbf{1 0 0} \mathbf{l b} \text { or kg; }}$ similarly, $\mathbf{1}$ or $\mathbf{1 0 0}$ moles is often a good choice for a gas.


## Example 17

Gas mixture $10.0 \% \mathrm{H}_{2}, 40.0 \% \mathrm{CH}_{4}, 30.0 \% \mathrm{CO}$, and $20.0 \% \mathrm{CO}_{2}$, what is the average molecular weight of the gas?

## Solution

Basis: 100 kg mol or lb mol of gas

| Component | Percent $=\mathbf{k g}$ <br> mol or lb mol | Mol wt. | Kg or lb |
| :--- | :---: | :---: | :---: |
| $\mathrm{CO}_{2}$ | 20.0 | 44.0 | 880 |
| CO | 30.0 | 28.0 | 840 |
| $\mathrm{CH}_{4}$ | 40.0 | 16.04 | 642 |
| $\mathrm{H}_{2}$ | 10.0 | 2.02 | $\frac{20}{2382}$ |
| Total | 100.0 |  |  |

Average molecular weight $=\frac{2382 \mathrm{~kg}}{100 \mathrm{~kg} \mathrm{~mol}}=23.8 \mathrm{~kg} / \mathrm{kg} \mathrm{mol}$
Other Method for Solution:
Average molecular weight $=0.2 * 44+0.3 * 28+0.4 * 16.04+0.1 * 2.02=23.8 \mathrm{~kg} / \mathrm{kg} \mathrm{mol}$

## Example 18

A liquefied mixture has the following composition: $n-\mathrm{C}_{4} \mathrm{H}_{10} 50 \%$ ( $\mathrm{MW}=58$ ), $\mathrm{n}-\mathrm{C}_{5} \mathrm{H}_{12} 30 \%$ (MW=72), and $\mathrm{n}-\mathrm{C}_{6} \mathrm{H}_{14} 20 \%$ (MW=86). For this mixture, calculate: (a) mole fraction of each component. (b) Average molecular weight of the mixture.

## Solution

Basis: 100 kg

|  | $\%=\mathrm{kg}$ | wt fr | MW | kg mol | mol fr |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| $\mathrm{n}-\mathrm{C}_{4} \mathrm{H}_{10}$ | 50 | $\mathbf{0 . 5 0}$ | 58 | 0.86 | $\mathbf{0 . 5 7}$ |
| $\mathrm{n}-\mathrm{C}_{5} \mathrm{H}_{12}$ | 30 | $\mathbf{0 . 3 0}$ | 72 | 0.42 | $\mathbf{0 . 2 8}$ |
| $\mathrm{n}-\mathrm{C}_{6} \mathrm{H}_{14}$ | $\mathbf{2 0}$ | $\mathbf{0 . 2 0}$ | 86 | 0.23 | $\mathbf{0 . 1 5}$ |
| 100 | $\mathbf{1 . 0 0}$ |  | 1.51 | $\mathbf{1 . 0 0}$ |  |

$$
\text { Average molecular weight }=\frac{\text { total mass }}{\text { total mol }}=\frac{100 \mathrm{~kg}}{1.51 \mathrm{~kg} \mathrm{~mol}}=66
$$

## Example 19

In a ternary alloy such as $\mathrm{Nd}_{4.5} \mathrm{Fe}_{77} \mathrm{~B}_{18.5}$ the average grain size is about 30 nm . By replacing 0.2 atoms of Fe with atoms of Cu , the grain size can be reduced (improved) to 17 nm .
(a) What is the molecular formula of the alloy after adding the Cu to replace the Fe ?
(b) What is the mass fraction of each atomic species in the improved alloy?

## Solution

Basis: 100 g mol (or atoms) of $\mathrm{Nd}_{4.5} \mathrm{Fe}_{77} \mathrm{~B}_{18.5}$
(a) The final alloy is $\mathrm{Nd}_{4.5} \mathrm{Fe}_{76.8} \mathrm{~B}_{18.5} \mathrm{Cu}_{0.2}$.
(b) Use a table to calculate the respective mass fractions.

| Component | Original g mol | Final g mol | MW | $\mathbf{g}$ | Mass fraction |
| :--- | :---: | :---: | ---: | ---: | :---: |
| Nd | 4.5 | 4.5 | 144.24 | 649.08 | 0.126 |
| Fe | 77 | 76.8 | 55.85 | 4289.28 | 0.833 |
| B | $\underline{18.5}$ | 18.5 | 10.81 | 199.99 | 0.039 |
| Cu |  | 0.2 | 63.55 | $\underline{12.71}$ | $\underline{0.002}$ |
| Total | 100.0 | 100.0 |  | 5151.06 | 1.000 |

## Example 20

A medium-grade bituminous coal analyzes as follows:

| Component | Percent |
| :--- | :---: |
| S | 2 |
| N | 1 |
| O | 6 |
| Ash | 11 |
| Water | 3 |
| Residuum | 77 |

The residuum is C and H , and the mole ratio in the residuum is $\mathrm{H} / \mathrm{C}=9$. Calculate the weight (mass) fraction composition of the coal with the ash and the moisture omitted (ash - and moisture - free).

## Solution

Take as a basis 100 kg of coal because then percent = kilograms.

$$
\text { Basis: } 100 \mathrm{~kg} \text { of coal }
$$


We need to determine the individual kg of $\mathbf{C}$ and of $\mathbf{H}$ in the $\mathbf{7 7} \mathbf{~ k g}$ total residuum.

To determine the kilograms of C and H , you have to select a new basis.
Basis: 100 kg mol (Because the $\mathrm{H} / \mathrm{C}$ ratio is given in terms of moles, not weight)

| Component | Mole fraction | kg mol | Mol. wt. | kg | Mass fraction |
| :--- | :--- | :---: | :---: | :---: | :---: |
| H | $\frac{9}{1+9}=0.90$ | 90 | 1.008 | 90.7 | 0.43 |
| C | $\frac{1}{1+9}=\frac{0.10}{1.00}$ | $\frac{10}{100}$ | 12 | $\frac{120}{210.7}$ | $\frac{0.57}{1.00}$ |

$\mathrm{H}:(77 \mathrm{~kg})(0.43)=33.15 \mathrm{~kg}$
C: $(77 \mathrm{~kg})(0.57)=43.85 \mathrm{~kg}$
Finally, we can prepare a table summarizing the results on the basis of $\mathbf{1 k g}$ of the coal ash-free and water-free.

| Component | $\mathbf{k g}$ | Wt. fraction |
| :---: | :---: | :---: |
| C | 43.85 | 0.51 |
| H | 33.15 | 0.39 |
| S | 2 | 0.02 |
| N | 1 | 0.01 |
| O | $\underline{86.0}$ | $\underline{0.07}$ |
| Total |  | 1.00 |

### 1.4 Temperature

* Temperature is a measure of the energy (mostly kinetic) of the molecules in a system. This definition tells us about the amount of energy.
* Other scientists prefer to say that temperature is a property of the state of thermal equilibrium of the system with respect to other systems because temperature tells us about the capability of a system to transfer energy (as heat).


## Four types of temperature:

Two based on a relative scale, degrees Fahrenheit ( ${ }^{\circ} \mathbf{F}$ ) and Celsius ( ${ }^{\circ} \mathbf{C}$ ), and two based on an absolute scale, degree

## Rankine ( ${ }^{\circ} \mathbf{R}$ ) and Kelvin (K).

## Temperature Conversion

$$
\begin{aligned}
& \Delta^{\circ} \mathrm{F}=\Delta^{\circ} \mathrm{R} \\
& \Delta^{\circ} \mathrm{C}=\Delta \mathrm{K}
\end{aligned}
$$

Also, the $\Delta^{\circ} \mathrm{C}$ is larger than the $\Delta^{\circ} \mathrm{F}$

$$
\begin{array}{lll}
\frac{\Delta^{\circ} \mathrm{C}}{\Delta^{\circ} \mathrm{F}}=1.8 & \text { or } & \Delta^{\circ} \mathrm{C}=1.8 \Delta^{\circ} \mathrm{F} \\
\frac{\Delta \mathrm{~K}}{\Delta^{\circ} \mathrm{R}}=1.8 & \text { or } & \Delta \mathrm{K}=1.8 \Delta^{\circ} \mathrm{R}
\end{array}
$$

The proper meaning of the symbols ${ }^{\circ} \mathbf{C},{ }^{\circ} \mathbf{F}, \mathbf{K}$, and ${ }^{\circ} \mathbf{R}$, as either the temperature or the unit temperature difference, must be interpreted from the context of the equation or sentence being examined.

Suppose you have the relation:

$$
T_{{ }^{\circ} \mathrm{F}}=a+b T_{{ }^{\circ} \mathrm{C}}
$$

What are the units of $\mathbf{a}$ and $\mathbf{b}$ ? The units of $\mathbf{a}$ must be ${ }^{\circ} \mathbf{F}$ for consistency. The correct units for $\mathbf{b}$ must involve the conversion factor ( $\mathbf{1 . 8} \boldsymbol{\Delta}^{\circ} \mathbf{F} \backslash \Delta^{\circ} \mathbf{C}$ ), the factor that converts the size of a interval on one temperature scale

$$
T_{{ }_{\mathrm{F}}}=a_{{ }^{\circ} \mathrm{F}}+(\frac{\underbrace{1.8 \Delta^{\circ} \mathrm{F}}_{b}}{\Delta^{\circ} \mathrm{C}}) T_{{ }^{\circ} \mathrm{C}}
$$

Unfortunately, the units for $b$ are usually ignored; just the value of $b(1.8)$ is employed.

* The relations between ${ }^{\circ} \mathbf{C},{ }^{\circ} \mathbf{F}, \mathbf{K}$, and ${ }^{\circ} \mathbf{R}$ are:

$$
\begin{array}{lll}
T_{{ }^{\circ} \mathrm{R}}=T_{{ }^{\circ} \mathrm{F}}\left(\frac{1 \Delta^{\circ} \mathrm{R}}{1 \Delta^{\circ} \mathrm{F}}\right)+460^{\circ} \mathrm{R} & \underline{\mathrm{Or}} & \mathrm{~T}_{{ }^{\mathrm{R}}}=\mathrm{T}_{{ }_{\mathrm{F}}}+460 \\
T_{\mathrm{K}}=T^{\circ} \mathrm{C}\left(\frac{1 \Delta \mathrm{~K}}{1 \Delta^{\circ} \mathrm{C}}\right)+273 \mathrm{~K} & \underline{\mathrm{Or}} & \mathrm{~T}_{\mathrm{K}}=\mathrm{T}_{{ }^{\circ} \mathrm{C}}+273 \\
T_{{ }^{\circ} \mathrm{F}}-32^{\circ} \mathrm{F}=T^{\circ} \mathrm{C}\left(\frac{1.8 \Delta^{\circ} \mathrm{F}}{1 \Delta^{\circ} \mathrm{C}}\right) & & \\
T_{{ }^{\circ} \mathrm{C}}=\left(T_{{ }^{\circ} \mathrm{F}}-32^{\circ} \mathrm{F}\right)\left(\frac{1 \Delta^{\circ} \mathrm{C}}{1.8 \Delta^{\circ} \mathrm{F}}\right) & \underline{\mathrm{Or}} & \mathrm{~T}_{{ }^{\circ} \mathrm{F}}=1.8 \mathrm{~T}_{{ }^{\circ} \mathrm{C}}+32
\end{array}
$$

## Example 21

Convert $100^{\circ} \mathrm{C}$ to (a) K , (b) ${ }^{\circ} \mathrm{F}$, and (c) ${ }^{\circ} \mathrm{R}$.

## Solution

(a) $(100+273)^{\circ} \mathrm{C} \frac{1 \Delta \mathrm{~K}}{1 \Delta^{\circ} \mathrm{C}}=373 \mathrm{~K}$
or with suppression of the $\Delta$ symbol,
$(100+273)^{\circ} \mathrm{C} \frac{1 \mathrm{~K}}{1^{\circ} \mathrm{C}}=373 \mathrm{~K}$
(b) $\left(100^{\circ} \mathrm{C}\right) \frac{1.8 \Delta^{\circ} \mathrm{F}}{1 \Delta^{\circ} \mathrm{C}}+32^{\circ} \mathrm{F}=212^{\circ} \mathrm{F}$
(c) $(212+460)^{\circ} \mathrm{F} \frac{1 \Delta^{\circ} \mathrm{R}}{1 \Delta^{\circ} \mathrm{F}}=672^{\circ} \mathrm{R}$
or

$$
(373 \mathrm{~K}) \frac{1.8 \Delta^{\circ} \mathrm{R}}{1 \Delta \mathrm{~K}}=672^{\circ} \mathrm{R}
$$

## Example 22

The heat capacity of sulfuric acid has the units $\mathrm{J} /(\mathrm{g} \mathrm{mol})\left({ }^{\circ} \mathrm{C}\right)$, and is given by the relation

$$
\text { Heat capacity }=139.1+1.56 * 10^{-1} \mathrm{~T}
$$

where T is expressed in ${ }^{\circ} \mathrm{C}$. Modify the formula so that the resulting expression has the associated units of $\mathrm{Btu} /(\mathrm{lb} \mathrm{mol})$ $\left({ }^{\circ} \mathrm{R}\right)$ and T is in ${ }^{\circ} \mathrm{R}$.

## Solution

$\mathrm{T}_{\mathrm{O}_{\mathrm{F}}}=1.8 \mathrm{~T}_{{ }^{\circ} \mathrm{C}}+32 \longrightarrow \mathrm{~T}_{{ }^{\circ} \mathrm{C}}=\left(\mathrm{T}_{\mathrm{OF}_{\mathrm{F}}}-32\right) / 1.8$
$\mathrm{T}_{{ }^{\mathrm{R}}}=\mathrm{T}_{\mathrm{o}_{\mathrm{F}}}+460 \longrightarrow \mathrm{~T}_{{ }^{\circ} \mathrm{F}}=\mathrm{T}_{{ }^{\mathrm{R}}}-460$
$\therefore \mathrm{T}_{{ }^{\mathrm{C}}}=\left[\mathrm{T}_{{ }_{\mathrm{R}}}-460-32\right] / 1.8$

$$
\text { heat capacity }=\{\left.139.1+1.56 \times 10^{-1}[\overbrace{\left.\left(T_{{ }^{\circ} \mathrm{R}}-460-32\right) \frac{1}{1.8}\right]}^{T_{\mathrm{o}} \mathrm{C}}\} \times \underbrace{(\mathrm{g} \mathrm{~mol})\left({ }^{\circ} \mathrm{C}\right)}_{\text {conversion factors }}\left|\frac{1 \mathrm{~J}}{1055 \mathrm{~J}}\right| \frac{454}{1 \mathrm{lb} \mathrm{~mol}} \frac{\mathrm{~g} \mathrm{~mol}}{1.8^{\circ} \mathrm{R}} \right\rvert\, \frac{1^{\circ} \mathrm{C}}{1 .} \underbrace{1}=
$$

$$
=23.06+\dot{2.07} \times 10^{-2} \mathrm{~T}_{\mathrm{o}}
$$

Note the suppression of the $\Delta$ symbol in the conversion between ${ }^{\circ} \mathrm{C}$ and ${ }^{\circ} \mathrm{R}$.

## Problems

1. Complete the following table with the proper equivalent temperatures:

2. The heat capacity of sulfur is $C_{p}=15.2+2.68 \mathrm{~T}$, where $\mathrm{C}_{\mathrm{p}}$ is in $\mathrm{J} /(\mathrm{g} \mathrm{mol})(\mathrm{K})$ and T is in K . Convert this expression so that $\mathrm{C}_{\mathrm{p}}$ is in $\mathrm{cal} /(\mathrm{g} \mathrm{mol})\left({ }^{\circ} \mathrm{F}\right)$ with T in ${ }^{\circ} \mathrm{F}$.

## Answers:

1. 

| ${ }^{\circ} \mathbf{C}$ | ${ }^{\circ} \mathbf{F}$ | $\mathbf{K}$ | ${ }^{\circ} \mathbf{R}$ |
| :--- | :---: | :---: | :---: |
| -40.0 | -40.0 | 233 | 420 |
| 25.0 | 77.0 | 298 | 537 |
| 425 | 796 | 698 | 1256 |
| -234 | -390 | 38.8 | 69.8 |

2. $\mathrm{C}_{\mathrm{p}}=93.2+0.186 \mathrm{~T}_{\mathrm{F}}$

### 1.5 Pressure

## Pressure and Its Units

Pressure is defined as "the normal (perpendicular) force per unit area (Figure 5.1). The pressure at the bottom of the static (nonmoving) column of mercury exerted on the sealing plate is

$$
\begin{equation*}
p=\frac{F}{A}=\rho g h+p_{0} \tag{5.1}
\end{equation*}
$$

Where $\mathrm{p}=$ pressure at the bottom of the column of the fluid, $\mathrm{F}=$ force, $\mathrm{A}=$ area, $\rho=$ density of fluid $g=$ acceleration of gravity, $h=$ height of the fluid column, and $p_{0}=$ pressure at the top of the column of fluid


Figure 5.1 Pressure is the normal force per unit area. Arrows show the force exerted on the respective areas

For Example, suppose that the cylinder of fluid in Figure 5.1 is a column of mercury that has an area of $1 \mathrm{~cm}^{2}$ and is 50 cm high. The density of the Hg is $13.55 \mathrm{~g} / \mathrm{cm}^{3}$. Thus, the force exerted by the mercury alone on the $1 \mathrm{~cm}^{2}$ section of the bottom plate by the column of mercury is

$$
\begin{aligned}
F & \left.=\frac{13.55 \mathrm{~g}}{\mathrm{~cm}^{3}}\left|\frac{980 \mathrm{~cm}}{\mathrm{~s}^{2}}\right| \frac{50 \mathrm{~cm}}{1 \mathrm{~cm}^{2}}\left|\frac{1 \mathrm{~kg}}{1000 \mathrm{~g}}\right| \frac{1 \mathrm{~m}}{100 \mathrm{~cm}} \right\rvert\, \frac{1(\mathrm{~N})\left(\mathrm{s}^{2}\right)}{1(\mathrm{~kg})(\mathrm{m})} \\
& =6.64 \mathrm{~N}
\end{aligned}
$$

The pressure on the section of the plate covered by the mercury is the force per unit area of the mercury plus the pressure of the atmosphere

$$
\left.p=\frac{6.64 \mathrm{~N}}{1 \mathrm{~cm}^{2}}\left|\left(\frac{100 \mathrm{~cm}}{1 \mathrm{~m}}\right)^{2}\right| \frac{\left(1 \mathrm{~m}^{2}\right)(1 \mathrm{~Pa})}{(1 \mathrm{~N})} \right\rvert\, \frac{1 \mathrm{kPa}}{1000 \mathrm{~Pa} .}+p_{0}=66.4 \mathrm{kPa}+p_{0}
$$

If we had started with units in the AE system, the pressure would be computed as [the density of mercury is 845.5 $\left.\mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}\right]$

$$
\begin{aligned}
p & \left.=\frac{845.5 \mathrm{lb}_{\mathrm{m}}}{1 \mathrm{ft}^{3}}\left|\frac{32.2 \mathrm{ft}}{\mathrm{~s}^{2}}\right| \frac{50 \mathrm{~cm}}{}\left|\frac{1 \mathrm{in} .}{2.54 \mathrm{~cm}}\right| \frac{1 \mathrm{ft}}{12 \mathrm{in} .} \right\rvert\, \frac{(\mathrm{s})^{2}\left(\mathrm{lb}_{\mathrm{f}}\right)}{32.174(\mathrm{ft})\left(\mathrm{lb}_{\mathrm{m}}\right)}+p_{0} \\
& =1388 \frac{1 \mathrm{lb}_{\mathrm{f}}}{\mathrm{ft}^{2}}+p_{0}
\end{aligned}
$$

## Measurement of Pressure

Pressure, like temperature, can be expressed using either an absolute or a relative scale.


Figure 5.2 (a) Open-end manometer showing a pressure above atmospheric pressure. (b) Manometer measuring an absolute pressure.

The relationship between relative and absolute pressure is given by the following expression:
Gauge Pressure + Barometer Pressure = Absolute Pressure

The standard atmosphere is defined as the pressure (in a standard gravitational field) equivalent to 1 atm or 760 mm Hg at $0^{\circ} \mathrm{C}$ or other equivalent.

The standard atmosphere is equal to

- 1.00 atmospheres (atm)
- 33.91 feet of water $\left(\mathrm{ft}_{2} \mathrm{O}\right)$
- 14.7 pounds (force) per square inch absolute (psia)
- 29.92 inches of mercury (in. Hg )
- 760.0 millimeters of mercury ( mm Hg )
- $1.013 * 10^{5}$ pascal $(\mathrm{Pa})$ or newtons per square meter $\left(\mathrm{N} / \mathrm{m}^{2}\right)$; or 101.3 kPa

For Example, convert 35 psia to inches of mercury and kPa .

$$
\begin{aligned}
& \frac{35 \mathrm{psia}}{} \left\lvert\, \frac{29.92 \mathrm{in.} \mathrm{Hg}}{14.7 \mathrm{psia}}=71.24 \mathrm{in} \mathrm{Hg}\right. \\
& \underline{35 \mathrm{psia}} \left\lvert\, \frac{101.3 \mathrm{kPa}}{14.7 \mathrm{psia}}=241 \mathrm{kPa}\right.
\end{aligned}
$$

For Example, What is the equivalent pressure to $\mathbf{1} \mathbf{~ k g} / \mathbf{c m}^{\mathbf{2}}$ (i.e., $\mathbf{k g}_{\mathrm{f}} / \mathbf{c m}^{\mathbf{2}}$ ) in pascal ( $\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{\mathbf{2}}$ )
$\left[1 \mathrm{~kg} / \mathrm{cm}^{2}\right] *\left[9.8 \mathrm{~m} / \mathrm{s}^{2}\right] *\left[(100 \mathrm{~cm} / 1 \mathrm{~m})^{2}\right]=9.8 * 10^{4} \mathrm{~N} / \mathrm{m}^{2}$ (or Pa )

## Example 23

What is the equivalent pressure to 60 Gpa (gigapascal) in
(a) atmospheres
(b) psia
(c) inches of Hg
(d) mm of Hg

## Solution

$$
\text { Basis: } 60 \mathrm{GPa}
$$

(a) $\frac{60 \mathrm{GPa}}{}\left|\frac{10^{6} \mathrm{kPa}}{1 \mathrm{GPa}}\right| \frac{1 \mathrm{~atm}}{101.3 \mathrm{kPa}}=0.59 \times 10^{6} \mathrm{~atm}$
(b) $\frac{60 \mathrm{GPa}}{}\left|\frac{10^{6} \mathrm{kPa}}{1 \mathrm{GPa}}\right| \frac{14.696 \mathrm{psia}}{101.3 \mathrm{kPa}}=8.70 \times 10^{6} \mathrm{psia}$
(c) $\frac{60 \mathrm{GPa}}{}\left|\frac{10^{6} \mathrm{kPa}}{1 \mathrm{GPa}}\right| \frac{29.92 \mathrm{in} . \mathrm{Hg}}{101.3 \mathrm{kPa}}=1.77 \times 10^{7} \mathrm{in} . \mathrm{Hg}$
(d) $\frac{60 \mathrm{GPa}}{}\left|\frac{10^{6} \mathrm{kPa}}{1 \mathrm{GPa}}\right| \frac{760 \mathrm{~mm} \mathrm{Hg}}{101.3 \mathrm{kPa}}=4.50 \times 10^{8} \mathrm{~mm} \mathrm{Hg}$

## Example 24

The pressure gauge on a tank of $\mathrm{CO}_{2}$ used to fill soda-water bottles reads 51.0 psi . At the same time the barometer reads $28.0 \mathrm{in} . \mathrm{Hg}$. What is the absolute pressure in the tank in psia? See Figure E5.2.


Figure E5.2

## Solution

$$
\text { Atmospheric pressure }=\frac{28.0 \mathrm{in.} \mathrm{Hg}}{} \left\lvert\, \frac{14.7 \mathrm{psia}}{29.92 \mathrm{in} \mathrm{Hg}}=13.76 \mathrm{psia}\right.
$$

The absolute pressure in the tank is

$$
51.0 \mathrm{psia}+13.76 \mathrm{psia}=64.8 \mathrm{psia}
$$

## Example 25

Small animals such as mice can live (although not comfortably) at reduced air pressures down to 20 kPa absolute. In a test, a mercury manometer attached to a tank, as shown in Figure E5.3, reads 64.5 cm Hg and the barometer reads 100 kPa . Will the mice survive?


Figure E5.3

