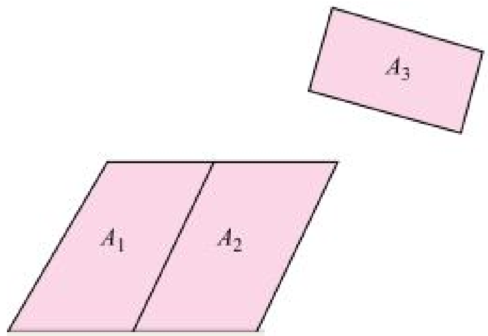
Relation between Shape Factors

For the surfaces 1,2 and 3 shown



The total shape factor is the sum of its parts.

$$F_{3-1,2} = F_{3-1} + F_{3-2}$$

It can also be written as

$$A_3 F_{3-1,2} = A_3 F_{3-1} + A_3 F_{3-2}$$

Since

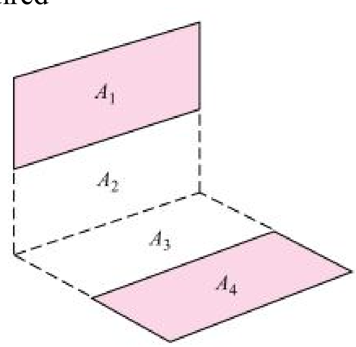
$$A_3 F_{3-1,2} = A_{1,2} F_{1,2-3}$$
$$A_3 F_{3-1} = A_1 F_{1-3}$$
$$A_3 F_{3-2} = A_2 F_{2-3}$$

The expression could be rewritten

$$A_{1,2} F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3}$$

This means that the total radiation arriving at surface 3 is the sum of the radiations from surfaces 1 and 2.

For example, surfaces 1, 2, 3 &4 as shown below. If $F_{1\text{-}4}$ is required



$$A_{1,2} F_{1,2-3,4} = A_1 F_{1-3,4} + A_2 F_{2-3,4}$$

And

$$A_1 F_{1-3,4} = A_1 F_{1-3} + A_1 F_{1-4}$$
$$A_{1,2} F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3}$$

$$\therefore\ A_{1,2} F_{1,2-3,4} = A_{1,2} F_{1,2-3} - A_2 F_{2-3} + A_1 F_{1-4} + A_2 F_{2-3,4} \ \Rightarrow$$

$$F_{1-4} = \frac{1}{A_1}\left(A_{1,2} F_{1,2-3,4} + A_2 F_{2-3} - A_{1,2} F_{1,2-3} - A_2 F_{2-3,4}\right)$$

For concave curved surfaces, the body may see it self and the shape factor for the body it self must be taken into accounts. The general solution used is

$$\sum_{j=1}^{n} F_{ij} = 1.0$$

Where F_{ij} is the fraction of the total energy leaving surface i that arrives at surface j.

For 3-body problem

$$F_{11} + F_{12} + F_{13} = 1, \qquad \textit{For body 1}$$
$$F_{21} + F_{22} + F_{23} = 1, \qquad \textit{For body 2}$$
$$F_{31} + F_{32} + F_{33} = 1, \qquad \textit{For body 3}$$

Shape-Factor Algebra for Open Ends of Cylinders

EXAMPLE 8-3

Two concentric cylinders having diameters of 10 and 20 cm have a length of 20 cm. Calculate the shape factor between the open ends of the cylinders.

■ Solution

We use the nomenclature of Figure 8-15 for this problem and designate the open ends as surfaces 3 and 4. We have $L/r_2 = 20/10 = 2.0$ and $r_1/r_2 = 0.5$; so from Figure 8-15 or Table 8-2 we obtain

$$F_{21} = 0.4126 \qquad F_{22} = 0.3286$$

Using the reciprocity relation [Equation (8-18)] we have

$$A_1F_{12} = A_2F_{21} \quad \text{and} \quad F_{12} = (d_2/d_1)F_{21} = (20/10)(0.4126) = 0.8253$$

For surface 2 we have

$$F_{21} + F_{22} + F_{23} + F_{24} = 1.0$$

From symmetry $F_{23} = F_{24}$ so that

$$F_{23} = F_{24} = \left(\tfrac{1}{2}\right)(1 - 0.4126 - 0.3286) = 0.1294$$

Using reciprocity again,

$$A_2F_{23} = A_3F_{32}$$

and

$$F_{32} = \frac{\pi(20)(20)}{\pi(20^2 - 10^2)/4} 0.1294 = 0.6901$$

We observe that $F_{11} = F_{33} = F_{44} = 0$ and for surface 3

$$F_{31} + F_{32} + F_{34} = 1.0 \qquad [a]$$

So, if F_{31} can be determined, we can calculate the desired quantity F_{34}. For surface 1

$$F_{12} + F_{13} + F_{14} = 1.0$$

and from symmetry $F_{13} = F_{14}$ so that

$$F_{13} = \left(\frac{1}{2}\right)(1 - 0.8253) = 0.0874$$

Using reciprocity gives

$$A_1 F_{13} = A_3 F_{31}$$

$$F_{31} = \frac{\pi(10)(20)}{\pi(20^2 - 10^2)/4} 0.0874 = 0.233$$

Then, from Equation (a)

$$F_{34} = 1 - 0.233 - 0.6901 = 0.0769$$