

- Difference between Kinematics and Dynamics:

**Kinematic:** The term kinematics means motion. It is the study of motion without regard for the cause. Calculations in Kinematic involve masses.

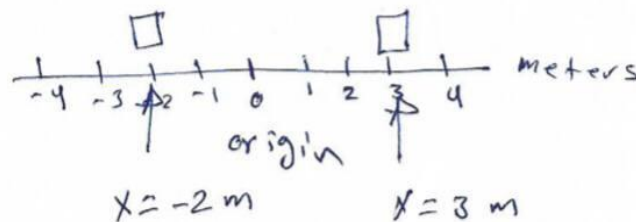
**Dynamics:** On the other hand, it is the study of the causes of motion. Calculations in Dynamics, therefore, involve masses and forces.

Kinematic and dynamic are the basics of mechanics, the study of force and motion.

- Kinematics take into account quantities such as displacement, velocity and acceleration.

**Position:** the location of an object. In physics, it is a number on an axis.

For example:



Where:

X: represents position.

Unit: meters (m).

**Displacement:** the direction and distance of the shortest path between an initial and final position.

$$\Delta X = X_f - X_i$$

Where:

$\Delta X$ : displacement.

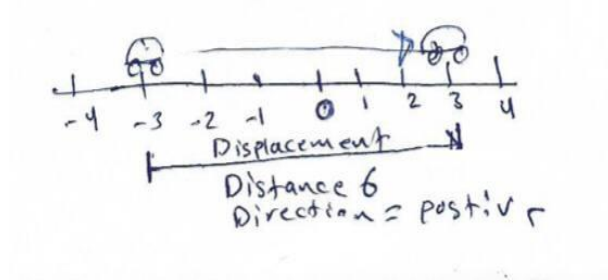
$X_f$ : final position.

$X_i$ : initial position.

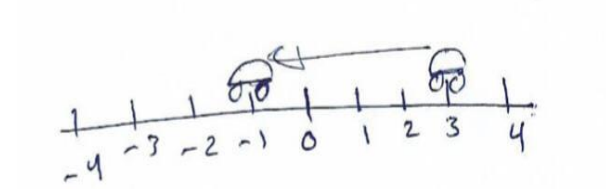
Units: meters (m).

Examples:

$$\begin{aligned}\Delta X &= X_f - X_i \\ &= 3 - (-3) \\ &= 6 \text{ m.}\end{aligned}$$



$$\begin{aligned}\Delta X &= X_f - X_i \\ &= -1 - 3 \\ &= -4 \text{ m.}\end{aligned}$$



**Velocity:** speed and direction.

You are familiar with concept of speed; it tells you how fast something is going (55 mile/h). The speedometer in a car measures speed but does not indicate direction.

When you need to know both speed and direction, you use velocity.

Velocity is a vector. It is the measure of how fast and in which direction the motion is occurring. ( $\bar{V}$ ) Represents it. In this section, we focus on average velocity, which is represented by  $\bar{V}$  with a bar over it ( $\bar{V}$ ).

$$\bar{V} = \Delta X/t$$

Where:

$\bar{V}$ : velocity.

$\Delta X$ : displacements.

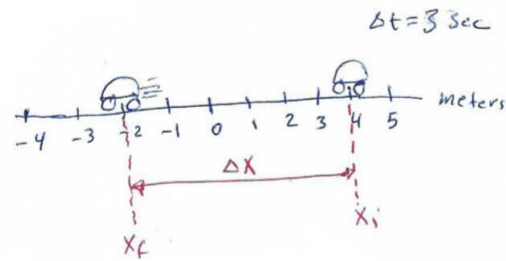
t: time.

Units: meters/seconds (m/s).

Example: - what is the car's velocity?

$$V = \frac{\Delta X}{\Delta t} = \frac{-2-4}{3}$$

$$= \frac{-6}{3} = -2 \text{ m/s}$$



- **Average velocity:** displacement divided by elapsed time.
- **Instantaneous velocity:** velocity at a specific moment.

For example: - if you drop an egg of a 40-floor building, the egg's velocity will change, it will move faster as it falls. Someone on the 39<sup>th</sup> floor would see it pass with a different velocity than would someone on the 30<sup>th</sup> floor. Therefore, when we use the word instantaneous, we describe an object's velocity at a particular instant.

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta X}{\Delta t}$$

Where:

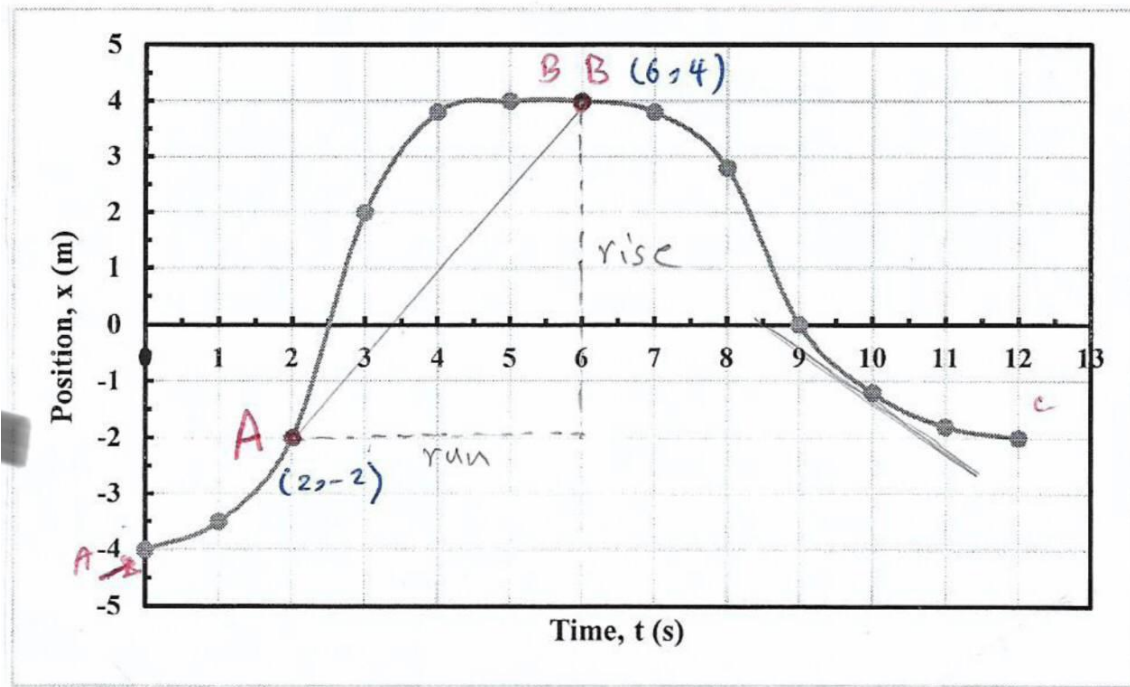
V: instantaneous velocity.

$\Delta X$ : displacement (m).

$\Delta t$ : elapsed time (sec.).

### Position – time graph and velocity:

It shows position of object over time. A graph of an object's position over time is a useful tool for analyzing motion.



You can see from the above graph that the car started at position ( $X = -4\text{m}$ ), then moved to the position ( $X = +4\text{m}$ ) at about ( $t = 4.5\text{s}$ ), stay there for a couple of seconds, and then reached the position ( $X = -2\text{m}$ ) again after a total of 12 seconds of motion.

- Where the graph is horizontal, as at point B, it indicates that the car's position is not changing, which means that the car is not moving.
- Where the graph is steep, position is changing rapidly with respect to time and the car is moving quickly.
- Displacement and velocity are mathematically related, and a position-time graph can be used to find the average or instantaneous velocity of an object.
- The slope of a straight line between any two points of the graph is object's average velocity between them.

Average velocity: slope of line between two points.

$$\bar{V} = \frac{\text{rise}}{\text{run}} = \frac{\Delta X}{\Delta t}$$

- The slope of the tangent line for any point on a straight-line segment of a position-time graph is constant. When the slope is constant, the velocity is constant. An example of constant velocity is the horizontal section of the graph that includes the point B in the above figure.
- The slope of a tangent line at different points on a curve is not constant. The slope at a single point on a curve is determined by the slope of the tangent line to the curve at that point.
- The slope that measured by the tangent line equals the instantaneous velocity at that point.

**Instantaneous velocity:** is the slope of a tangent line at specific point.

Example: - what is the average velocity between points A and B in the graph above?

Sol.:-

$$\bar{V} = \frac{\text{rise}}{\text{run}} = \frac{\Delta X}{\Delta t}$$

$$\bar{V} = \frac{6}{4} = 1.5 \text{ m/s}$$

$$\text{Or, } \bar{V} = \frac{4 - (-2)}{6 - 2} = \frac{6}{4} = 1.5 \text{ m/s}$$

Example: - consider the points A, B and C in the graph above? Where is the instantaneous velocity equal zero? Where is it positive? Where is it negative?

Sol.:-

According to the figure above, we can conclude that:

It equals zero at B

It is positive at A

It is negative at C

**Acceleration:** change in the velocity.

When an object's velocity changes, it accelerates. Acceleration measures the rate at which an object speeds up, slows down or changes direction. Any of these variations constitutes a change in velocity. The letter (a) represents acceleration.

$$\bar{a} = \frac{\Delta V}{\Delta t}$$

Where:

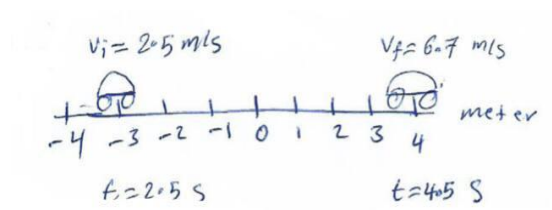
$\bar{a}$ : (average) acceleration.

$\Delta V$ : change in instantaneous velocity.

$\Delta t$ : elapsed time.

Unit:  $m/s^2$ .

Example: - What is the average acceleration of car between 2.5 and 4.5 seconds?



Sol.:-

$$\bar{a} = \frac{\Delta V}{\Delta t} = \frac{6.7 - 2.5}{4.5 - 2.5} = \frac{4.2}{2} = 2.1 m/s^2.$$

**Instantaneous acceleration:** acceleration at a particular moment.

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t}$$

Where:

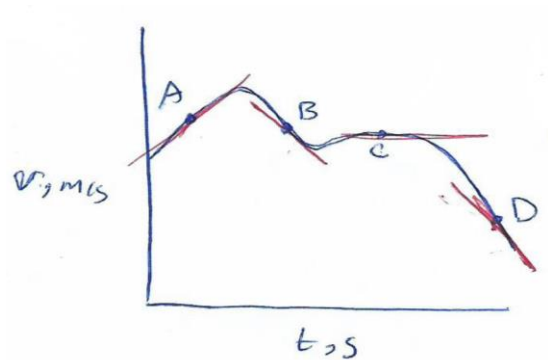
a: instantaneous acceleration.

$\Delta V$ : velocity.

$\Delta t$ : elapsed time.

Earlier, we discussed how the slope of the tangent line at any point on a position-time graph equals the instantaneous velocity at that point. The same way can be applied here to conclude that the instantaneous acceleration at any point on a velocity-time equals the slope of the tangent line.

Example: - The graph shows the car's velocity versus time. Describe the instantaneous acceleration at A, B, C and D as positive, negative or zero.



Sol.:-

a is positive at A

a is negative at B

a equals zero at C

a is negative at D

Example: - the car goes 10.3m in 4.15s at constant velocity, and then accelerates at  $1.22 \text{ m/s}^2$  for 5.34s. What is its final velocity?



Sol.:-

$$V_i = \frac{\Delta X}{\Delta t} = \frac{10.3}{4.15} = 2.48 \text{ m/s}$$

$$a = \frac{V_f - V_i}{\Delta t}$$

$$1.22 = \frac{V_f - 2.48}{5.34}$$

$$V_f = 8.99 \text{ m/s}$$

**Derivation: creating new equation.**

Deriving motion equation for constant acceleration:

Assumption:

- Constant acceleration, since the acceleration is constant, the velocity increases at constant rate. This means the average velocity is the sum of the initial and final velocities divided by two.

$$\bar{V} = \frac{V_i + V_f}{2}$$

From the definition of acceleration

$$a = \frac{V_f - V_i}{t} \quad \dots (1)$$

$$V_f - V_i = at$$

$$V_f = V_i + at \quad \dots (2)$$

By square both sides, we obtain:

$$(V_f)^2 = (V_i + at)^2$$

$$V_f^2 = V_i^2 + 2V_i at + a^2 t^2$$

$$V_f^2 = V_i^2 + at(2V_i + at)$$

$$V_f^2 = V_i^2 + at(V_i + V_i + at)$$

Since

$$V_f = V_i + at$$

Therefore

$$V_f^2 = V_i^2 + at(V_i + V_f) \quad \dots (3)$$

Since

$$\bar{V} = \frac{V_f + V_i}{2} \quad \& \quad \bar{V} = \frac{\Delta X}{t}$$

$$\frac{V_f + V_i}{2} = \frac{\Delta X}{t}$$



$t (V_f + V_i) = 2 \Delta X$  sub. Into eq. (3)

$$V_f^2 = V_i^2 + a (2 \Delta X)$$

$$V_f^2 = V_i^2 + 2 a \Delta X$$

- We have now accomplished our goal. We can calculate the final velocity of an object if we know its initial velocity, its acceleration and its displacement but we do not know the elapsed time.

We can also find the following equation

$$\bar{V} = \frac{V_f + V_i}{2} \quad \& \quad \bar{V} = \frac{\Delta X}{t}$$

$$\frac{V_f + V_i}{2} = \frac{\Delta X}{t}$$

$$V_f = \frac{2\Delta X}{t} - V_i$$

Since

$$V_f = V_i + at$$

$$V_i + at = \frac{2\Delta X}{t} - V_i$$

$$\frac{2\Delta X}{t} = V_i + at + V_i = 2 V_i + at$$

$$(2\Delta X = 2 V_i t + at^2)/2$$

$$\Delta X = V_i t + \frac{1}{2} at^2$$

Example: - what acceleration will stop the car exactly at the stop sign for the below figure?

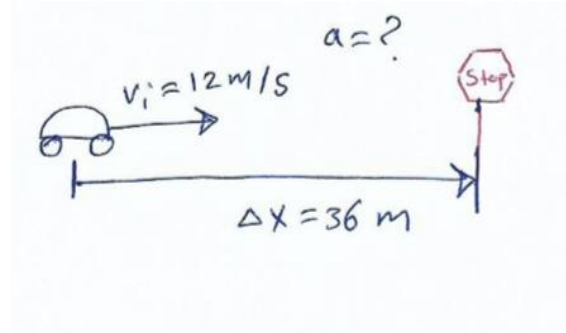
Sol.:-

Since

$$V_f^2 = V_i^2 + 2 a \Delta X$$

$$V_f^2 - V_i^2 = 2 a \Delta X$$

$$a = \frac{V_f^2 - V_i^2}{2 \Delta X} = \frac{(0)^2 - (12)^2}{2 * 36} = -2 \text{ m/s}^2$$



Example: - what is the runner's velocity at the end of a 100-meter dash? The acceleration equals  $0.528 \text{ m/s}^2$ .

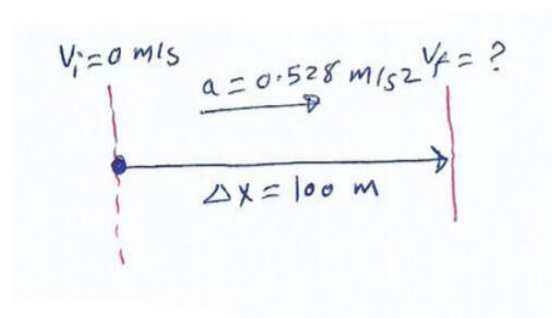
Sol.:-

$$V_f^2 = V_i^2 + 2 a \Delta X$$

$$V_f^2 - (0)^2 = 2 (0.528) * (100)$$

$$V_f^2 = 105.6$$

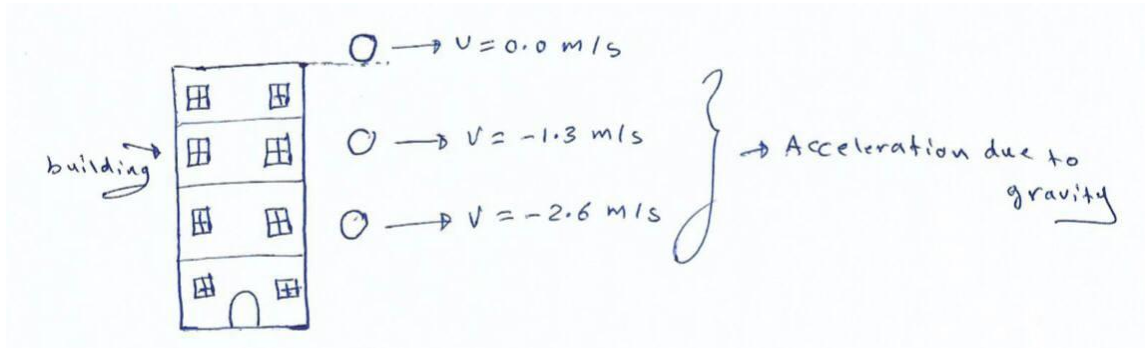
$$V_f = 10.3 \text{ m/s}$$



**Free – Fall acceleration,  $g$  ( $m/s^2$ ):**

Rate of acceleration due to the force of earth's gravity.

For example, a ball is dropped from the top of the building.



$$g = 9.8 \text{ m/s}^2$$

$g$  = magnitude of free fall acceleration.

Example: - what is the egg's velocity after falling from rest for 0.1 seconds?

Sol.:-

Since

$$V_f = V_i + at$$

$$V_f = (0) + (-9.8) * (0.1)$$

$$V_f = -0.98 \text{ m/s.}$$

Example: - the ball is thrown straight up, with velocity 10.0 m/s. when will its velocity be  $-5.60 \text{ m/s}$ ?

Sol.:-

$$V_f = V_i + at$$

$$V_f - V_i = at$$

$$t = \frac{V_f - V_i}{a} = \frac{-5.6 - 10}{-9.80}$$

$$t = 1.6 \text{ s.}$$

