Heat Exchange between Non-Black Bodies

For nonblackbodies, we have to define the following two terms:

G = irradiation = total radiation incident upon a surface per unit time and per unit area.
J = radiosity = total radiation that leaves a surface per unit time and per unit area.

$$J = \epsilon E_b + \rho G$$

where ϵ is the emissivity and E_b is the blackbody emissive power. Since the transmissivity is assumed to be zero, the reflectivity may be expressed as

$$\rho = 1 - \alpha = 1 - \epsilon$$

so that

$$J = \epsilon E_b + (1 - \epsilon)G \qquad \textbf{[8-37]}$$

The net energy leaving the surface is the difference between the radiosity and the irradiation:

$$\frac{q}{A} = J - G = \epsilon E_b + (1 - \epsilon)G - G$$

Solving for G in terms of J

$$q = \frac{\epsilon A}{1 - \epsilon}(E_b - J)$$

or

$$q = \frac{E_b - J}{(1 - \epsilon)/\epsilon A}$$

The terms $(1 - \epsilon)/\epsilon A$ is called the surface resistance.

(Eb − J) represents the potential difference.

- For two surface problems:

The energy leaving surface 1 and arriving at surface 2 = $J_1 A_1 F_{12}$

The energy leaving surface 2 and arriving at surface 1 = $J_2 A_2 F_{21}$

The net interchange between the two surfaces is

$$q_{1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

But

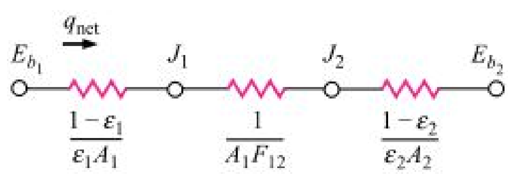
$$A_1 F_{12} = A_2 F_{21}$$

so that

$$q_{1-2} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$$

or

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}} \quad \textbf{[8-39]}$$

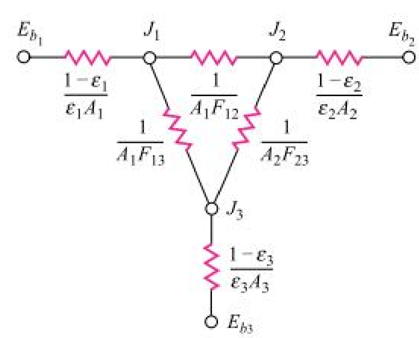


The term $1/A_iF_{ij}$ is called the space resistance.

$Eb1 = \sigma T_1^4$, $Eb2 = \sigma T_2^4$ In this case the net heat transfer would be the overall potential difference divided by the sum of the resistances:

$$q_{net} = \frac{E_{b1} - E_{b2}}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2} = \frac{\sigma(T_1^4 - T_2^4)}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2}$$

- <u>For three surface problems:</u>



For three bodies (as shown in Figure) exchange heat each body exchange heat with the other two. The heat exchange between body 1 and body 2 would be

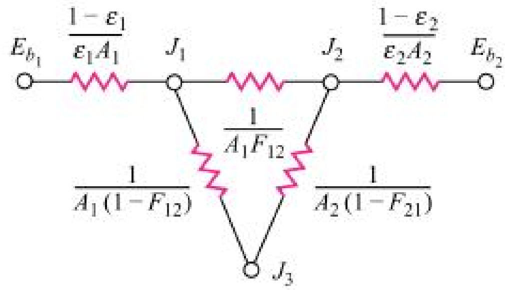
$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

and that between body 1 and body 3,

$$q_{1-3} = \frac{J_1 - J_3}{1/A_1 F_{13}}$$

We can solve the problem by applying of Kirchhoff's current law to the circuit, which states that the sum of the currents entering a node is zero.

A special case, if the three body problems with one body dose not exchange heat (insulated), the net heat work in this case is

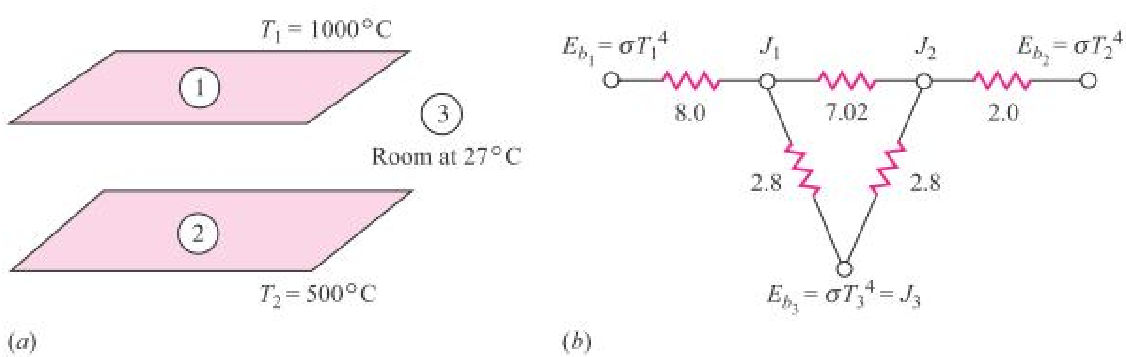


$$q_{net} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\dfrac{A_1 + A_2 - 2A_1 F_{12}}{A_2 - A_1 (F_{12})^2} + \left(\dfrac{1}{\epsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)}$$

EXAMPLE 8-6 Hot Plates Enclosed by a Room

Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart, as shown in Figure Example 8-6. One plate is maintained at 1000°C and the other at 500°C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27°C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.

Figure Example 8-6 | (*a*) Schematic. (*b*) Network.



■ Solution

This is a three-body problem, the two plates and the room, so the radiation network is shown in Figure 8-27. From the data of the problem

$$T_1 = 1000°\text{C} = 1273 \text{ K} \qquad A_1 = A_2 = 0.5 \text{ m}^2$$
$$T_2 = 500°\text{C} = 773 \text{ K} \qquad \epsilon_1 = 0.2$$
$$T_3 = 27°\text{C} = 300 \text{ K} \qquad \epsilon_2 = 0.5$$

Because the area of the room A_3 is very large, the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ may be taken as zero and we obtain $E_{b_3} = J_3$. The shape factor F_{12} was given in Example 8-2:

$$F_{12} = 0.285 = F_{21}$$
$$F_{13} = 1 - F_{12} = 0.715$$
$$F_{23} = 1 - F_{21} = 0.715$$

The resistances in the network are calculated as

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.5)} = 8.0 \qquad \frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.5)} = 2.0$$
$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.5)(0.285)} = 7.018 \qquad \frac{1}{A_1 F_{13}} = \frac{1}{(0.5)(0.715)} = 2.797$$
$$\frac{1}{A_2 F_{23}} = \frac{1}{(0.5)(0.715)} = 2.797$$

Taking the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

node J_1:

$$\frac{E_{b_1} - J_1}{8.0} + \frac{J_2 - J_1}{7.018} + \frac{E_{b_3} - J_1}{2.797} = 0 \qquad \textbf{[a]}$$

node J_2:

$$\frac{J_1 - J_2}{7.018} + \frac{E_{b_3} - J_2}{2.797} + \frac{E_{b_2} - J_2}{2.0} = 0 \qquad \textbf{[b]}$$

Now

$$E_{b_1} = \sigma T_1^4 = 148.87 \text{ kW/m}^2 \quad [47{,}190 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_2} = \sigma T_2^4 = 20.241 \text{ kW/m}^2 \quad [6416 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_3} = \sigma T_3^4 = 0.4592 \text{ kW/m}^2 \quad [145.6 \text{ Btu/h} \cdot \text{ft}^2]$$

Inserting the values of E_{b_1}, E_{b_2}, and E_{b_3} into Equations (a) and (b), we have two equations and two unknowns J_1 and J_2 that may be solved simultaneously to give

$$J_1 = 33.469 \text{ kW/m}^2 \qquad J_2 = 15.054 \text{ kW/m}^2$$

The total heat lost by plate 1 is

$$q_1 = \frac{E_{b_1} - J_1}{(1-\epsilon_1)/\epsilon_1 A_1} = \frac{148.87 - 33.469}{8.0} = 14.425 \text{ kW}$$

and the total heat lost by plate 2 is

$$q_2 = \frac{E_{b_2} - J_2}{(1-\epsilon_2)/\epsilon_2 A_2} = \frac{20.241 - 15.054}{2.0} = 2.594 \text{ kW}$$

The total heat received by the room is

$$q_3 = \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_2 - J_3}{1/A_2F_{23}}$$

$$= \frac{33.469 - 0.4592}{2.797} + \frac{15.054 - 0.4592}{2.797} = 17.020 \text{ kW} \quad [58{,}070 \text{ Btu/h}]$$

From an overall-balance standpoint we must have

$$q_3 = q_1 + q_2$$

because the net energy lost by both plates must be absorbed by the room.