Turbulent Heat Transfer in a Tube — EXAMPLE 6-1

Air at 2 atm and 200°C is heated as it flows through a tube with a diameter of 1 in (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3-m length of the tube?

■ Solution

We first calculate the Reynolds number to determine if the flow is laminar or turbulent, and then select the appropriate empirical correlation to calculate the heat transfer. The properties of air at a bulk temperature of 200°C are

$$\rho = \frac{p}{RT} = \frac{(2)(1.0132\times10^5)}{(287)(473)} = 1.493\ \text{kg/m}^3 \qquad [0.0932\ \text{lb}_m/\text{ft}^3]$$

$$\text{Pr} = 0.681$$

$$\mu = 2.57\times10^{-5}\ \text{kg/m}\cdot\text{s} \qquad [0.0622\ \text{lb}_m/\text{h}\cdot\text{ft}]$$

$$k = 0.0386\ \text{W/m}\cdot{}^\circ\text{C} \qquad [0.0223\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}]$$

$$c_p = 1.025\ \text{kJ/kg}\cdot{}^\circ\text{C}$$

$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57\times10^{-5}} = 14{,}756$$

so that the flow is turbulent. We therefore use Equation (6-4*a*) to calculate the heat-transfer coefficient.

$$\text{Nu}_d = \frac{hd}{k} = 0.023\,\text{Re}_d^{0.8}\,\text{Pr}^{0.4} = (0.023)(14{,}756)^{0.8}(0.681)^{0.4} = 42.67$$

$$h = \frac{k}{d}\text{Nu}_d = \frac{(0.0386)(42.67)}{0.0254} = 64.85\ \text{W/m}^2\cdot{}^\circ\text{C} \qquad [11.42\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}]$$

The heat flow per unit length is then

$$\frac{q}{L} = h\pi d(T_w - T_b) = (64.85)\pi(0.0254)(20) = 103.5\ \text{W/m} \qquad [107.7\ \text{Btu/ft}]$$

We can now make an energy balance to calculate the increase in bulk temperature in a 3.0-m length of tube:

$$q = \dot{m}c_p\Delta T_b = L\left(\frac{q}{L}\right)$$

We also have

$$\dot{m} = \rho u_m \frac{\pi d^2}{4} = (1.493)(10)\pi\frac{(0.0254)^2}{4}$$

$$= 7.565\times10^{-3}\ \text{kg/s} \qquad [0.0167\ \text{lb}_m/\text{s}]$$

so that we insert the numerical values in the energy balance to obtain

$$(7.565\times10^{-3})(1025)\Delta T_b = (3.0)(103.5)$$

and

$$\Delta T_b = 40.04^\circ\text{C} \qquad [104.07^\circ\text{F}]$$

Heat Transfer in a Rough Tube — EXAMPLE 6-5

A 2.0-cm-diameter tube having a relative roughness of 0.001 is maintained at a constant wall temperature of 90°C. Water enters the tube at 40°C and leaves at 60°C. If the entering velocity is 3 m/s, calculate the length of tube necessary to accomplish the heating.

■ Solution

We first calculate the heat transfer from

$$q = \dot{m}c_p\Delta T_b = (989)(3.0)\pi(0.01)^2(4174)(60-40) = 77{,}812 \text{ W}$$

For the rough-tube condition, we may employ the Petukhov relation, Equation (6-7). The mean film temperature is

$$T_f = \frac{90+50}{2} = 70°\text{C}$$

and the fluid properties are

$$\rho = 978 \text{ kg/m}^3 \qquad \mu = 4.0\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$k = 0.664 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 2.54$$

Also,

$$\mu_b = 5.55\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$\mu_w = 2.81\times10^{-4} \text{ kg/m}\cdot\text{s}$$

The Reynolds number is thus

$$\text{Re}_d = \frac{(978)(3)(0.02)}{4\times10^{-4}} = 146{,}700$$

Consulting Figure 6-4, we find the friction factor as

$$f = 0.0218 \qquad f/8 = 0.002725$$

Because $T_w > T_b$, we take $n = 0.11$ and obtain

$$\text{Nu}_d = \frac{(0.002725)(146{,}700)(2.54)}{1.07+(12.7)(0.002725)^{1/2}(2.54^{2/3}-1)}\left(\frac{5.55}{2.81}\right)^{0.11}$$
$$= 666.8$$

$$h = \frac{(666.8)(0.664)}{0.02} = 22138 \text{ W/m}^2\cdot°\text{C}$$

The tube length is then obtained from the energy balance

$$q = \bar{h}\pi dL(T_w - \overline{T_b}) = 77{,}812 \text{ W}$$
$$L = 1.40 \text{ m}$$

Example: Estimate the heat transfer rate from internal water flow at a mean bulk temperature at 20°C to a 0.9m long by 0.15m diameter pipe at 0°C. The velocity is 0.16m/s.

Solution:

At 20°C for water, $\upsilon=1.006\times10^{-6}$ m^2/s, Pr=7002, k=0.597 W/m.K,

$$Re=\frac{u.d}{\upsilon}=23900 \quad \therefore turbulent$$

اذا لم يذكر نوعية السطح نعتبره smooth وكذلك نعتبره fully developed

$$\therefore Nu_d=\frac{h.d}{k}=0.023\,Re_d^{\,0.8}\,\Pr^{0.3}\Rightarrow h=0.023\,Re_d^{\,0.8}\,\Pr^{0.3}.k/d=523\,W/m^2.K$$

$$q=hAs(Tw-Tbm)=h\pi DL(Tw-Tbm)=523\times\pi\times0.15\times0.9(0-20)$$

$$=-5768W \quad (cooling)$$

Also see examples 6.1, 6.2, 6.3, 6.4 and 6.6