Moment of a Force:

Example (1): Find the moment of the force 200 N about the point (A) shown in figure.

Solution:

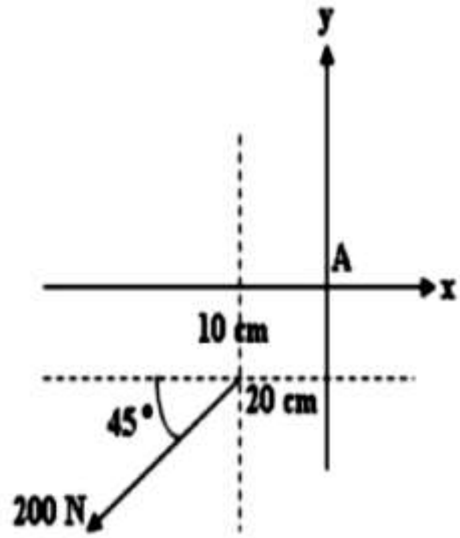
$F_x = F.\cos\theta = 200 \cos 45 = 200 * 0.707 = -141.42$ N

$F_Y = F.\sin\theta = 200 \sin 45 = 200 * 0.707 = -141.42$ N

$M1 = F_x * d = 141.42 * 10 = 1414.2$ N.cm

$M2 = F_Y * d = 141.2 * 20 = -2828.4$ N.cm

$M(A) = M_1 - M_2 = 1414.2 - 2828.4 = -1414.2$ N.cm



Example (2):

Determine the moment of the force (70 N) shown in fig. about the Point (A).

Solution

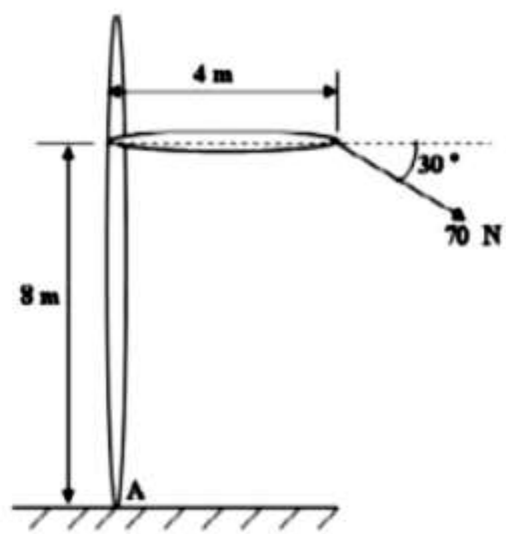
$Fx = F.\cos\theta = 70 \cos 30$
$= 70 * 0.866 = 60.62$ N

$Fy = F.\sin\theta = 70 \sin 30$
$= 70 * 0.5 = 35$ N

$M1 = Fx * d = 60.62 * 8 = 484.97$ N . m

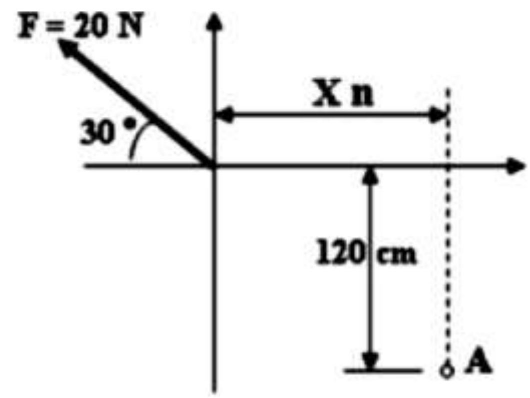
$M2 = Fy * d = 35 * 4 = 140$ N . m

$M(A) = M1 + M2 = 484.97 N + 140 = 624.97$ N . m



Example (3)

Find the distance (Xn) , if the moment of the force (F) about the point (A) is equal to **zero** .



Solution

Fx = F . cos θ = 20 cos 30

= 20 * 0.866 = 17.32 N

Fy = F . sin θ = 20 sin 30

= 20 * 0.5 = 10 N

M1 = Fx * d = 17.32 * 120 = - 2078.46 N . cm

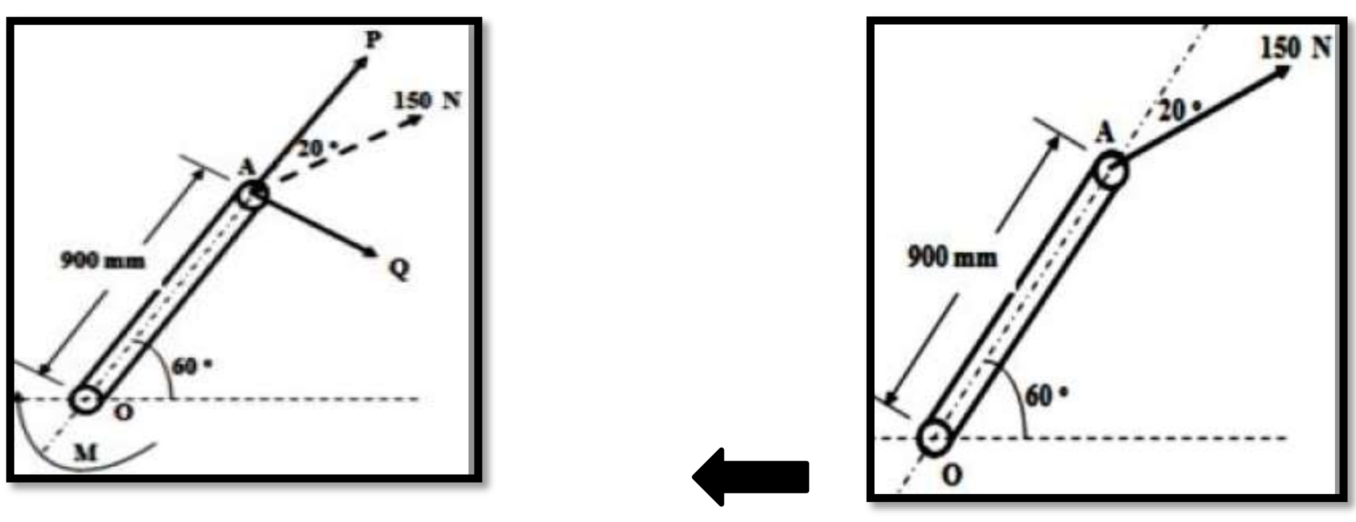
M2 = Fy * d = 10 * Xn = 10 Xn N .cm

M (A) = - M1+ M2

0 = - 2078.46 + 10 Xn

Xn = 2078.46 / 10 = 207.84 cm

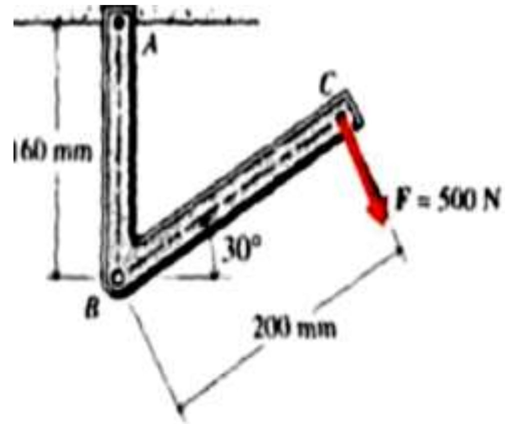
Example (4): A (150 N) force acts on the end of the (900 mm) lever as shown in figure below.Determine the moment of the force about (O).



Sol.: Q=150 sin 20= 51.3 , 900 mm=0.9 m

M= Q* 0.9 = 41.17 N.m

Example (5):
Determine the moment of the force 500 N about the point A and B.



$\text{Cos}\ (60) = \frac{200}{L}$

$\text{Cos}\ (60) = \frac{dac}{(L-160)}$

$L = \frac{200}{\text{Cos}(60)}$

$L = 160 + \frac{dac}{\text{COS}(60)}$

$d_{ac} = 200 - 160\ \text{Cos}\ (60) = 120\ mm$

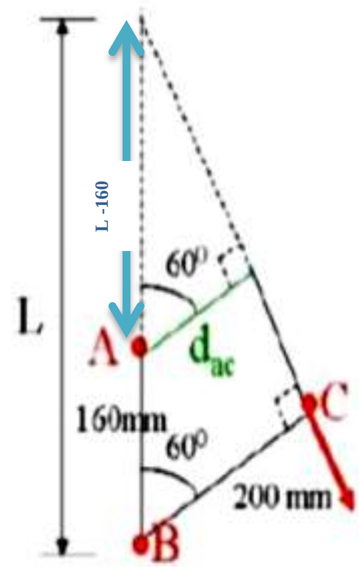
$d_{ac} = 120\ mm = 0.12\ m$

M at point(A) = MA = (F)* d_{AC}

= 500 *0.12 = 60 N.m

M at point (B)= MB = (F)*d_{CB}

= 500 * 0.2=100 N.m



Equilibrium for a Rigid Body:

Conditions for Rigid-Body Equilibrium :

The body is said to be in equilibrium when **resultant force** and **couple moment** are both **equal** to **zero**. Mathematically, the equilibrium of a body is expressed as:

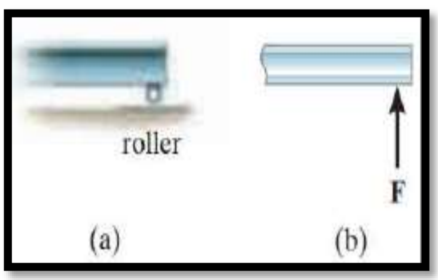
$\sum Fx = 0$

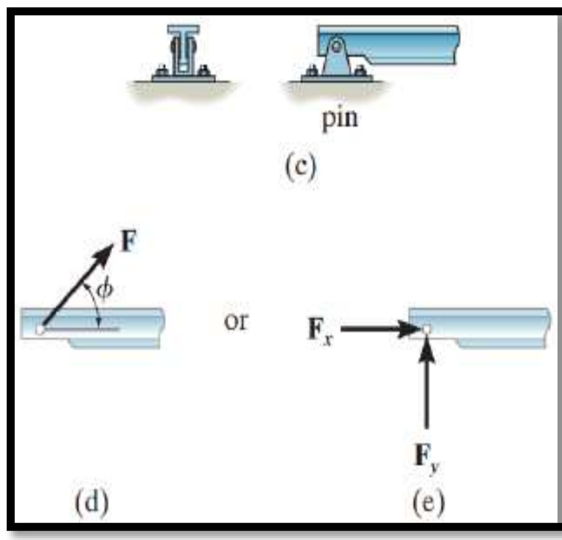
$\sum Fy = 0$

$\sum M = 0$

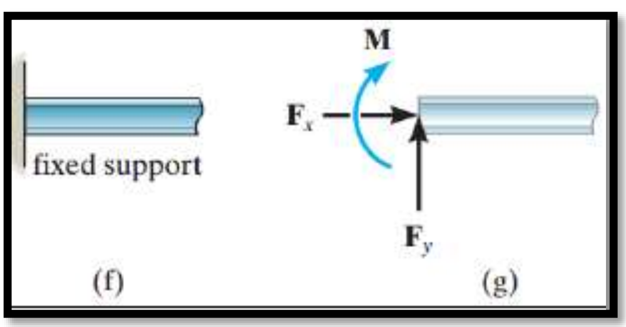
Main Support Reactions:

Roller :

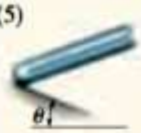
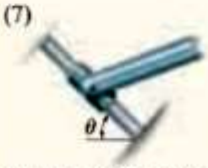
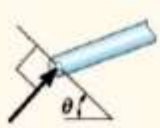
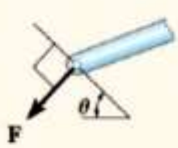




Pin(hinge)



(Fixed)

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Free-Body Diagrams

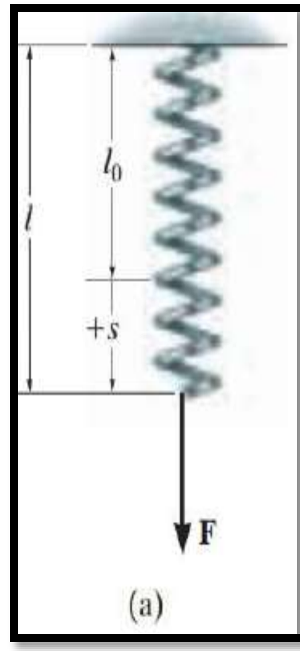
A free-body diagram is a sketch of the outlined shape of the body, which represents it as being isolated or "free" from its surroundings, i.e., a "free body." On this sketch it is necessary to show all the forces and moments that the surroundings exert on the body so that these effects can be accounted for when the equations of equilibrium are applied.

To construct a free-body diagram for a rigid body or any group of bodies considered as a :single system, **the following steps** should be performed

- Draw Outlined Shape .
- Show All Forces and Couple Moments .
- Identify Each Loading and Give Dimensions .

Springs

If a linearly elastic spring (or cord) of undeformed length l is used to support a particle, the length of the spring will change in direct proportion to the force F acting on it, Fig. a .



A characteristic that defines the "elasticity" of a spring is the spring constant or stiffness k. The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed (a distance $s = l - l_0$, measured from its unloaded position, is :

$$F = ks$$

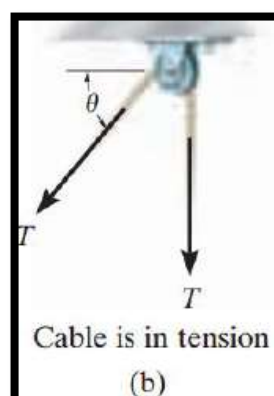
For example, if the spring in Fig. a has an unstretched length of 0.8 m and a stiffness k = 500 N/m and it is stretched to a length of 1 m, so that

$s = l - l_0 = 1\text{ m} - 0.8\text{ m} = 0.2\text{ m}$, then a force :

$F = ks = 500\text{ N/m}\ (0.2\text{ m}) = 100\text{ N}$.

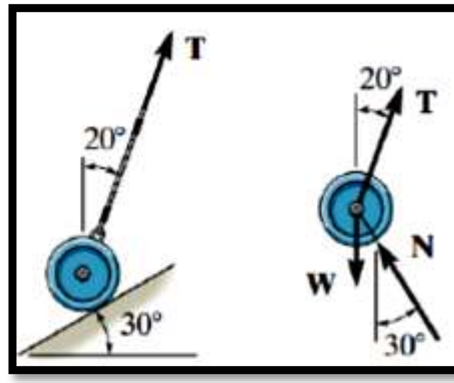
Cables and Pulleys:

A cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable. Hence, for any angle θ, shown in Fig. b, the cable is subjected to a constant tension T throughout its length.

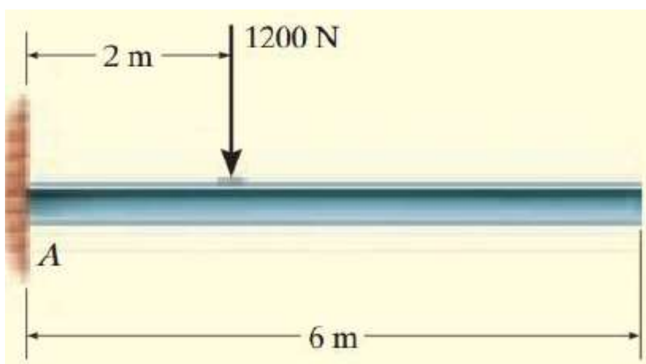


Smooth Contact:

If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact.

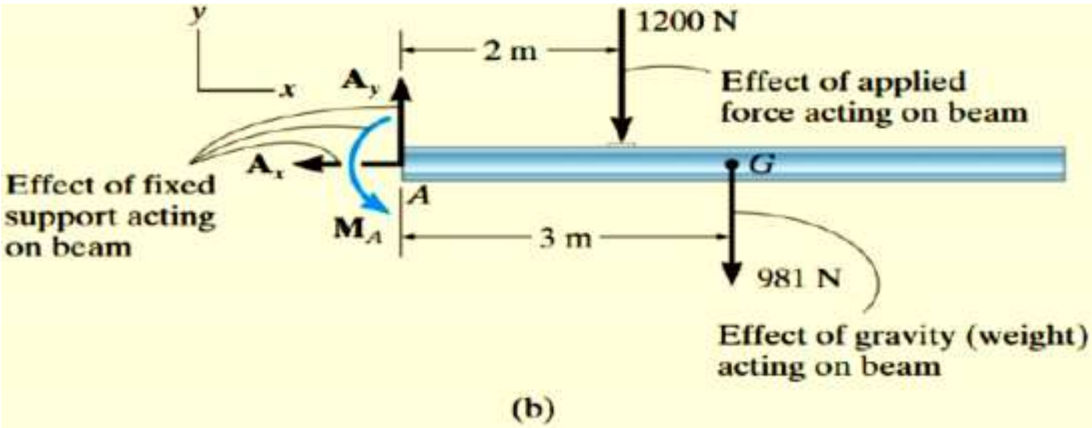


Example: Draw the free-body diagram of the uniform beam shown in Fig. below. The beam has a mass of 100 kg.



Solution:

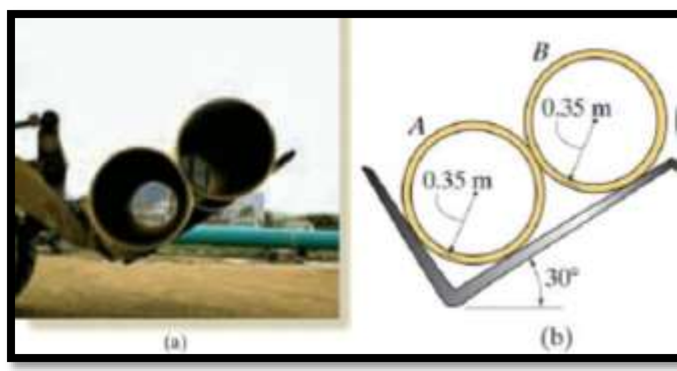
Since the support at A is fixed, the wall exerts three reactions on the beam, denoted as Ax, Ay, and MA. The magnitudes of these reactions are unknown, and their sense has been assumed. The weight of the beam, W = 100(9.81) N = 981 N, acts through the beam's center of gravity G, which is 3 m from A since the beam is uniform.

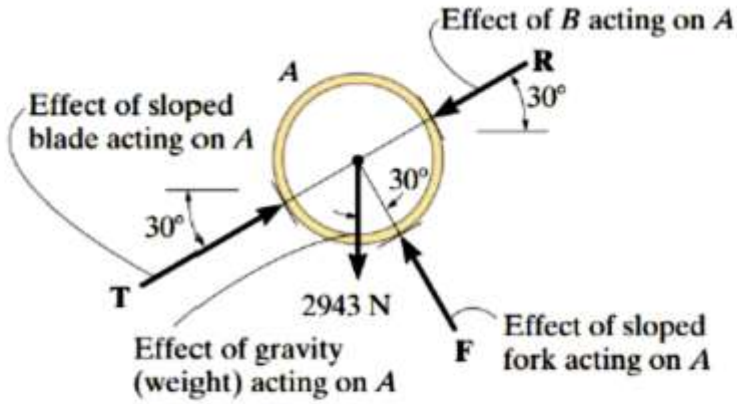


(b)

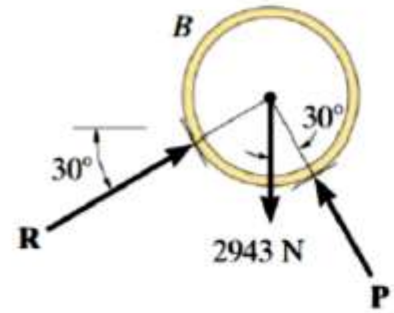
Example: Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. a and b below. Draw the free-body diagrams for each pipe and both pipes together

Solution: W = 300(9.81) N = 2943 N

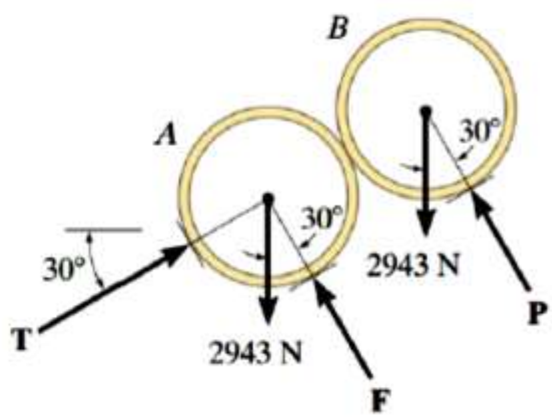



Effect of B acting on A
A
R
30°
Effect of sloped
blade acting on A
30°
30°
T
2943 N
Effect of gravity
(weight) acting on A
F
Effect of sloped
fork acting on A

(c)

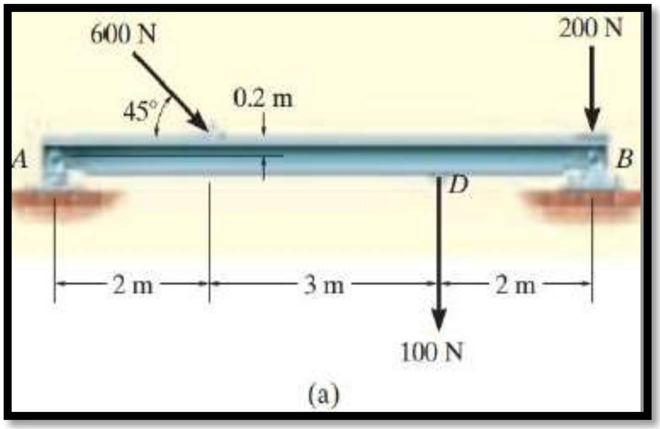

B
30°
30°
R
2943 N
P

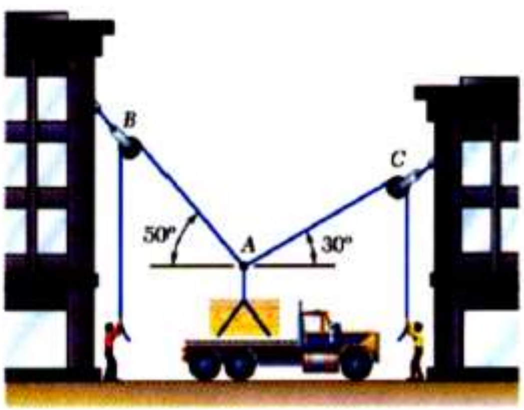
(d)


B
A
30°
30°
30°
2943 N
T
2943 N
P
F

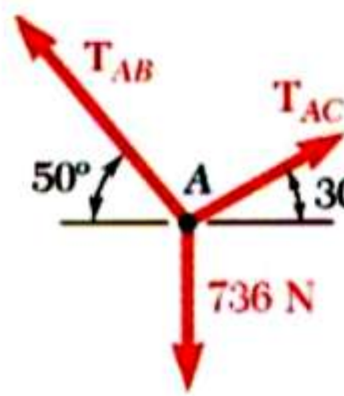
(e)

Home Work: Find the vertical and the horizontal components of reaction on the beam caused by the pin at point (**B)** and the roller at point **(A)** as shown in Figure (a). below. Neglect the weight of the beam.



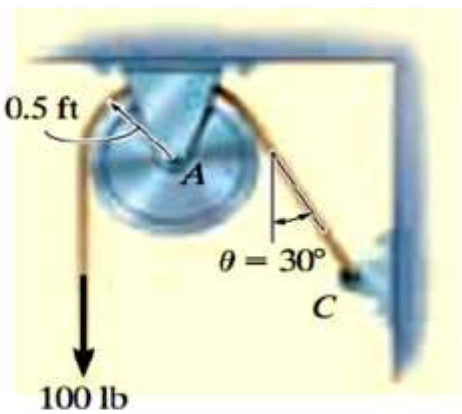


Space Diagram: A sketch showing the physical conditions of the problem.

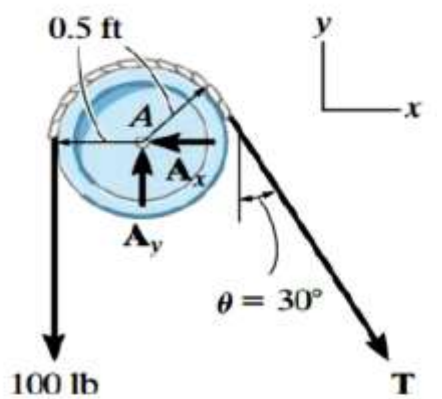


Free-Body Diagram: A sketch showing only the forces on the selected particle.

Example: The cord shown in Figure **a**. below supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin **A**.



(a)



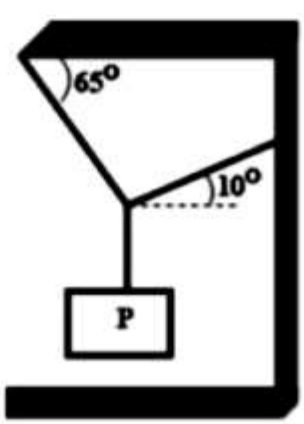
Solution:

$\Sigma M_A = 0$; $-100\,(0.5) + T\,(0.5) = 0$ ⟶ $T = 100$ Ib.

$\Sigma F_x = 0$; $-A_x + 100 \sin 30 = 0$ ⟶ $A_x = 50$ Ib.

$\Sigma F_y = 0$; $A_y - 100 - 100 \cos 30 = 0$ ⟶ $A_y = 187$ Ib.

Draw Free – body diagram for the ropes system shown in fig.



Solution