Chapter Two

Material Balances

2.1 Introduction to Material Balances

A **material balance** is nothing more than the application of the law of the **conservation of mass**: **"Matter is neither created nor destroyed"**

Open and Closed Systems

a. System

By **system** we mean any arbitrary portion of or a whole **process** that you want to consider for analysis. You can define a **system** such as a **reactor**, a **section of a pipe**. Or, you can define the **limits** of the **system** by drawing the **system boundary**, namely a line that encloses the portion of the process that you want to analyze.

b. Closed System

Figure 1 shows a two-dimensional view of a three-dimensional vessel holding **1000 kg of H_2O**. **Note that material neither enters nor leaves the vessel**, that is, **no material crosses the system boundary**. Changes can take place **inside the system**, but for a **closed system**, **no mass exchange occurs with the surroundings**.

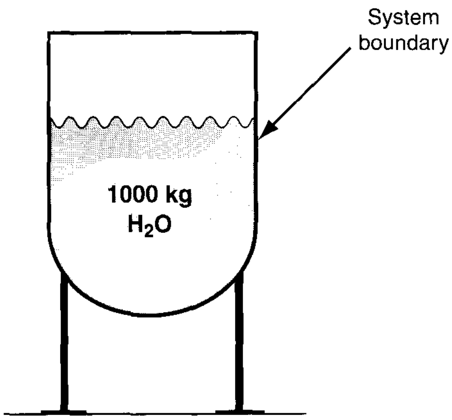

Figure 1 Closed system.

c. Open System

Figure 2 is an example of an **open system** (also called a **flow system**) because material crosses the system boundary.

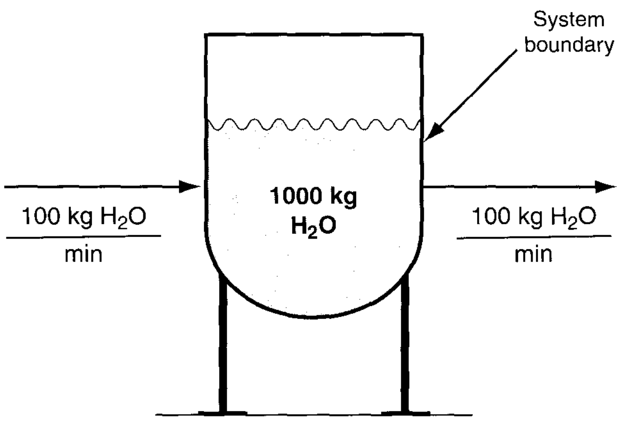

Figure 2 Open steady – state system.

Steady-State and Unsteady-State Systems

a. Steady – State System

Because the rate of addition of water is equal to the rate of removal, the amount of water in the vessel shown in **Figure 2** remains constant at its original value (**1000 kg**). We call such a **process** or **system** a **steady – state process** or a **steady – state system** because

1. The **conditions** inside the process (specifically the amount of water in the vessel in Figure 2) **remain unchanged with time**, and
2. The **conditions** of the flowing streams **remain constant with time**.

✶ Thus, in a **steady-state process**, by definition all of the conditions in the process **(e.g., temperature, pressure, mass of material, flow rate, etc.)** remain constant with time. A **continuous process** is one in which material enters and/or leaves the system without interruption.

b. Unsteady – State System

Because the amount of water in the system **changes with time (Figure 3)**, the **process** and **system** are deemed to be an **unsteady – state** (**transient**) **process**.

✶ For an **unsteady-state process**, not all of the **conditions** in the **process (e.g., temperature, pressure, mass of material, etc.)** remain constant with time, and/or the **flows** in and out of the **system** can **vary with time**.

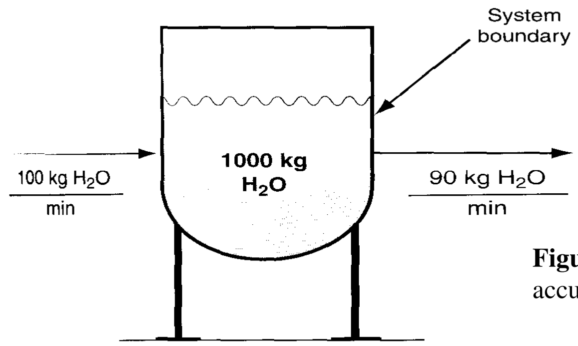

Figure 3 Initial conditions for an open unsteady – state system with accumulation.

✶ Figure 4 shows the system after 50 minutes of accumulation (Fifty minutes of accumulation at 10 kg/min amounts to 500 kg of total accumulation).

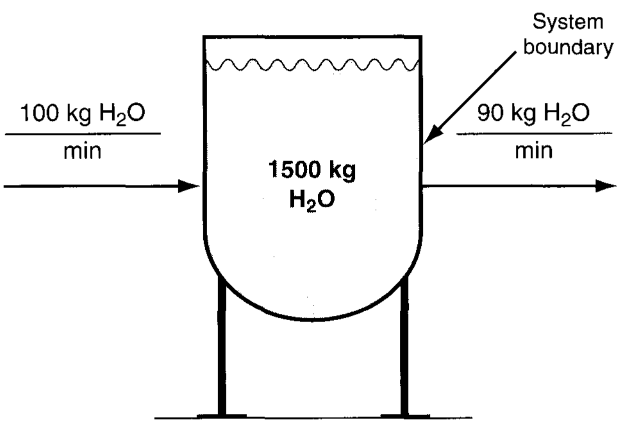

Figure 4 The condition of the open unsteady – state system with accumulation after 50 minutes.

✶ **Figures 5** and **6** demonstrate **negative accumulation**.

Note that the amount of water in the system decreases with time at the rate of **10 kg/min**. Figure 6.6 shows the system after **50 minutes** of operation.

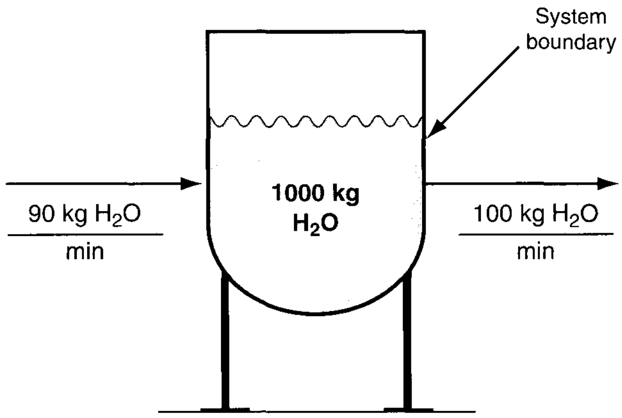

Figure 5 Initial conditions for an unsteady – state process with negative accumulation.

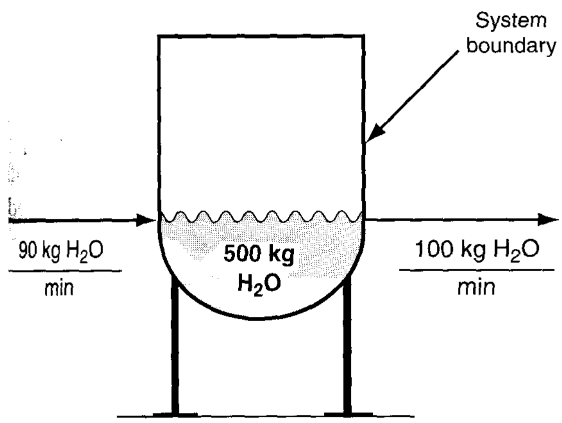

Figure 6 Condition of the open unsteady – state system with negative accumulation after 50 minutes.

✶ **The material balance for a single component process is**

$$\left\{\begin{matrix}\textbf{Accumulation of material}\\ \textbf{within the system}\end{matrix}\right\} = \left\{\begin{matrix}\textbf{Total flow into}\\ \textbf{the system}\end{matrix}\right\} - \left\{\begin{matrix}\textbf{Total flow out}\\ \textbf{of the system}\end{matrix}\right\} \quad ...6.1$$

Equation 6.1 can apply to **moles** or any **quantity** that is **conserved**. As an example, look at **Figure 6.7** in which we have converted all of the mass quantities in **Figure 2** to their equivalent values in moles.

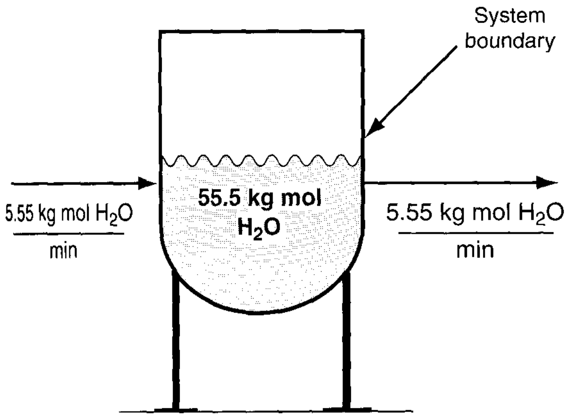

Figure 7 The system in Figure 2 with the flow rates shown in kg mol.

If the process is in the **steady state**, the **accumulation** term by definition is **zero**, and **Equation 6.1** simplifies to a famous truism

What goes in must come out (In = Out) ...6.2

If you are analyzing an unsteady-state process, the accumulation term over a time interval can be calculated as

$$\{\textbf{Accumulation}\} = \left\{\begin{matrix}\textbf{Final material}\\ \textbf{in the system}\end{matrix}\right\} - \left\{\begin{matrix}\textbf{Initial material}\\ \textbf{in the system}\end{matrix}\right\} \quad (6.3)$$

The **times** you select for the final and initial conditions can be anything, but you usually select an **interval** such as **1 minute** or **1 hour** rather than specific times.

✶ When you combine **Equations 6.1 and 6.3** you get the **general material balance** for a component in the system in the **absence of reaction**

$$\left\{\begin{matrix}\textbf{Final material}\\ \textbf{in the system}\\ \textbf{at } t_2\end{matrix}\right\} - \left\{\begin{matrix}\textbf{Initial material}\\ \textbf{in the system}\\ \textbf{at } t_1\end{matrix}\right\} = \left\{\begin{matrix}\textbf{Flow into}\\ \textbf{the system}\\ \textbf{from } t_1 \textbf{ to } t_2\end{matrix}\right\} - \left\{\begin{matrix}\textbf{Flow out of}\\ \textbf{the system}\\ \textbf{from } t_1 \textbf{ to } t_2\end{matrix}\right\} \quad ...6.4$$

Example 1

Will you save money if instead of buying premium 89 octane gasoline at $1.269 per gallon that has the octane you want, you blend sufficient 93 octane supreme gasoline at $l.349 per gallon with 87 octane regular gasoline at $1.149 per gallon?

Solution

Choose a **basis** of **1 gallon of 89 octane gasoline**, the desired product. The system is the gasoline tank.

- For simplicity, assume that **no gasoline exists** in the tank at the start of the blending, and **one gallon exists** in the tank at the end of the blending.
- This arrangement corresponds to an **unsteady-state process**. Clearly it is an **open system**.

The **initial number of gallons** in the system is **zero** and the **final number of gallons** is **one**.

Let **x** = the number of gallons of **87** octane gasoline added, and **y** = the number of gallons of **93** octane added to the blend. Since **x + y = 1** is the total flow into the tank,

$\therefore$ y = 1 – x

According to Equation (6.4) the balance on the octane number is

$$\underset{\textit{Accumulation}}{\left|\frac{89\text{ octane}}{1\text{ gal}}\right|\frac{1\text{ gal}}{}} - 0 = \underset{\textit{Inputs}}{\left|\frac{87\text{ octane}}{1\text{ gal}}\right|\frac{x\text{ gal}}{} + \left|\frac{93\text{ octane}}{1\text{ gal}}\right|\frac{(1-x)\text{ gal}}{}}$$

The solution is x = 2/3 gal and thus y = 1 – x = 1/3 gal.

The cost of the blended gasoline is (2/3) ($1.149) + (1/3) ($1.349) = $ 1.216

A value less than the cost of the 89 octane gasoline ($1.269).

Multiple Component Systems

Suppose the input to a vessel contains **more than one component**, such as 100 kg/min of a 50% water and 50% sugar (sucrose, $C_{12}H_{22}O_{11}$, MW = 342.3) mixture (see Figure 8). The mass balances with respect to the **sugar and water**, balances that we call **component balances**.

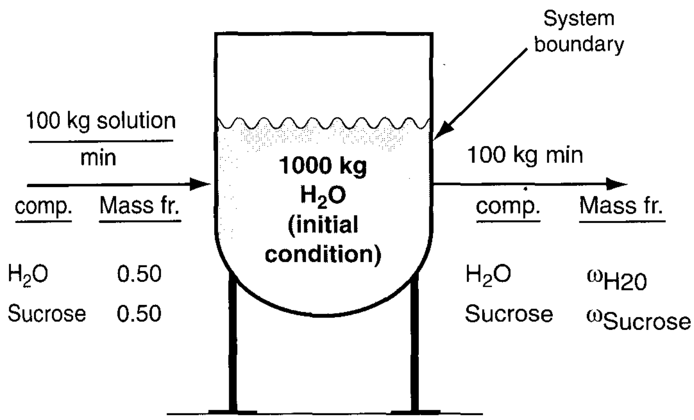

Figure 8 An open system involving two components.

<u>For Example</u>, look at the mixer shown in Figure 9, an apparatus that mixes two streams to increase the concentration of NaOH in a dilute solution. **The mixer is a steady – state open system**. Initially the mixer is empty, and after 1 hour it is empty again.

<u>Basis = 1 hour</u> for convenience. As an alternate to the **basis** we selected you could select **<u>F_1 = 9000 kg/hr as the basis, or F_2 = 1000 kg/hr as the basis;</u>** the **numbers** for this example would not change – just the **units** would change. Here are the components and total balances in **kg**:

Balances	Flow in F_1	Flow in F_2	Flow out	Accum.
NaOH	450	500	950	= 0
H_2O	8,550	500	9,050	= 0
Total	9,000	1,000	10,000	= 0

We can convert the kg shown in Figure 6.9 to kg moles by dividing each compound by its respective molecular weight (NaOH = 40 and H_2O = 18).

$$\text{NaOH:} \quad \frac{450}{40} = 11.25 \qquad \frac{500}{40} = 12.50 \qquad \frac{950}{40} = 23.75$$

$$H_2O: \quad \frac{8550}{18} = 475 \qquad \frac{500}{18} = 27.78 \qquad \frac{9050}{18} = 502.78$$

Then the component and total balances in **kg mol** are:

	Flow in			
Balances	F_1	F_2	**Flow out**	**Accum.**
NaOH	11.25	12.50	23.75	= 0
H_2O	475	27.78	502.78	= 0
Total	486.25	40.28	536.53	= 0

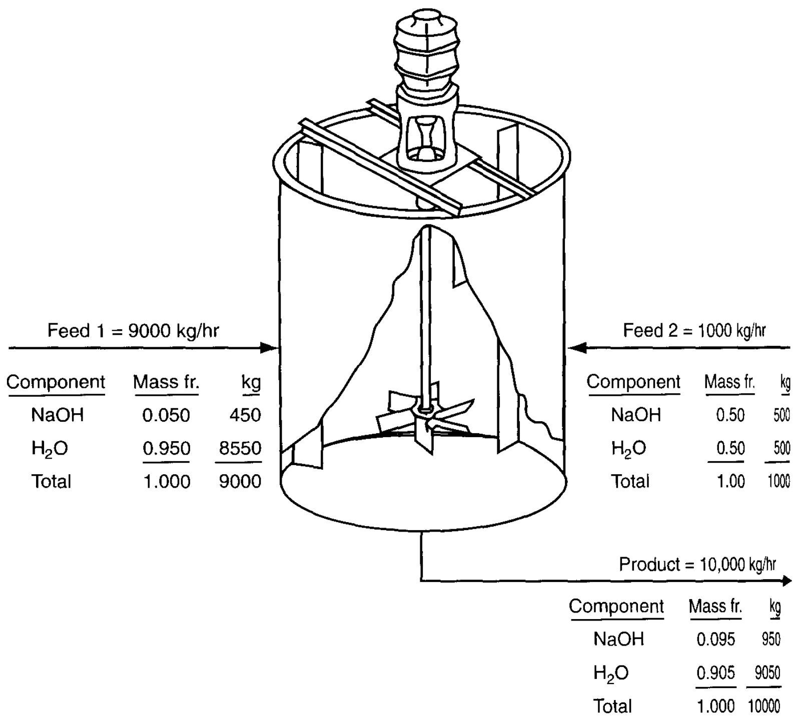

Figure 9 Mixing of a dilute stream of NaOH with a concentrated stream of NaOH. Values below the stream arrows are based on 1 hour of operation.

Example 2

Centrifuges are used to separate particles in the range of 0.1 to 100 μm in diameter from a liquid using centrifugal force. Yeast cells are recovered from a broth (a liquid mixture containing cells) using a tubular centrifuge (a cylindrical system rotating about a cylindrical axis). Determine the amount of the cell-free discharge per hour if 1000 L/hr is fed to the centrifuge, the feed contains 500 mg cells/L, and the product stream contains 50 wt.% cells. Assume that the feed has a density of 1 g/cm^3.

Solution

This problem involves a **steady state**, **open (flow)** system **without reaction**.

Basis = 1 hour

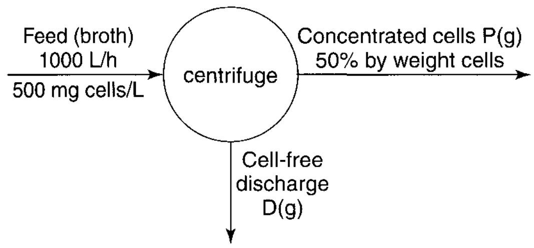

Figure E6.2

M.B. on cells

In (mass) = Out (mass)

$$\frac{1000\text{ L feed}}{}\left|\frac{500\text{ mg cells}}{1\text{ L feed}}\right|\frac{1\text{ g}}{1000\text{ mg}} = \frac{0.5\text{ g cells}}{1\text{ g }P}\left|\frac{P\text{ g}}{}\right.$$

P = 1000 g

M.B. on fluid

In (mass) = Out (mass)

$$\frac{1000\text{ L}}{}\left|\frac{1000\text{ cm}^3}{1\text{ L}}\right|\frac{1\text{ g fluid}}{1\text{ cm}^3} = \frac{1000\text{ g }P}{}\left|\frac{0.50\text{ g fluid}}{1\text{ g }P}\right. + D\text{ g fluid}$$

D = $(10^6 - 500)$ g

Accounting for Chemical Reactions in Material Balances

Chemical reaction in a system requires the augmentation of **Equation 6.4** to take into account the **effects of the reaction**. To illustrate this point, look at **Figure 10**, which shows a steady – state system in which **HC1** reacts with **NaOH** by the following reaction:

NaOH + HCl $\longrightarrow$ NaC1 + H_2O

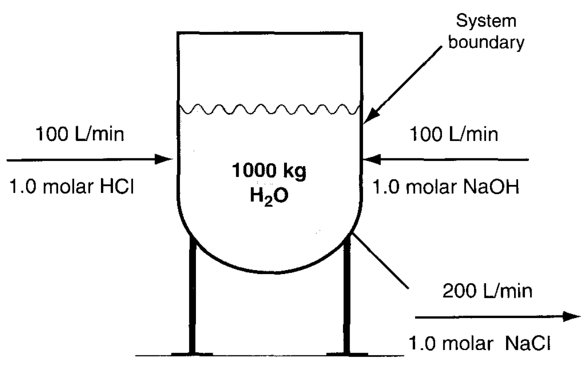

Figure 10 Reactor for neutralizing HC1 with NaOH.

Equation 4 must be augmented to include terms for the **<u>generation</u>** and **<u>consumption</u>** of components by the **chemical reaction** in the system as follows

$$\begin{Bmatrix}\text{Accumulation}\\ \text{within the}\\ \text{system}\end{Bmatrix} = \begin{Bmatrix}\text{Input}\\ \text{through}\\ \text{the system}\\ \text{boundaries}\end{Bmatrix} - \begin{Bmatrix}\text{Output}\\ \text{through}\\ \text{the system}\\ \text{boundaries}\end{Bmatrix} + \begin{Bmatrix}\text{Generation}\\ \text{within the}\\ \text{system}\end{Bmatrix} - \begin{Bmatrix}\text{Consumption}\\ \text{within the}\\ \text{system}\end{Bmatrix} \quad \ldots 6.5$$