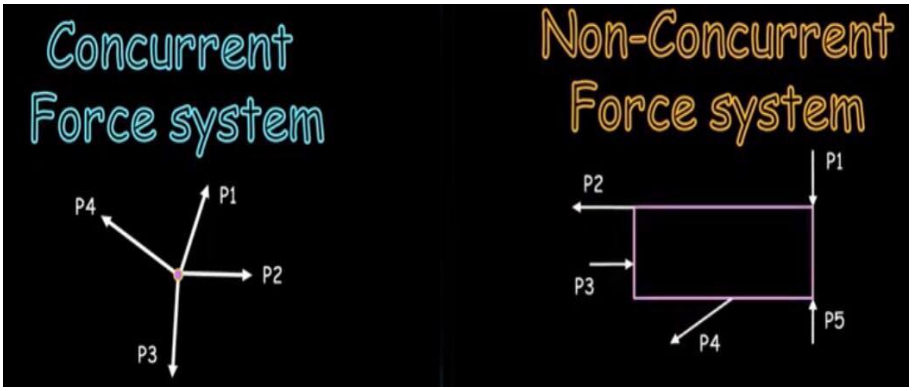
Resultant of Concurrent Force system:

Concurrent: all forces pass through a common point.



يمكن إيجاد محصلة القوى بطريقتين:

1. Parallelogram method:

- Only two concurrent forces.
- The angle between the two forces is known.

2. Resolution of forces method:

- More than two concurrent forces.

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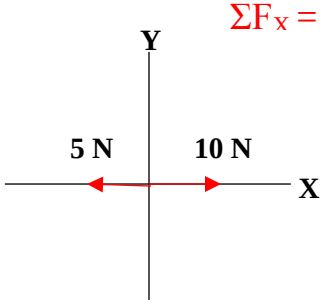
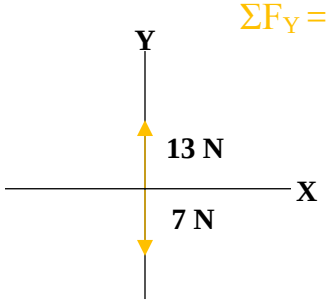
1. Parallelogram method:

$\Sigma F_X = 10 - 5 = 5$ N

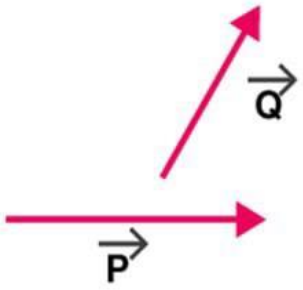


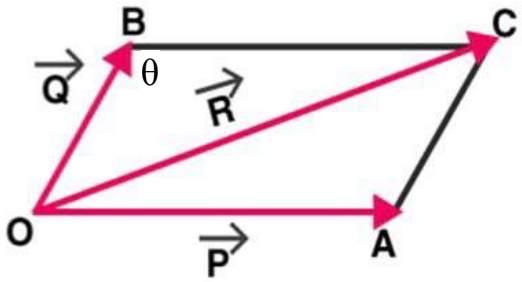
$\Sigma F_Y = 13 - 7 = 6$ N



Parallelogram law states that if two forces acting at a point are represented, in magnitude and direction by two adjacent sides of a Parallelogram drawn from a point; their resultant, in magnitude and direction is given by the diagonal passing through the same point.

ينص هذا القانون على انه لو كان هناك قوتان معلومتا القيمة والاتجاه تؤثران على نقطة واحدة ويمكن تمثيلهما بضلعين متجاورين لمتوازي الاضلاع، فان محصلة هاتين القوتين يمكن تمثيلها بقطر متوازي الاضلاع المار من خلال نفس النقطة.

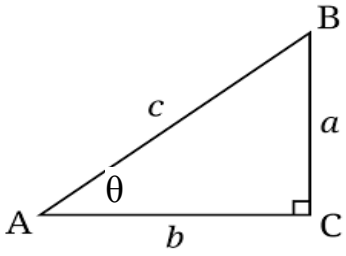




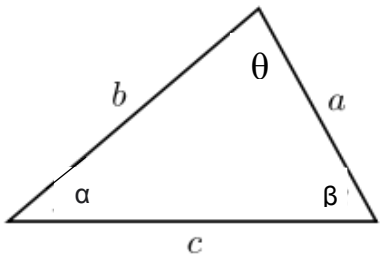
$$R = \sqrt{(P^2 + Q^2 - 2.P.Q.(\cos\theta)}$$

Notes:

1. Cosine law



$$\cos\theta = \frac{b}{c}$$



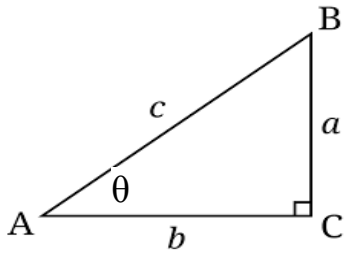
$$a = \sqrt{(b^2 + c^2 - 2.b.c.(\cos\alpha)}$$

$$b = \sqrt{(a^2 + c^2 - 2.a.c.(\cos\beta)}$$

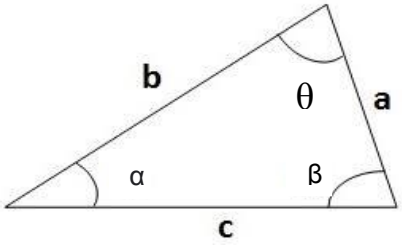
$$c = \sqrt{(a^2 + b^2 - 2.a.b.(\cos\theta)}$$

1. ضلعين والزاوية بينهما معلومة.
2. كل الاضلاع معلومة.

2. Sine law



$$\sin\theta = \frac{a}{c}$$

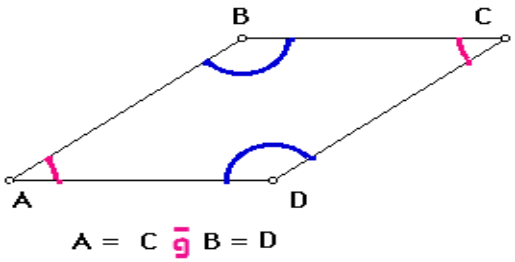


$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\theta}$$

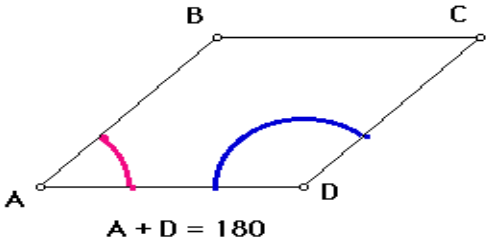
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\theta}{c}$$

1. المعلوم زاويتين وضلع.
2. المعلوم ضلعين وزاوية تقابل أحدهما.

3.

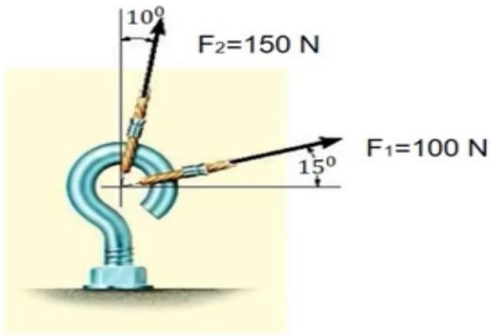


كل زاويتين متقابلتين متساويتين بالقياس



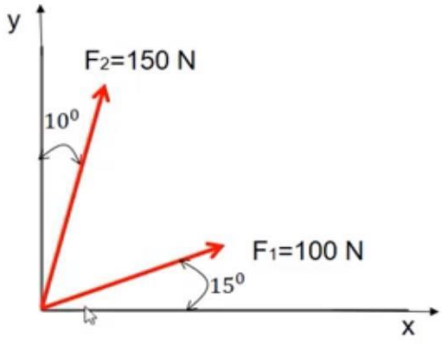
مجموع الزوايا المتتابعة = 180

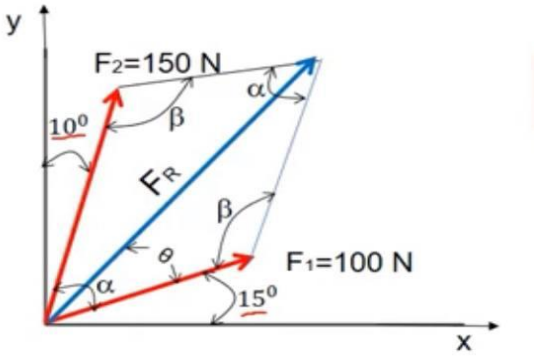
Example (1): the screw eye is subjected to two forces F1 and F2. Determine the magnitude and direction of resultant force?



Sol.:

- Determine the magnitude of F_R:





$F_R = \sqrt{(F_1^2 + F_2^2 - 2.F_1.F_2.(\cos \beta)}$

$\alpha = 90° - (10°+15°) = 65°$

$\beta = \frac{360° - 2*65°}{2} = 115°$

$F_R = \sqrt{(100^2 + 150^2 - 2.100.150.(\cos 115°))}$

$F_R = 212.6$ N

- Determine the direction of F_R:

Law of sine:

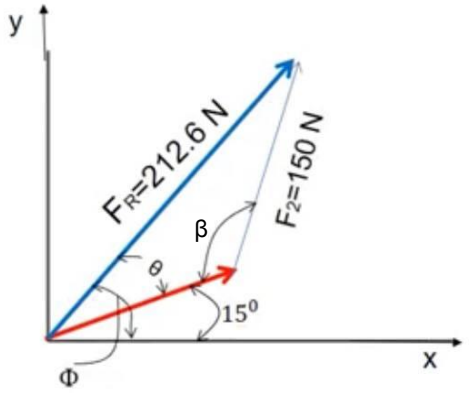
$$\frac{F2}{\sin\theta} = \frac{FR}{\sin\beta}$$

$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115}$$

$$sin\ \theta = \frac{150 * \sin 115}{212.6} = 0.639$$

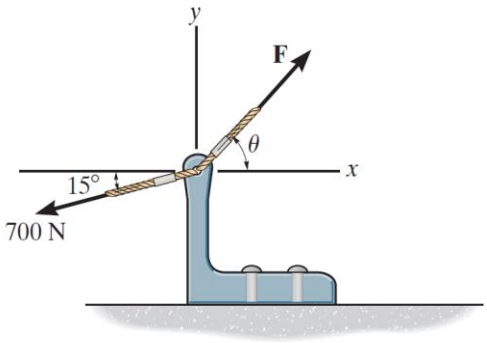
$\theta = \sin^{-1} 0.639 = 39.7°$

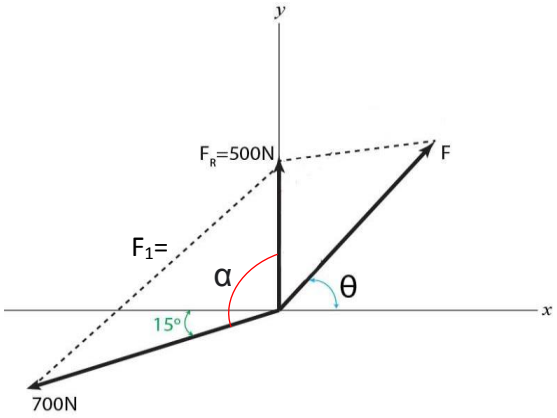
$\Phi = \theta + 15° = 39.7 + 15 = 54.7°$



Example (2): if the magnitude of the resultant force is to be 500 N, directed along the positive y-axis. Determine the magnitude of force F and its direction?

Sol.:





$\alpha = 90° + 15° = 105°$

Law of cosine:

$F_1 = \sqrt{(F_2^2 + F_R^2 - 2.F_2.F_R.(\cos\alpha)}$

$F_1 = \sqrt{(700^2 + 500^2 - 2.700.500.(\cos 105°)}$

$F_1 = 959.77$ N.

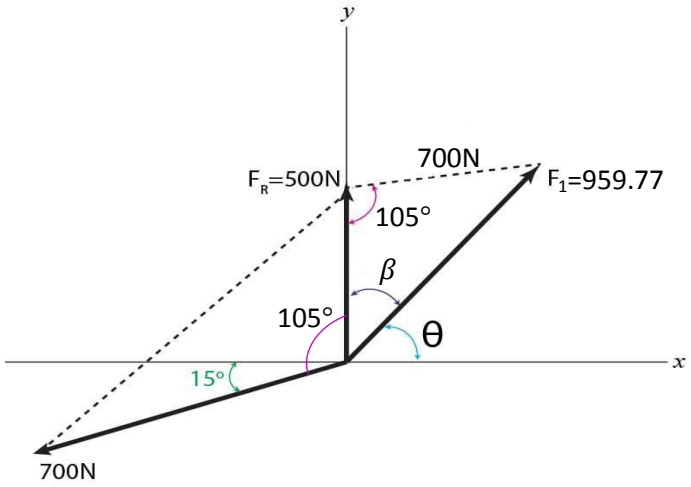
$$\frac{F1}{\sin\theta} = \frac{F2}{\sin\beta}$$

$$\frac{959.77}{\sin 105^\circ} = \frac{700}{\sin\beta}$$

$$\beta = \sin^{-1}\left(\frac{700 * \sin 105^\circ}{959.77}\right)$$

$$\beta = 44.79^\circ$$

$$\theta = 90^\circ - \beta = 45.21^\circ$$

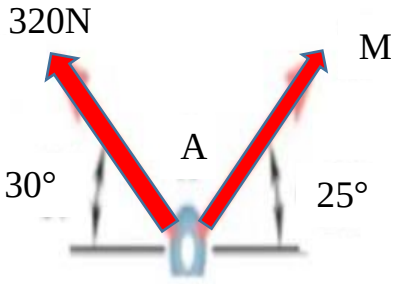


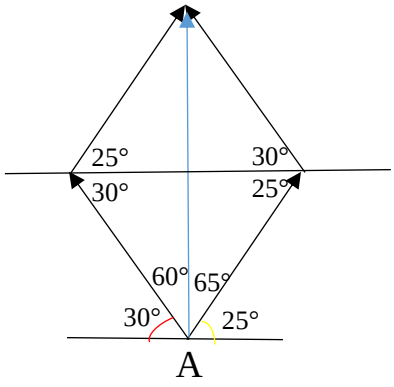
Example (3): two forces of magnitude 320N and M are applied to a bolt at A calculate:

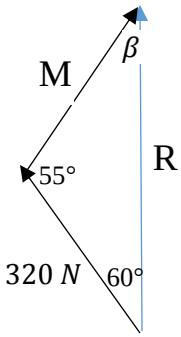
a. the magnitude of M, if resultant R of the two forces applied at A is to be vertical.

b. the corresponding resultant force.

Sol.:







$\beta + 55 + 60 = 180$

$\beta = 180 - 115 = 65$

$$\frac{M}{\sin 60} = \frac{320}{\sin \beta}$$

$$\frac{M}{\sin 60} = \frac{320}{\sin 65}$$

$$M = \frac{320 * \sin 60}{\sin 65} = 305.8N$$

$$\frac{R}{\sin 55} = \frac{320}{\sin 65}$$

$$R = \frac{320 * \sin 55}{\sin 65} = 289.2\ N$$