6.3 RECYCLE REACTOR

In certain situations it is found to be advantageous to divide the product stream from a plug flow reactor and return a portion of it to the entrance of the reactor. Let the *recycle ratio* $\boldsymbol{R}$ be defined as

$$\boldsymbol{R} = \frac{\text{volume of fluid returned to the reactor entrance}}{\text{volume leaving the system}} \tag{15}$$

This recycle ratio can be made to vary from zero to infinity. Reflection suggests that as the recycle ratio is raised the behavior shifts from plug flow ($\boldsymbol{R} = 0$) to mixed flow ($\boldsymbol{R} = \infty$). Thus, recycling provides a means for obtaining various degrees of backmixing with a plug flow reactor. Let us develop the performance equation for the recycle reactor.

Consider a recycle reactor with nomenclature as shown in Fig. 6.13. Across the reactor itself Eq. 5.18 for plug flow gives

$$\frac{V}{F'_{A0}} = \int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A} \tag{16}$$

where F'_{A0} would be the feed rate of A if the stream entering the reactor (fresh feed plus recycle) were unconverted. Since F'_{A0} and X_{A1} are not known directly, they must be written in terms of known quantities before Eq. 16 can be used. Let us now do this.

The flow entering the reactor includes both fresh feed and the recycle stream. Measuring the flow split at point L (point K will not do if $\varepsilon \neq 0$) we then have

$$\begin{aligned} F'_{A0} &= \begin{pmatrix} \text{A which would enter in an} \\ \text{unconverted recycle stream} \end{pmatrix} + \begin{pmatrix} \text{A entering in} \\ \text{fresh feed} \end{pmatrix} \\ &= \boldsymbol{R}F_{A0} + F_{A0} = (\boldsymbol{R} + 1)F_{A0} \end{aligned} \tag{17}$$

Now to the evaluation of X_{A1}: from Eq. 4.5 we may write

$$X_{A1} = \frac{1 - C_{A1}/C_{A0}}{1 + \varepsilon_A C_{A1}/C_{A0}} \tag{18}$$

Because the pressure is taken to be constant, the streams meeting at point K may be added directly. This gives

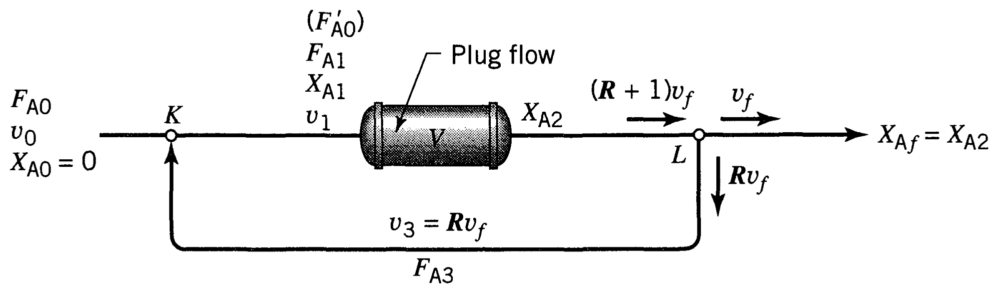

Figure 6.13 Nomenclature for the recycle reactor.

$$C_{A1} = \frac{F_{A1}}{v_1} = \frac{F_{A0} + F_{A3}}{v_0 + \boldsymbol{R}v_f} = \frac{F_{A0} + \boldsymbol{R}F_{A0}(1 - X_{Af})}{v_0 + \boldsymbol{R}v_0(1 + \varepsilon_A X_{Af})} = C_{A0}\left(\frac{1 + \boldsymbol{R} - \boldsymbol{R}X_{Af}}{1 + \boldsymbol{R} + \boldsymbol{R}\varepsilon_A X_{Af}}\right) \quad \textbf{(19)}$$

Combining Eqs. 18 and 19 gives X_{A1} in terms of measured quantities, or

$$X_{A1} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R} + 1}\right) X_{Af} \quad \textbf{(20)}$$

Finally, on replacing Eqs. 17 and 20 in Eq. 16 we obtain the useful form for the performance equation for recycle reactors, good for any kinetics, any ε value and for $X_{A0} = 0$.

$$\boxed{\frac{V}{F_{A0}} = (\boldsymbol{R} + 1) \int_{\left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right) X_{Af}}^{X_{Af}} \frac{dX_A}{-r_A} \quad \ldots \text{any } \varepsilon_A} \quad \textbf{(21)}$$

For the special case where density changes are negligible we may write this equation in terms of concentrations, or

$$\boxed{\tau = \frac{C_{A0}V}{F_{A0}} = -(\boldsymbol{R} + 1) \int_{\frac{C_{A0} + \boldsymbol{R}C_{Af}}{\boldsymbol{R}+1}}^{C_{Af}} \frac{dC_A}{-r_A} \quad \ldots \varepsilon_A = 0} \quad \textbf{(22)}$$

These expressions are represented graphically in Fig. 6.14.

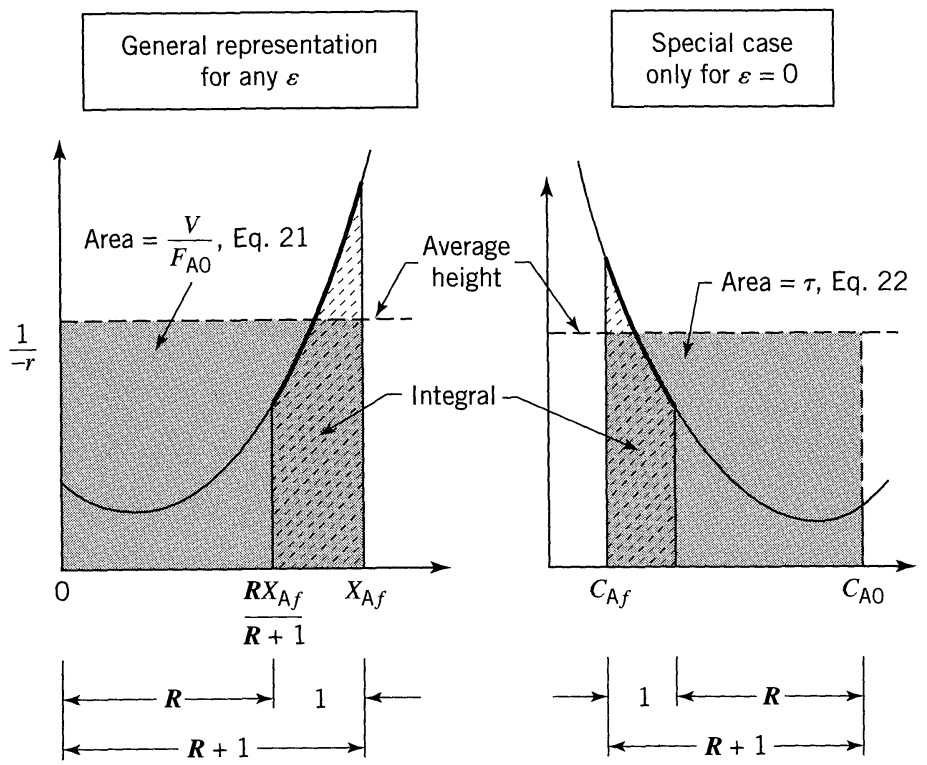

Figure 6.14 Representation of the performance equation for recycle reactors.

For the extremes of negligible and infinite recycle the system approaches plug flow and mixed flow, or

$$\frac{V}{F_{A0}} = (\boldsymbol{R}+1)\int_{\frac{\boldsymbol{R}}{\boldsymbol{R}+1}X_{Af}}^{X_{Af}} \frac{dX_A}{-r_A}$$

$\boldsymbol{R} = 0$ ↓

$$\frac{V}{F_{A0}} = \int_{A}^{X_{Af}} \frac{dX_A}{-r_A}$$

plug flow

$\boldsymbol{R} = \infty$ ↓

$$\frac{V}{F_{A0}} = \frac{X_{Af}}{-r_{Af}}$$

mixed flow

The approach to these extremes is shown in Fig. 6.15.

Integration of the recycle equation gives, for *first-order reaction*, $\varepsilon_A = 0$,

$$\boxed{\frac{k\tau}{\boldsymbol{R}+1} = \ln\left[\frac{C_{A0}+\boldsymbol{R}C_{Af}}{(\boldsymbol{R}+1)C_{Af}}\right]} \qquad \textbf{(23)}$$

and for *second-order reaction,* 2A → products, $-r_A = kC_A^2$, $\varepsilon_A = 0$,

$$\boxed{\frac{kC_{A0}\tau}{\boldsymbol{R}+1} = \frac{C_{A0}(C_{A0}-C_{Af})}{C_{Af}(C_{A0}+\boldsymbol{R}C_{Af})}} \qquad \textbf{(24)}$$

The expressions for $\varepsilon_A \neq 0$ and for other reaction orders can be evaluated, but are more cumbersome.

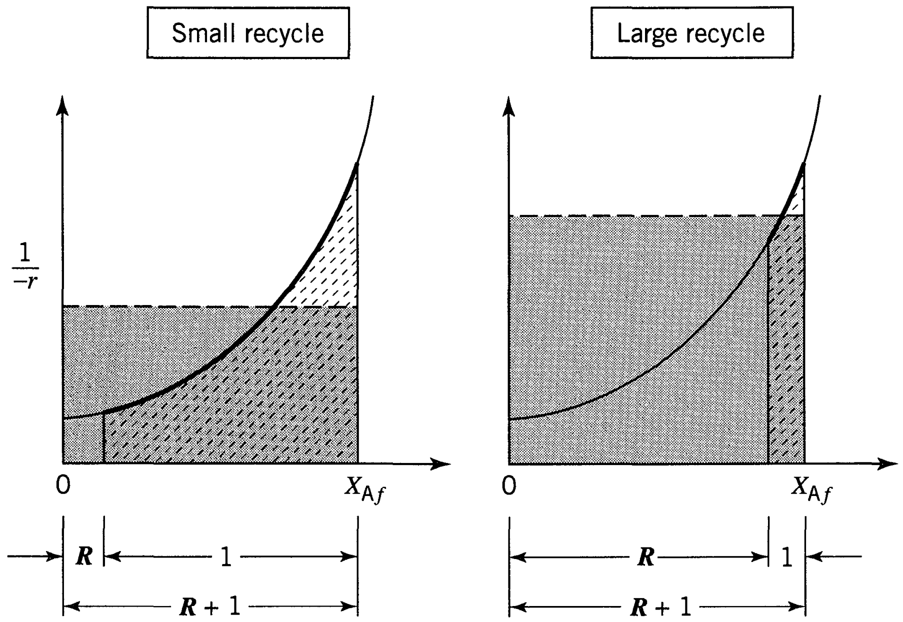

Figure 6.15 The recycle extremes approach plug flow ($\boldsymbol{R} \to 0$) and mixed flow ($\boldsymbol{R} \to \infty$).

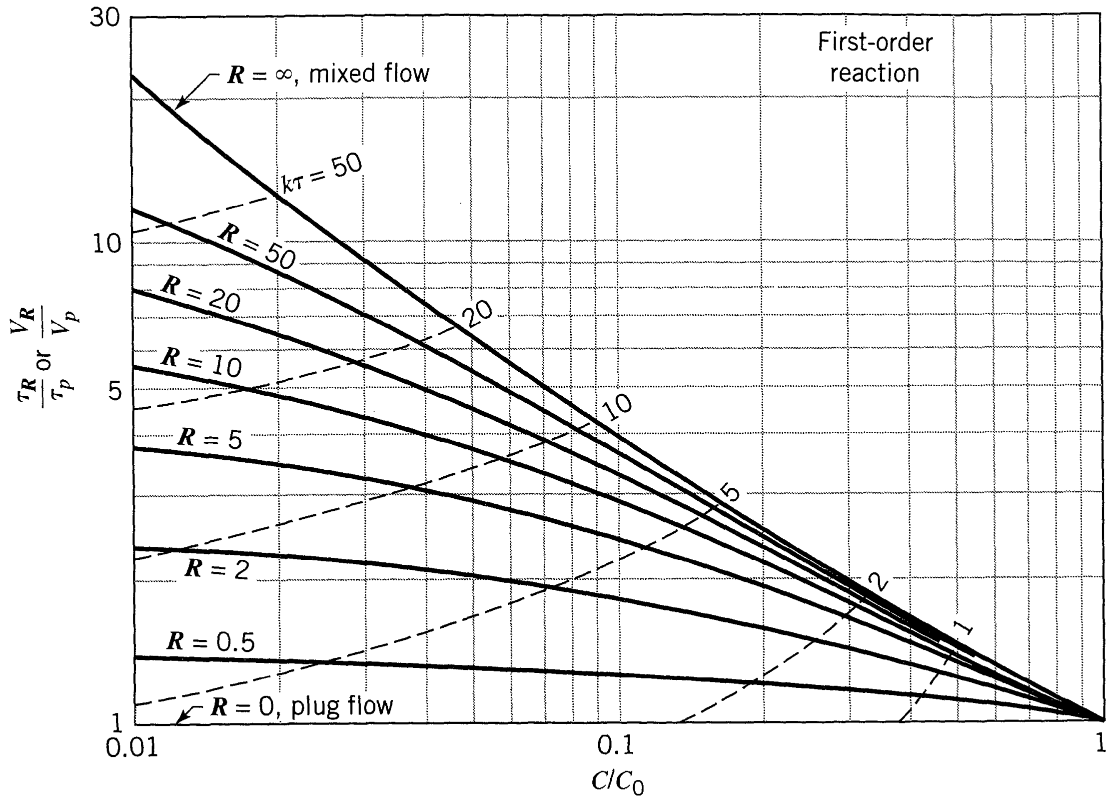

Figure 6.16 Comparison of performance of recycle and plug flow for first-order reactions

$$A \rightarrow R, \qquad \varepsilon = 0.$$

Figures 6.16 and 6.17 show the transition from plug to mixed flow as ***R*** increases, and a match of these curves with those for N tanks in series (Figs. 6.5 and 6.6) gives the following rough comparison for equal performance:

No. of tanks	***R*** for first-order reaction at X_A = 0.5	0.90	0.99	***R*** for second-order reaction at X_A = 0.5	0.90	0.99
1	∞	∞	∞	∞	∞	∞
2	1.0	2.2	5.4	1.0	2.8	7.5
3	0.5	1.1	2.1	0.5	1.4	2.9
4	0.33	0.68	1.3	0.33	0.90	1.7
10	0.11	0.22	0.36	0.11	0.29	0.5
∞	0	0	0	0	0	0

The recycle reactor is a convenient way for approaching mixed flow with what is essentially a plug flow device. Its particular usefulness is with solid catalyzed reactions with their fixed bed contactors. We meet this and other applications of recycle reactors in later chapters.

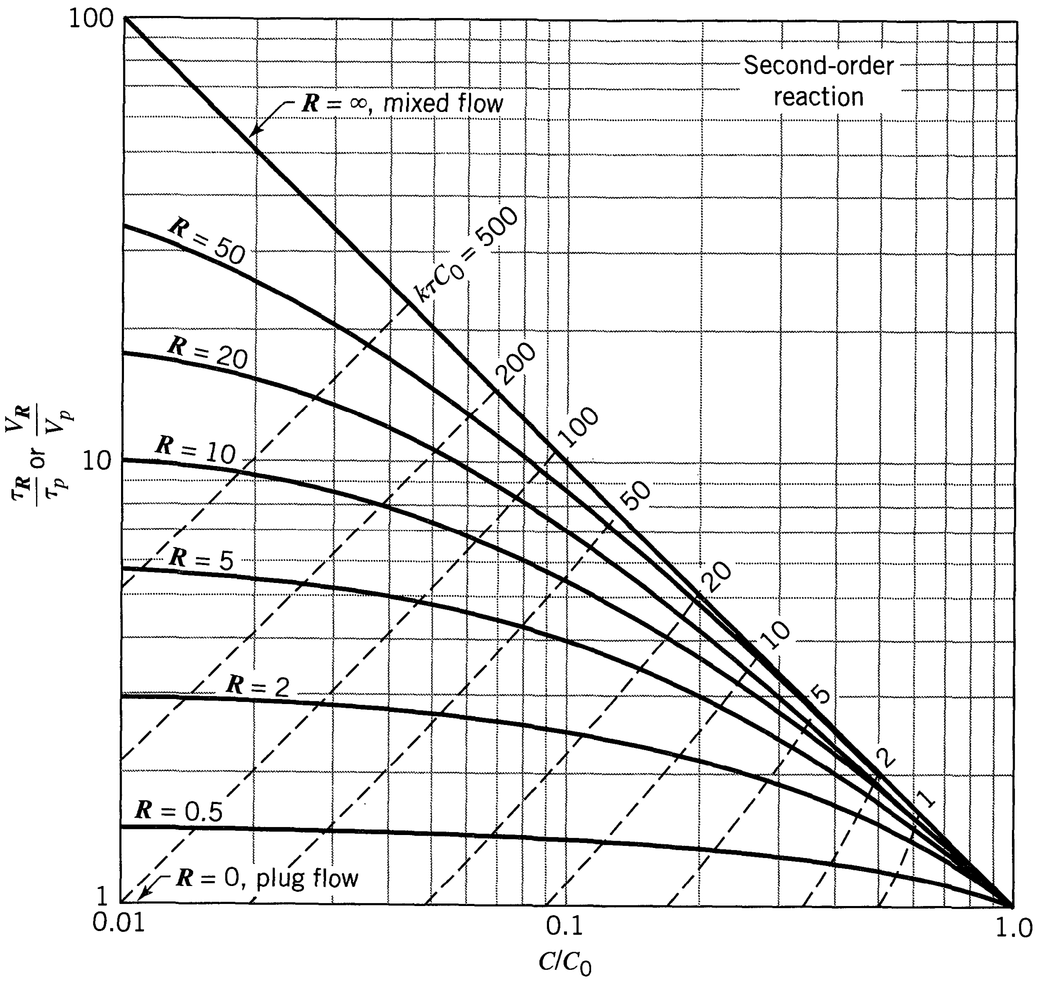

Figure 6.17 Comparison of performance of recycle reactors with plug flow reactors for elementary second-order reactions (Personal communication, from T. J. Fitzgerald and P. Fillesi):

$$2A \rightarrow \text{products}, \qquad \varepsilon = 0$$

$$A + B \rightarrow \text{products}, \qquad C_{A0} = C_{B0} \text{ with } \varepsilon = 0$$

6.4 AUTOCATALYTIC REACTIONS

When a material reacts away by any nth order rate ($n > 0$) in a batch reactor, its rate of disappearance is rapid at the start when the concentration of reactant is high. This rate then slows progressively as reactant is consumed. In an autocatalytic reaction, however, the rate at the start is low because little product is present; it increases to a maximum as product is formed and then drops again to a low value as reactant is consumed. Figure 6.18 shows a typical situation.

Reactions with such rate-concentration curves lead to interesting optimization problems. In addition, they provide a good illustration of the general design method presented in this chapter. For these reasons let us examine these reactions in some detail. In our approach we deal exclusively with their $1/(-r_A)$ versus X_A curves with their characteristic minima, as shown in Fig. 6.18.