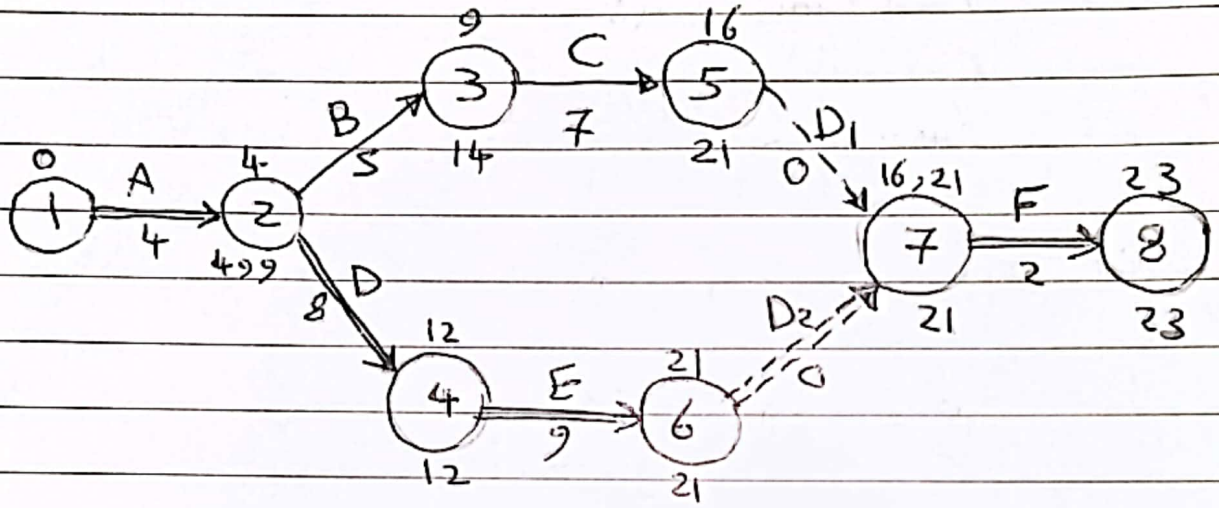
- CPM by circles and arrows ◦ 15

- For the same network in the previous example

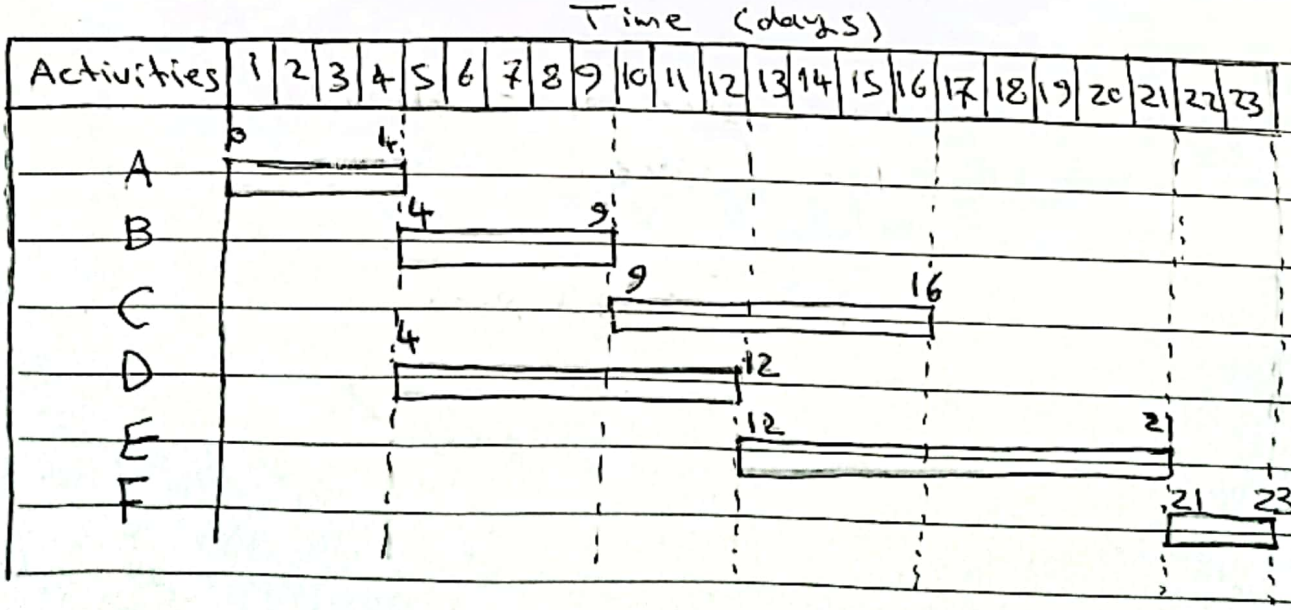


- Critical Path : A D E F
  Duration = 23 days

- Activities : D, D_2 : Dummy

- Bar Chart ◦ 16

- For the project above :



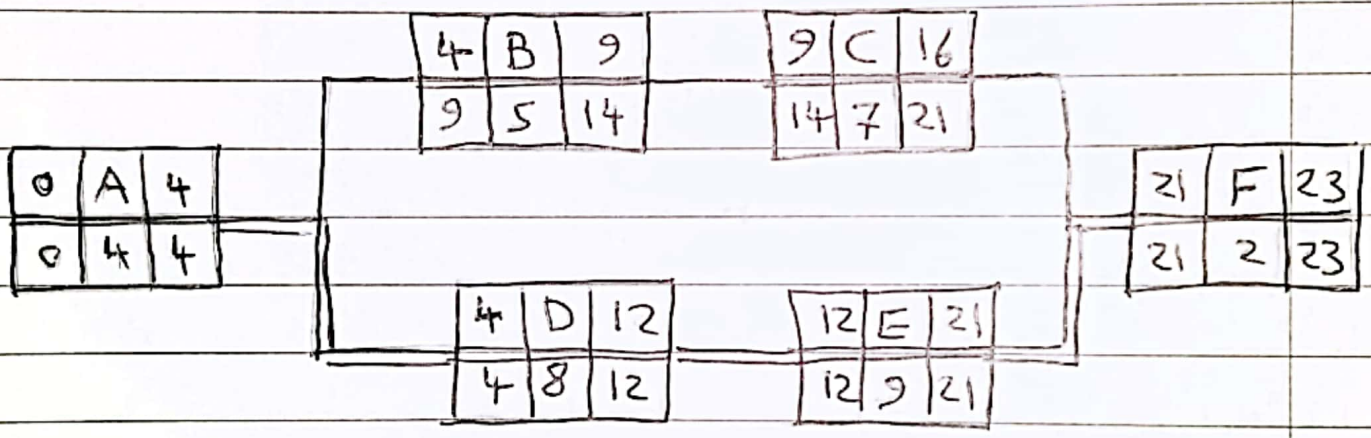
• Critical Path Method • 14
CPM

• Precedence method (rectangles)

as example for Activity A
4 = duration (days)

ES	A	EF
0		4
0	4	4
LS		LF

ES = Early Start
EF = Early Finish
LS = Late Start
LF = Late Finish



• Critical Path : ADEF
Project Duration : 23 days

Activity	Duration (days)	Precedence
A	4	—
B	5	A
C	7	B
D	8	A
E	9	D
F	2	C, E

• Index & Measure • 22

- Factors affecting the project
  Weight for each factor. ($\sum$ = 100%)
  Ranking each alternative (0 → 10 ... from 10)
  Alternative mark = (weight × Ranking value)
  Take the largest value.

- Example: Locations for a mill

Factors	wt.	A	B	C
Materials	40%	9 ⇒ 3.60	7 ⇒ 2.80	5 ⇒ 2.00
Labors	15%	7 ⇒ 1.05	8 ⇒ 1.20	3 ⇒ 0.45
Market	20%	2 ⇒ 0.40	5 ⇒ 1.00	9 ⇒ 1.80
Energy	20%	1 ⇒ 0.80	4 ⇒ 0.80	7 ⇒ 1.40
Weather	5%	8 ⇒ 0.40	6 ⇒ 0.30	3 ⇒ 0.15
		5.65	6.10	5.80

Decision: Choose B

Ordering: B — C — A

The best is B
If not available then C
If not then A

• Graphical Method LP • 2S

الطريقة الخطية بيانياً

- Two variables متغيرات
- Maximize $Z = 4x_1 + 5x_2$ تعظيم الربح

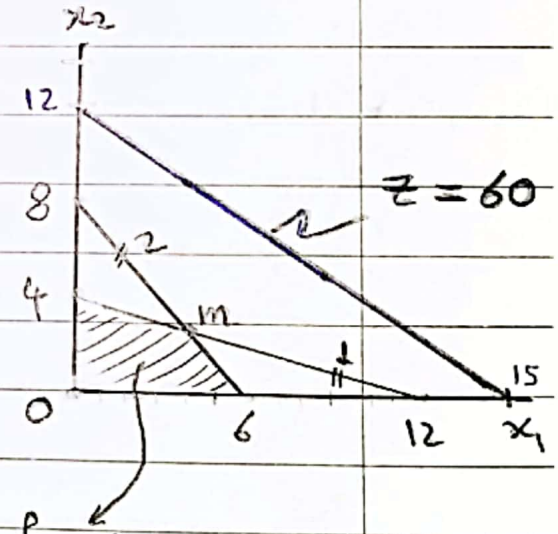
subject to

Mechine time $x_1 + 3x_2 \leq 12$ كلفة الوقت

Raw materials $4x_1 + 3x_1 \leq 24$ كلفة المواد الخام

$x_1, x_2 \geq 0$ شرط اللاسالبية

- Steps خطوات الحل
  - * plot constraints ارسم المتراجحات
  - * Shade feasible zone ظلل المنطقة الجدية
  - * Plot objective line (family) ارسم مجموعة من خطوط دالة الهدف
  - * Determine where Z just touches the feasible zone. حدد اين يمس خط دالة الهدف Z لاول مرة المنطقة الجدية ضد المنطقة (الثاني) في الحل.



- In this example :

Solution is 2 (M)

$x_1 + 3x_2 = 12$

$4x_1 + 3x_2 = 24$

$4x_1 + 12x_2 = 48$

$9x_2 = 24 \Rightarrow x_2 = \boxed{2\frac{2}{3}}$, $x_1 = \boxed{4}$

$Z_{max} = 4(4) + 5\left(\frac{8}{3}\right) = \boxed{\frac{88}{3}}$

• PERT • 27

طريقة بيرت

- Program Evaluation and Review Technique

- Probabilistic Method

Three (durations) for each activity :

Optimistic (a) متفائل

Most likely (m) الأكثر احتمالاً

Pessimistic (b) متشائم

te = expected time (duration) to be used in CPM

(te هو الزمن نستخدمه في الشبكة، نضعه CPM)

- Example : Calculate Probability of completing Project in 35 days

variance التباين ← σ^2

Activity	a	m	b (days)	Precedence	te	σ	σ^2
Critical ← (A)	2	4	8	—	4.33	1.00	1.00
(B)	4	7	12	A	7.33	1.33	1.77
C	4	7	9	B	6.83	0.83	
(D)	5	9	11	B	8.67	1.00	1.00
E	1	2	4	C	2.17	0.50	
(F)	2	3	5	D	3.17	0.50	0.25
(G)	2	4	5	E,F	3.83	0.50	0.25
(H)	2	3	5	G	3.17	0.50	0.25
							4.52 ← مجموع فقط للخط الحرج

TD = 915

- Using equations: $te = \frac{a+4m+b}{6}$ (B distribution)

توزيع بيتا

$$\sigma = \frac{b-a}{6}$$

$$\sigma_{project} = \sqrt{\sum \sigma^2}$$

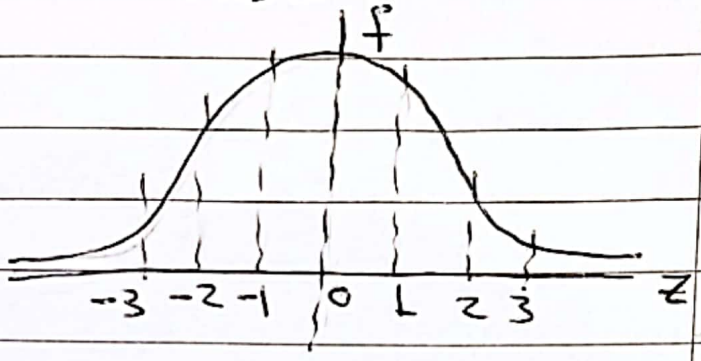
* $\sigma_{project} = \sqrt{\sum \sigma^2} = \sqrt{4.52} = \boxed{2.13}$

Normal distribution now to be used

$$Z = \frac{x - \mu}{\sigma_{project}}$$

$$= \frac{35 - 30.5}{2.13}$$

$$= 2.11$$

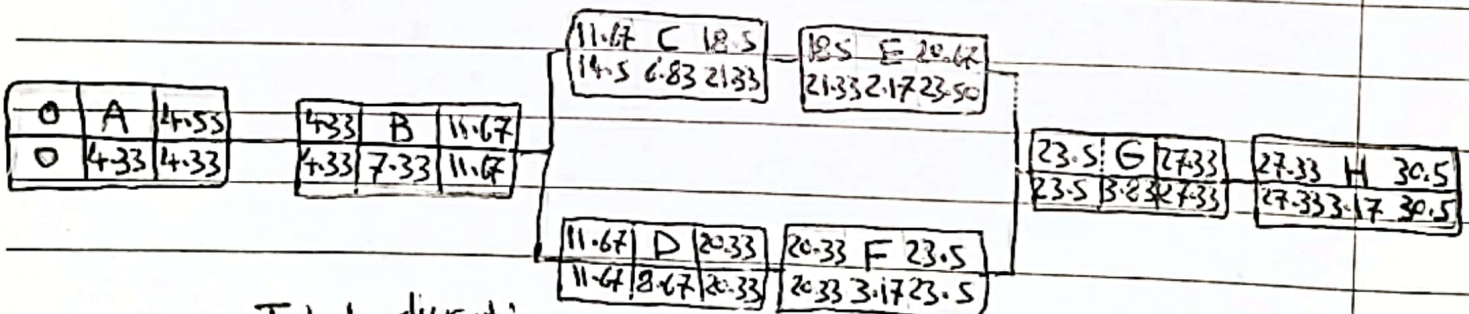


From table: $P = \boxed{98.26\ \%}$

* As another example:

for $P = 85\%$ from table $Z = 1.04$

$$\frac{x - 30.5}{2.13} = 1.04 \Rightarrow x = \boxed{32.17}\ \text{days}$$



Total duration = 30.5 days

CP: ABDFGH

That is the probability of 80% to complete the project is corresponding to $\boxed{32.17}$ day to be the project duration.

• Sensitivity Analysis • 32

تحليل الحساسية

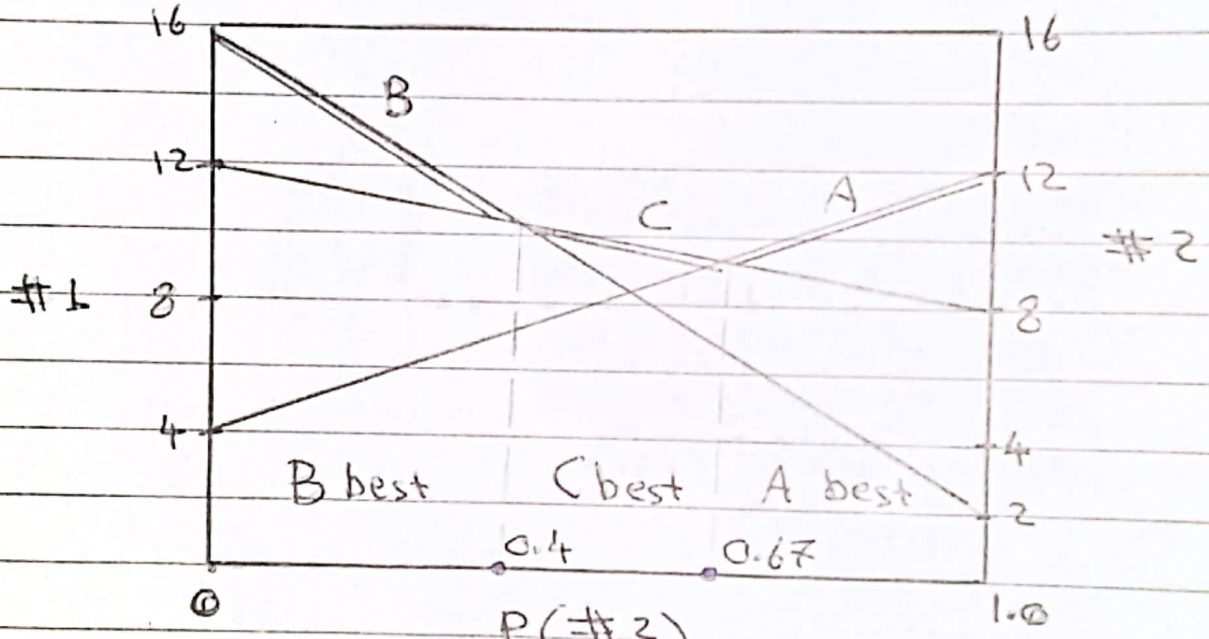
State

Alternatives	#1	#2
A	4	12
B	16	2
C	12	8

Revenue in $1000

#1 : High demand

#2 : Low demand



* For A to be the best alternative :

$$0.67 \leq P(\#2) \leq 1.0$$

For B to be the best :

$$0.0 \leq P(\#2) \leq 0.4$$

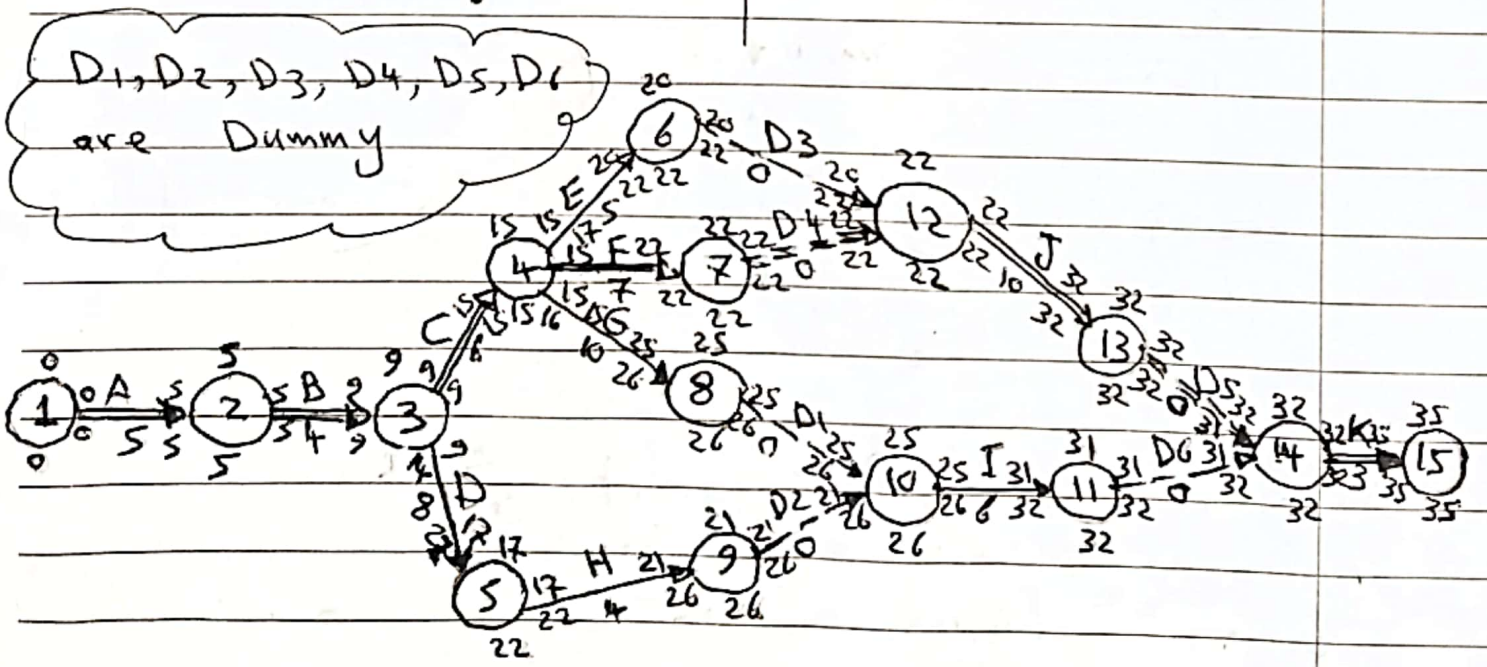
For C to be the best :

$$0.40 \leq P(\#2) \leq 0.67$$

• CPM Example • 43

مثال حول طريقة المسار الحرج

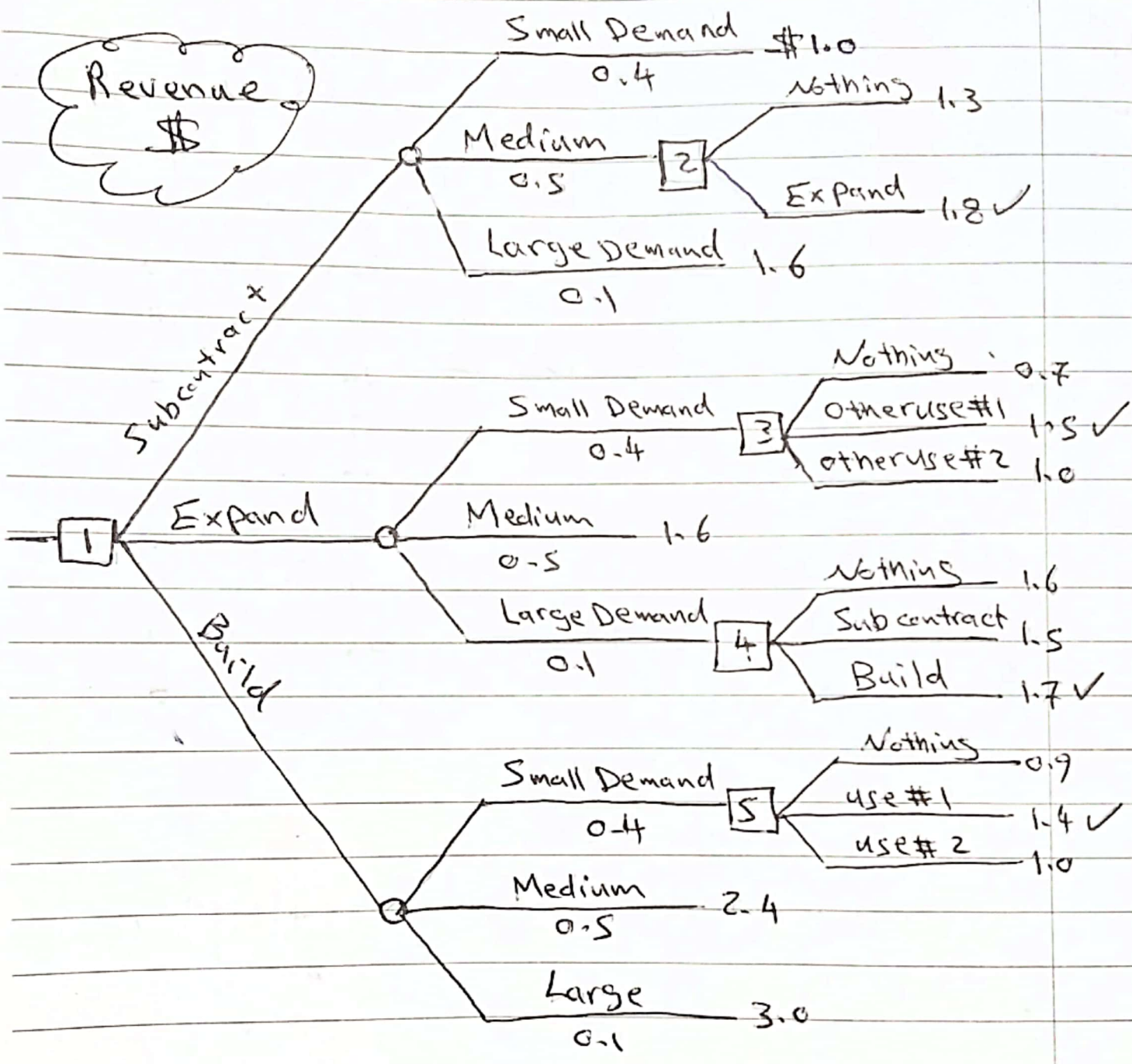
Activities	(days) D	Precedence	ES	LS	EF	LF	Float
A	5	—	0	0	5	5	0
B	4	A	5	5	9	9	0
C	6	B	9	9	15	15	0
D	8	B	9	14	17	22	5
E	5	C	15	17	20	22	2
F	7	C	15	15	22	22	0
G	10	C	15	16	25	26	1
H	4	D	17	22	21	26	5
I	6	G,H	25	26	31	32	1
J	10	E,F	22	22	32	32	0
K	3	I,J	32	32	35	35	0



* Critical Path : A-B-C-F-J-K
* Project Duration = 35 days

• Decision Tree Example • 44

مثال على شجرة القرارات



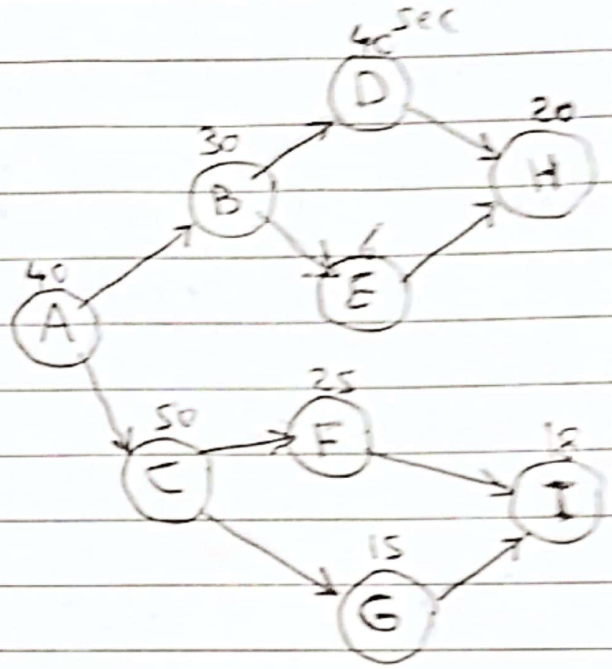
- Subcontract : $1.0(0.4) + 1.8(0.5) + 1.6(0.1) = \boxed{1.46}$
  Expand : $1.5(0.4) + 1.6(0.5) + 1.7(0.1) = \boxed{1.57}$
  Build : $1.4(0.4) + 2.4(0.5) + 3(0.1) = \boxed{2.06}$ ✓

∴ Choose **Build**

Balancing Line Example

مثال حول موازنة خط

Task	Time (sec)	Predecessors
A	40	—
B	30	A
C	50	A
D	40	B
E	6	B
F	25	C
G	15	C
H	20	D, E
I	18	F, G
	244	



- Given: 2400 units/wk
  40 hrs/wk

- $\frac{\text{Available time}}{\text{total time}} = \frac{\text{Available units}}{\text{minimum St.}}$

$$\frac{40 \times 3600}{244} = \frac{2400}{St.} \Rightarrow \text{min St.} = 4.07 \rightarrow \boxed{5.0}$$

- Cycle time $= \frac{40 \times 3600}{2400} = \boxed{60}$ sec

- Av. idle time $= \frac{56}{5} = \boxed{11.2}$

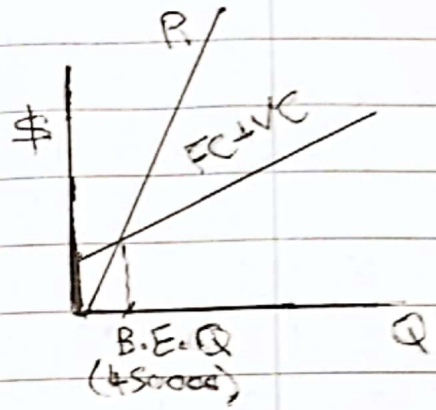
- % lost $= \frac{11.2}{60} \times 100\% = \boxed{18.7}\ \%$

- efficiency $= \boxed{81.3}\ \%$

St.	tasks	idle time (sec)
I	A	20
II	B & E	24
III	D, H	0
IV	C	10
V	F, G & I	2
		56

- Break-Even Q = 60
  نقطة التعادل

- A manager wants to add new machine
  Fixed Cost = 90000 $ / month
  Variable Cost = 0.6 $ / unit
  Expected Unit Price = 0.80 $/unit



- Now: $\overline{FC} + \overline{VC} = R \times Q$
  ثابت متغير

$\Rightarrow 90000 + 0.6Q = 0.8Q$

$\Rightarrow Q = 450\,000$ unit/month

- If Production: 600 000 units / month:

$$\text{Profit} = P = R \times Q - FC - VC \times Q$$
$$= 0.8(600000) - 90\,000 - 0.6(600000)$$
$$= \$\boxed{30\,000}$$

- What Q to make $P = 20\,000$ $?

$20\,000 = 0.8Q - 90000 - 0.6Q \Rightarrow$

$Q = \boxed{550\,000}$ units

- What Q to make Income : 50000 $?

$$\text{Income} = R \times Q = 50000 = 0.8Q$$

$\therefore Q = \boxed{62\,500}$

• Simplex Method • 65

طريقة سمبلكس

• Maximize : $Z = 80x_1 + 100x_2$

subject to : $x_1 + 2x_2 \leq 720$

$5x_1 + 4x_2 \leq 1800$

$3x_1 + x_2 \leq 900$

$x_1, x_2 \geq 0$

		x_1 ↓	x_2 ↓	S_1	S_2	S_3	M	
→	S_1	1	(2)	1	0	0	720	$\frac{720}{2} = 360$ ✓
	S_2	5	4 ✓	0	1	0	1800	$\frac{1800}{4} = 450$
	S_3	3	1 ✓	0	0	1	900	$\frac{900}{1} = 900$
	W	−80	(−100) ✓	0	0	0	0	
	x_2	$\frac{1}{2}$ ✓	1	$\frac{1}{2}$	0	0	360	$\frac{360}{\frac{1}{2}} = 720$
→→	S_2	(3)	0	−2	1	0	360	$\frac{360}{3} = 120$ ✓
	S_3	$\frac{5}{2}$ ✓	0	$\frac{-1}{2}$	0	1	540	$\frac{540}{\frac{5}{2}} = 216$
	W	(−30) ✓	0	50	0	0	36000	
	x_1	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	120	
	x_2	0	1	$\frac{5}{6}$	$\frac{1}{6}$	0	300	
	S_3	0	$-\frac{5}{2}$	$\frac{7}{6}$	$-\frac{5}{6}$	$-\frac{5}{2}$	240	
	W	0	0	30	10	0	39600	

∴ $x_1 = 120$, $x_2 = 300$, $x_3 = 0$, $S_3 = 240$, $Z_{max} = 39600$

$x_1 + 2x_2 \Rightarrow 720$ $(S_1 = 0)$ ✓

$5x_1 + 4x_2 \Rightarrow 1800$ $(S_2 = 0)$ ✓

$3x_1 + x_2 \Rightarrow 660 \Rightarrow S_3 = 900 - 660 = 240$ ✓

• Continuous Vs. Discrete Decay • 7.0

• Present amount = 100 $

• Present Value after 7 years with $i = 10\%$

(a) Continuous decay :-

$$PV = Ce^{-it}$$

$t = 0$, $PV = 100 \Rightarrow 100 = C \Rightarrow$

$$PV = 100e^{-0.1t}$$

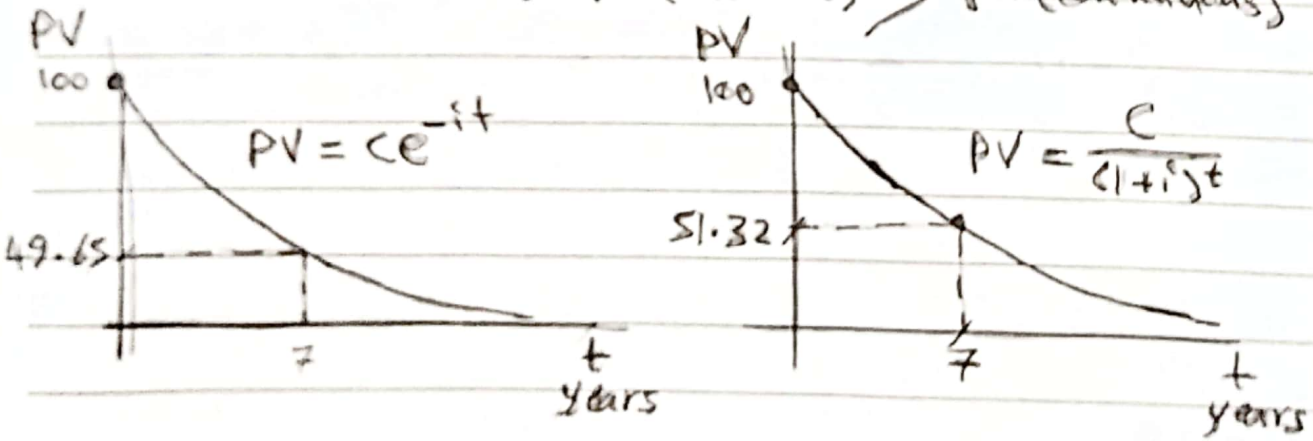
$$t = 7 \Rightarrow PV = 100e^{-0.1(7)} = \boxed{49.65}\ \$$$

(b) Discrete decay :-

$$PV = \frac{C}{(1+i)^t}$$

$$\therefore PV = \frac{100}{(1+0.1)^7} = \boxed{51.32}\ \$$$

• Note that : $PV(\text{discrete}) > PV(\text{continuous})$

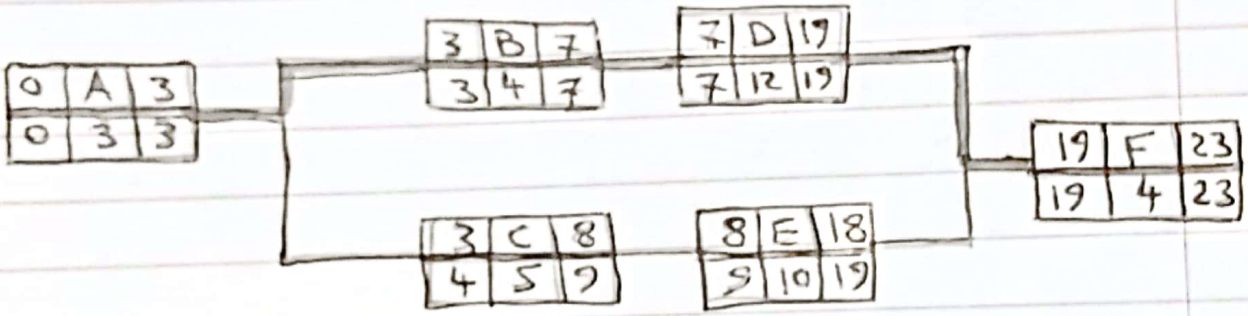


• PERT Example • مثال

• Find Probability of completing project in 25 months or less.

Activity	Durations a	m	b	Precedence	بالكل te	σ	σ²
A	1	3.25	4	—	3	0.5	0.25
B	3	3.50	7	A	4	0.67	0.44
C	3	5.25	6	A	5		
D	10	12.00	14	B	12	0.67	0.44
E	9	9.50	13	C	10		
F	3	4.00	5	D, E	4	0.33	0.11
							1.25

• $te = \frac{a + 4m + b}{6}$, $\sigma = \frac{b - a}{6}$



• $\sigma_{project} = \sqrt{\Sigma \sigma^2} = \sqrt{1.25} = 1.118$

• $Z = \frac{x - \mu}{\sigma} = \frac{25 - 23}{1.11} = 1.79$

∴ $P(x \leq 25) = 96.33\%$ من الجدول .