Fluid Mechanics

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Rate of Flow or Discharge

Rate of flow (or discharge) is defined as *the quantity of a liquid flowing per second through a section of pipe or channel*. It is generally denoted by Q.

Let us consider a liquid flowing through a pipe .

Let, A= Area of the pipe at section 1–1,

V= Average velocity of the liquid,

∴ Discharge, Q= Area x average velocity i.e., $Q = A.V$

If area is in m^2 and velocity is in m/s, then the discharge

$$Q = m^2 \times \frac{m}{s} = {}^{m^3}/_{S} = cumecs$$

Continuity Equation

The continuity equation is based on the principle of conservation of mass. It states as follows:-

"If no fluid is added or removed from the pipe in any length then the mass passing across different section shall be same ".

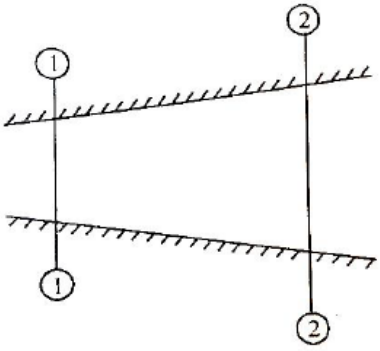
Consider two cross–section of a pipe as shown in the following Figure:-

Let , A_1 = Area of the pipe at section 1–1,

V_1 = velocity of the fluid at section 1–1,

ρ_1 = density of the fluid at section 1–1,

And, A_2, V_2, ρ_2 are corresponding values at sections 2–2.



.... total quantity of fluid passing through section 1–1= $\rho_1\ A_1\ V_1$ and, the total quantity of fluid passing through section 2–2= $\rho_2\ A_2\ V_2$

From the law of conservation of matter (theorem of continuity). It has:-

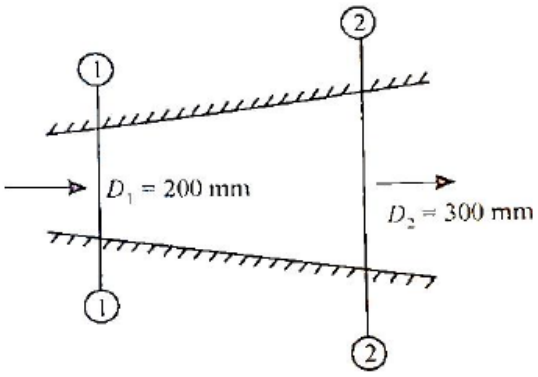
$$\rho_1\ A_1\ V_1 = \rho_2\ A_2\ V_2 \qquad \text{.....(21)}$$

Eq. (21) is applicable to the compressible as well as incompressible fluids and is called ***Continuity Equation***. In case of incompressible fluids, $\rho_1 = \rho_2$ and the continuity Eq. (21) reduces to $A_1V_1 = A_2V_2$ ..... (22)

Example **14/** The diameters of a pipe at the section 1-1and 2-2 are 200mm and 300mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4m/s, find:

(i) Discharge through the pipe, and

(ii) Velocity of water at section 2-2.



Solution: diameter of the pipe at section 1-1, D_1=200mm=0.2m

$\therefore Area \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} 0.2^2 = 0.0314\, m^2$

$Velocity\ ,\ V_1 = 4\, m/s$

Diameter of the pipe at section 2–2, $D_2 = 300mm$

$\therefore Area, A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} 0.3^2 = 0.0707 m^2$

i. **Discharge through the pipe, Q:**

Using the relation,

$Q = A_1 V_1 \ \rightarrow \ Q = 0.0314 \times 4 = 0.1256\, m^3/s$ .....Ans.

ii. **Velocity of water at section 2-2, V_2:**

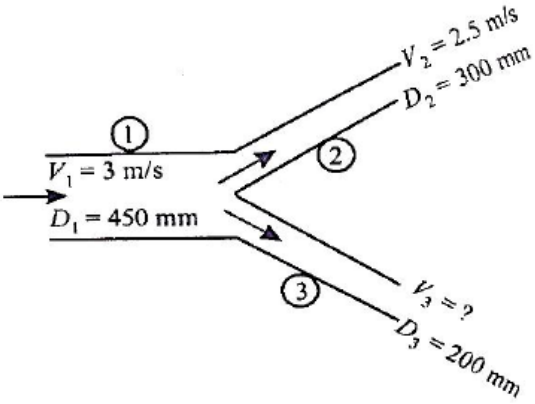
Using the relation,

$A_1 V_1 = A_2 V_2 \ \rightarrow \ V_2 = \frac{A_1 V_1}{A_2} = \frac{\mathbf{0.0314 \times 4}}{\mathbf{0.0707}} = 1.77$m/s ..... Ans.

Example **15/** A pipe (1) 450mm in diameter branches into two pipes (2 and 3) of diameters 300mm and 200mm respectively as shown in the following figure. If the average velocity in 450mm diameter pipe is 3m/s find:

i. Discharge through 450mm diameter pipe,

ii. Velocity in 200mm diameter pipe if the average velocity in 300mm pipe is 2.5m/s.



Solution:

Diameter D_1=450mm=0.45m

$\therefore Area \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} 0.45^2 =$

$0.159\ m^2$

$Velocity \quad V_1 = 3m/s$

Diameter $D_2 = 300mm = 0.3m$

$\therefore Area \quad A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} 0.3^2 =$

$0.0707\ m^2$

$Velocity \quad V_2 = 2.5m/s$

Diameter $D_3 = 200mm = 0.2m$

$\therefore Area \quad A_3 = \frac{\pi}{4} 0.2^2 = 0.0314\ m^2$

i. **Discharge through pipe (1) Q_1:**

Using the relation: $Q_1=A_1V_1=0.159\text{x}3=0.477 m^3/s$ .....Ans.

ii. **Velocity in pipe of diameter 200mm i.e. V_3:**

Let Q_1, Q_2 and Q3 be the discharge in pipes 1,2 and 3 respectively:

Then according to continuity equation:

$Q_1= Q_2+ Q_3 \rightarrow Q_1=0.447\ m^3/s$ **and** $Q_2= A_2V_2=0.0707\text{x}2.5=0.1767 m^3/s$

$\therefore\ 0.477=0.1767+Q_2$

or $Q_3=0.477–0.1767=0.3 m^3/s$

But $Q_3= A_3V_3$

$\therefore V_3 = \frac{Q_3}{A_3} = \frac{0.3}{0.0314} = 9.55m/s \rightarrow V_3 = 9.55m/s$ ..... Ans.