Al-Mustaqbal University College

Department of Medical Instrumentation Technologies

Mathematics II / Second Stage

By

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Differential Equations

First order differential equations of first degree:

***Separable Equations*:**
When M is a function of x alone and N is a function of y alone, then equation (1) has the form:

$$M(x)dx + N(y)dy = 0$$

This is the standard form of a first order separable equation, we can find its solution by direct integration of each term, giving us:

$$\int M(x)dx + \int N(y)dy = C$$

***Example*:** Solve the differential equation

$$\frac{dy}{dx} = \frac{2x}{y^2}$$

***Solution*:**

$$y^2 dy = 2x dx$$

$$\int y^2 dy = \int 2x dx$$

$$\frac{y^3}{3} = \frac{2x^2}{2} + C \Longrightarrow \frac{y^3}{3} = x^2 + C$$

$$y^3 = 3(x^2 + C) \Longrightarrow y^3 = (3x^2 + 3C)$$

$$y = (3x^2 + 3C)^{1/3}$$

***Example*:** Solve the differential equation

$$\frac{dy}{dx} = (1 + y^2)x^2$$

***Solution*:**

$$\frac{dy}{1 + y^2} = x^2 dx$$

$$\int \frac{dy}{1 + y^2} = \int x^2 dx$$

$$\tan^{-1} y = \frac{x^3}{3} + C$$

Homogeneous Equations:

Occasionally a differential equation whose variables cannot be separated can be transformed by a change of variable into an equation whose variable can be separated. This is the case with any equation that can be put into the form:

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right) \qquad \text{........(3)}$$

Such an equation is called **homogeneous,** to transfer equation (3) into an equation whose variables may be separated, we introduce the new variable

$$v = \frac{y}{x} \qquad \ldots\ldots..(4)$$

then

$$y = vx \quad , \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Equation (5) can be solved by separation of variables:

$$\frac{dx}{x} + \frac{dv}{v - F(v)} = 0$$

***Example*:** Show that the equation

$$(x^2 + y^2)dx + 2xydy = 0$$

is homogeneous and solve it

***Solution*:** from the given equation, we have:

$$(x^2 + y^2) + 2xy\frac{dy}{dx} = 0 \Longrightarrow \frac{(x^2 + y^2)}{2xy} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{(x^2 + y^2)}{2xy}$$

$$\frac{dy}{dx} = -\frac{\left(\frac{x^2}{x^2} + \frac{y^2}{x^2}\right)}{\frac{2xy}{x^2}} \Longrightarrow \frac{dy}{dx} = -\frac{\left(1 + \left(\frac{y}{x}\right)^2\right)}{2\left(\frac{y}{x}\right)}$$

$$\frac{dx}{x} + \frac{dv}{v + \frac{1+v^2}{2v}} = 0$$

This has the form equation (3), with

$$F(v) = -\frac{1+v^2}{2v} \quad , \text{ where } v = \frac{y}{x}$$

Then equation (6) becomes:

$$\frac{dx}{x}+\frac{2vdv}{1+3v^2}=0 \quad \Rightarrow \quad \frac{dx}{x}+\frac{2vdv}{1+3v^2}=0$$

The solution of this equation can be written as:

$$\ln|x|+\frac{1}{3}\ln(1+3v^2)=C$$

$$\ln|x|+\frac{1}{3}\ln\left(1+3\left(\frac{y}{x}\right)^2\right)=C$$