H.w 3

1-

If $y = \sin^{-1}\left(\frac{x-1}{x+1}\right)$ *find* $\frac{dy}{dx}$

Sol:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}} \cdot \frac{(x+1).1-(x-1).1}{(x+1)^2} = \frac{x+1-x+1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}\,(x+1)^2} = \frac{2}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}\,(x+1)^2}$$

$$y = \frac{\sqrt{x^2-4}}{x^2} + \frac{1}{2}\sec^{-1}\left(\frac{1}{2}x\right)$$

2) $\frac{dy}{dx} =$

مشتقة دالة

$$\frac{x^2 \cdot \frac{1}{2}(x^2-4)^{-\frac{1}{2}} \cdot 2x - \sqrt{x^2-4}\cdot 2x}{(x^2)^2} + \frac{1}{2}\left[\frac{1}{\left(\frac{1}{2}x\right)\sqrt{\left(\frac{1}{2}x\right)^2-1}} \cdot \frac{1}{2}\right]$$

مشتقة كسرية

مقام . مشتقة بسط − بسط . مشتقة مقام
(مقام)²

3) $y = \sin^{-1}\sqrt{2}\,t$

مشتقة $\sqrt{2}\,t = \sqrt{2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{2}\,t)^2}} \cdot \frac{\sqrt{2}}{1}$$

4) $y = \cos^{-1} x^2$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x \Rightarrow \frac{-2x}{\sqrt{1-x^4}}$$

h.w 4

1- $y = x^{\cos x}$

Sol: Take ln for both sides

$$y = x^{\cos x} \quad \rightarrow \ln y = lnx^{\cos x}$$

$$\ln y = \cos x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{x} - \sin x \ln x$$

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x \ln x\right] = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \ln x\right]$$

2- Solve for x if $2^x = 4^{x-1}$

Sol:

Take ln for both sides

$$\ln(2^x) = \ln(4^{x-1})$$

$$x \ln 2 = (x-1)\ln 4 = x \ln 4 - \ln 4$$

$$x\, ln\, 2 - x \ln 4 = -\ln 4$$

$$x\,(\ln 2 - \ln 4) = -\ln 4 \quad \rightarrow \quad \therefore x = \frac{-\ln 4}{\ln 2 - \ln 4}$$

3-

$3.\ e^{\sec x} \cdot \sec e^{x}$

$e^{\sec x} \cdot \sec e^{x} \cdot \tan e^{x} + \sec e^{x} \cdot e^{\sec x} \cdot \sec x \tan x$

مشتقة الاولى . دالة الثانية + مشتقة الثانية . دالة الاولى

الناتج النهائي

$e^{\sec x} \cdot \sec e^{x} \cdot \tan e^{x} + \sec e^{x} \cdot e^{\sec x} \cdot \sec x \cdot \tan x$

4-

$y = (x+1)^x = e^{x\ln(x+1)} \Longrightarrow y' = e^{x\ln(x+1)} \left(x * \frac{1}{x+1} + \ln(x+1) \right)$

$= (x+1)^x \left(\frac{x}{x+1} + \ln(x+1) \right.$