

H.w 3

1-

If $y = \sin^{-1}\left(\frac{x-1}{x+1}\right)$ find $\frac{dy}{dx}$

Sol:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}} \cdot \frac{(x+1) \cdot 1 - (x-1) \cdot 1}{(x+1)^2} = \frac{x+1-x+1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2} (x+1)^2} = \frac{2}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2} (x+1)^2}$$

Handwritten solutions for three problems:

2) $y = \frac{\sqrt{x^2-4}}{x^2} + \frac{1}{2} \sec^{-1}\left(\frac{1}{2}x\right)$

$\frac{dy}{dx} = \frac{x^2 \cdot \frac{1}{2}(x^2-4) \cdot 2x - \sqrt{x^2-4} \cdot 2x}{(x^2)^2} + \frac{1}{2} \left[\frac{1}{\left(\frac{1}{2}x\right) \sqrt{\left(\frac{1}{2}x\right)^2 - 1}} \cdot \frac{1}{2} \right]$

Handwritten notes: "plus minus minus plus" and "plus" are written below the derivative formula.

3) $y = \sin^{-1} \sqrt{2} t$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \frac{\sqrt{2}}{1}$

Handwritten notes: "sqrt 2 t" and "sqrt 2" are written next to the derivative.

4) $y = \cos^{-1} x^2$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x \Rightarrow \frac{-2x}{\sqrt{1-x^4}}$

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$$y = (x+1)^x = e^{x \ln(x+1)} \Rightarrow y' = e^{x \ln(x+1)} \left(x \cdot \frac{1}{x+1} + \ln(x+1) \right)$$
$$= (x+1)^x \left(\frac{x}{x+1} + \ln(x+1) \right)$$